Bound states of a large number of magnons in a three-dimensional ferromagnet (magnons drops)

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We consider the bound states of N magnons in a ferromagnet with easy-axis anisotropy, assuming that the energy of the magnetic anisotropy is small in comparison with the volume energy. It is shown that there always exist bound states with $N \geq N_* \gg 1$, where N_* is determined by the ratio of the exchange constant to the anisotropy constant. The investigated states can be treated as "magnon drops."

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It is known that, in contrast to the one-dimensional case[1–3], in a three-dimensional crystal bounds states of two quasiparticles are produced only when the amplitude of their attraction exceeds a certain critical value. Using

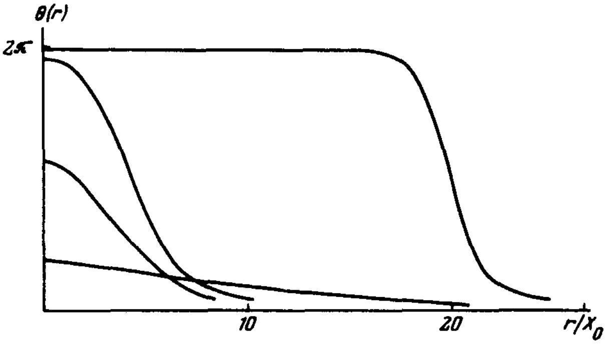

FIG. 1. Distribution of the magnetization $\theta(r)$: $a - \omega = 0.1\omega_0$; b$-\ \omega = 0.5\ \omega_0$; c$-\ \omega = \omega_* \approx 0.915\ \omega_0$; d$-\ 0.99\ \omega_0$.

magnons in a ferromagnet as an example, we shall show that the condition for the existence of a bound state of a large number N of Bose particles with a zero total quasimomentum assumes a different form and becomes less stringent.

In an investigation of states with a large number of spin deflections, we use a classical description in terms of the macroscopic density $M(r, t)$ of the magnetic moment. This approach, for a Bose system at $N \gg 1$, leads to the same results as the quantum-mechanical analysis. The number N of magnons can be naturally defined in such an approach as the number of spin deflections in the system

$$N\{\mathbf{M}(\mathbf{r}, t)\} = \frac{1}{2\mu_0} \int \{M_0 - M_z(\mathbf{r}, t)\} d\mathbf{r}, \tag{1}$$

where μ_0 is the Bohr magneton and M_0 is the saturation magnetization.

We write the energy of the ferromagnet in the form

$$W\{\mathbf{M}(\mathbf{r}, t)\} = \frac{1}{2} \int \left\{ a\left(\frac{\partial \mathbf{M}}{\partial x_i}\right)^2 - \beta M_z^2 \right\} d\mathbf{r}, \tag{2}$$

where β is the anisotropy constant[1)] $(\beta > 0)$, $a = Ia^2/2\mu_0 M_0$, is the exchange constant, I is the exchange integral of the ferromagnet, and a is the lattice constant. We assume that $\mu_0 \beta M_0 \ll I$.

The magnetization distribution corresponding to the bound state of N magnons realizes the minimum of the energy function $W\{\mathbf{M}\}$ at a given integer value $N\{\mathbf{M}\} = N = \text{const}$ and under the condition $M_x^2 + M_y^2 + M_z^2 = M_0^2 = \text{const}$.

The components of the vector M are conveniently written in the form

$$M_z = M_0 \cos\theta, \qquad M_x + iM_y = M_0 \sin\theta \exp\{i\phi\}. \tag{3}$$

It can be shown that the extremum of interest to us is realized at $\partial\phi/\partial x_i = 0$.

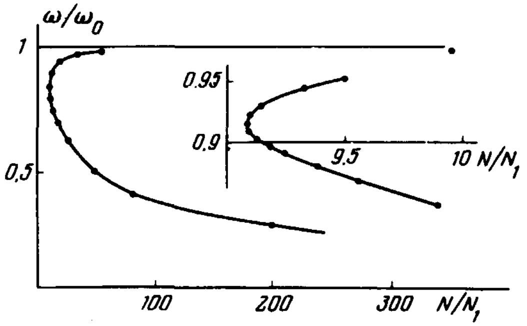

FIG. 2. Dependence of the quasiparticle energy $\hbar\omega(N)$ on the number of magnons in the bound state. The points designate the results of a numerical calculation.

In addition, one should expect the lowest energy to be possessed by the centrally symmetrical solution $\theta = \theta(r)$. The equation for the function $\theta(r)$ is

$$x_0^2\left(\frac{d^2\theta}{dr^2} + \frac{2}{r}\frac{d\theta}{dr}\right) - \sin\theta\cos\theta + \frac{\omega}{\omega_0}\sin\theta = 0, \qquad (4)$$

where $\omega = \omega(N)$ has the meaning of the Lagrange multiplier for the corresponding extremal problem, $x_0^2 = \alpha/\beta$, and $\omega_0 = 2\mu_0\beta M_0$.

