Influence of small magnetic impurity on the conductivity of a quasi-one-dimensional metal

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It is shown with the aid of the similarity hypothesis that the conductivity of a quasi-one-dimensional metal with a small admixture of magnetic atoms should depend on the concentration like $\sqrt{c}$. The behavior in a magnetic field is also determined. These predictions can be used to verify the similarity hypothesis on which calculations of a large number of quasi-one-dimensional models are based.

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The conductivity of a semimetal in a strong magnetic field was calculated in [1]. The finite conductivity results formally from the fact that the random fields of the impurities must be regarded as operators, and these operators do not commute at different points: $[\zeta_i, \zeta_k] \neq 0$. Although in the case of a long-range Coulomb interaction this commutator is indeed relatively small, expansion in its terms is impossible, since the conductivity is not analytic. To obtain a final formula, a sample of finite dimensions was therefore considered and similarity theory was used.

An experimental check on the derived formula is hindered by the fact that in real semimetals there are always neutral scattering centers for which, as shown in [1], the commutator is not small, and the result is appreciably altered.

There is, however, another method of verifying the main premise of this theory, namely the similarity hypothesis. We consider a quasi-one-dimensional metal with relatively large impurity concentration (or internal disorder). As shown in [2], the longitudinal conductivity of such a material will be connected with the low probability of hopping from one filament to the other, and will therefore be small. We now introduce into this metal magnetic impurities. The interaction of the conduction electrons with these impurities is described by the Hamiltonian

$$U_m = -J\sum_i (\mathbf{s}\mathbf{S}_i), \qquad (1)$$

where S_i are the spins of the impurities and s is the electron spin.

It is easy to see that the potentials in different points do not commute with one another, namely,

$$\tfrac{1}{2}\mathrm{Sp}_s[(\mathbf{s}\mathbf{S}_i)(\mathbf{s}\mathbf{S}_k)(\mathbf{s}\mathbf{S}_i)(\mathbf{s}\mathbf{S}_k) - (\mathbf{s}\mathbf{S}_i)(\mathbf{s}\mathbf{S}_i)(\mathbf{s}\mathbf{S}_k)(\mathbf{s}\mathbf{S}_k)] = -\tfrac{4}{3}[S(S+1)]^2. \qquad (2)$$

(It is assumed here that the spins S_i are free and the temperature is higher than the Kondo temperature.)

If the concentration of the magnetic impurity is such that $l_m \gg l_2$, where

$$l_m^{-1} = N_m\left(\int |J(k_z = 2p_0,\ \mathbf{k}_\perp)|^2 d^2\mathbf{k}_\perp/(2\pi)^2\right)S(S+1)/v^2, \qquad (3)$$

then we return to the situation with the magnetic field. By applying directly formula (45) of [1] to a sample of length L we have (taking into account to spin projections)

$$\sigma = \frac{e^2}{\pi S}\left[\frac{\pi^{5/2}}{2}\, e^{-L/4l_2} L^{-1/2} l_2^{3/2} + \frac{5\pi^3}{192}\frac{l_2^2}{l_m^2} e^{3L/4l_2} L\right], \qquad (4)$$

where S is the area of cell cross section perpendicular to the filaments.

Reasoning further as in [1], we assume that the sample can be regarded as "infinite" starting with a length at which both terms of (4) become of the same order, i.e., starting with

$$L \approx 2l_2 \ln\frac{l_m}{l_2}\ . \qquad (5)$$

The conductivity of such a sample is of the order of

$$\sigma \sim \frac{e^2}{\pi S}\, l_2^{3/2}\, l_m^{-1/2}\ . \qquad (6)$$

We obtain thus a finite conductivity proportional to $\sqrt{N_m}$. It is easiest to verify the similarity hypothesis by means of this concentration dependence.

As already noted, the result is valid in the case when the spins S_i are free. Let us see now what happens when a magnetic field is applied or when the spins become ordered. When a strong field $\mu H \gg T$ is applied, where μ is the magnetic moment of the impurity (or else in the case of ferromagnetic ordering and $\Theta \gg T$), the operators of the potentials take the form $s_z S$, i.e., they begin to commute, and consequently the effect vanishes.

The law governing the decrease of the conductivity cannot be found within the framework of the employed scheme, inasmuch as at $\mu H \sim T$ the scattering with electron spin flip becomes inelastic (at $T \gg \mu H$ such a scattering can be regarded as elastic, and at $T \ll \mu H$ it is suppressed). The only thing that can be stated on the basis of the general considerations is that expression (2) is replaced by

$$-\frac{4}{3}[S(S+1)]^2 f(S, \mu H/T), \tag{7}$$

where the function f has the asymptotic behavior

$$\begin{aligned} f &\approx 1 - a(S)(\mu H/T)^2, & \mu H &<< T \\ f &\approx b(S)\exp(-\mu H/T), & \mu H &>> T \end{aligned} \tag{8}$$

where $a(S) \sim 1$ and $b(S) \sim 1$ if $S \sim 1$. Since $\sigma \sim \sqrt{f}$, we can expect to have at low temperatures (but above the ordering temperature, i.e.,

$$\Theta << T << \mu H) \; \sigma \approx \sigma_0 \sqrt{b(S)}\exp(-\mu H/2T),$$

and at high temperatures $\sigma \approx \sigma_0[1 - \frac{1}{2}a(S)(\mu H/T)^2]$, where σ_0 is the value at $H = 0$.

We do not consider ferromagnetic ordering, since the most probable type of the low-temperature state will be "spin glass." For our purposes we can regard it as an assembly of fixed classical spins of magnitude $S_1 \leq S$ with random directions. Then formula (2) is replaced by

$$-2S_1^4 \overline{\sin^2\Theta} \approx -\frac{4}{3}S_1^4 .$$

It follows therefore that when "spin glass" is produced the conductivity decreases (even if $S_1 = S$), but does not vanish.

[1]A. A. Abrikosov and I. A. Ryzhkin, Fiz. Tverd. Tela, in press.
[2]A. A. Abrikosov and I. A. Ryzhkin, Zh. Eksp. Teor. Fiz. **72**, 225 (1977) [Sov. Phys. JETP **45**, No. 1 (1977)].