Using the Landau-Lifshitz equation for the magnetization, we can verify that the solutions of (4) correspond in the classical limit to circular precession of the magnetization, with frequency $\omega(N)$ and with a coordinate-dependent amplitude. To determine the quantum-mechanical meaning of the quantity ω, we obtain the relation

$$\frac{dE(N)}{dN} = \hbar\omega(N), \qquad (5)$$

where $E(N)$ is the energy of the bound state of N magnons. Thus, when the number of magnons is increased by unity, the energy of the bound state increases by $\hbar\omega(N)$. Consequently, $\hbar\omega(N)$ is the minimal energy of a quasiparticle in a ferromagnet containing a bound state of a large number of magnons.

By analyzing the phase plane for Eq. (3), we can show that at finite N it has solutions at $0 < \omega < \omega_0$. At $\omega \ll \omega_0$, it is easy to predict the qualitative form of the plot (see Fig. 1a) and obtain the following asymptotic expressions: $\cos\theta(r) = \tanh[(r - 2x_0\omega_0/\omega)/x_0]$, $\omega(N) = 2\omega_0(2N_1/3N)^{1/3}$, and

$$E(N) = 2\epsilon_0 N_1^{1/3}(3N/2)^{2/3}, \qquad (6)$$

where $\epsilon_0 = \hbar\omega_0$ is the energy of a free magnon with $\mathbf{k} = 0$, and $N_1 = 4\pi s(I/2\mu_0\beta M_0)^{3/2}$.

The results of the numerical analysis of the values of E and ω are shown in Figs. 2 and 3. It turned out that the functions $E(N)$ and $\omega(N)$ are doubly-valued functions of N, i.e., there exist two branches of bound states. A plot of the function $\omega(N)$ has a vertical tangent at $N = N_* \approx 9.08N_1$, and $\omega = \omega_* = 0.915\omega_0$.

It is easy to verify (see (5)), that at $N \gtrsim N_*$ we have $E(N) = E(N_*) + \hbar\omega_*(N - N_*) \pm A(N - N_*)^{3/2}$, and $E(N_*) \approx 1.034N_*$, where A is a constant. It turns out that

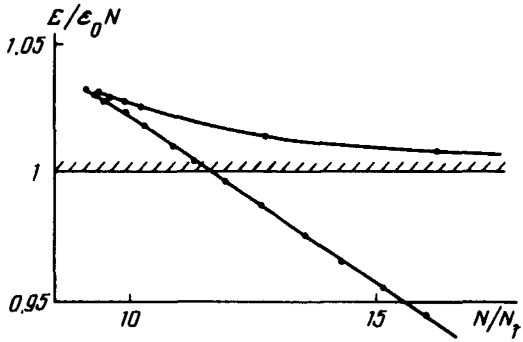

FIG. 3. Dependence of the energy per magnon of the bound state of N magnons on the number of magnons.

the bound states of N magnons pertaining to both branches are stable against small perturbations. It is clear that the states to arbitrary perturbations will be those for which $E(N)/N < \epsilon_0$, i.e., the transition to states of the continuous spectrum is forbidden (we recall that the Hamiltonian (2) commutes with the projection z of the total spin, i.e., it conserves the number of magnons). Thus, all the states of the upper branch of $E(N)$ and the states of the lower branch at $N < N_0$, $N_0 \approx 11.3N_1$, are metastable.

We have arrived at the conclusion that even if no bound states of two magnons can be produced in a ferromagnet, bound states of N magnons with $N \geq N_*$ can be produced. One can apparently make the following general statement: if the attraction potential U of the bosons is insufficient for the production of bound pairs ($U < U_c$), then bound states of N bosons can be produced at $N \geq N_c$, with $N_c \propto (U_c/U)^{3/2}$.

Returning to the interpretation of the macroscopic meaning of the considered states, we note the following. First, whereas an aggregate of "free" magnons can be regarded as a gas of quasiparticles with weak attraction (the energy of the easy-axis magnetic anisotropy corresponds to paired attraction of long-wave magnons), the bound state of a large number of magnons is a "magnon drop." This drop is in fact the embryo of a region with opposite direction of the magnetization in an infinite single-domain ferromagnet.[2)] It is clear that such a state of the magnet can exist only under conditions of external action that ensures a given number of spin deflections N. Second, for the existence of a magnon drop it is necessary that the average lifetime of the magnons exceed the time of their "condensation" into a drop, and the spreading out of the drop as a result of relaxation mechanisms in the magnet be offset by excitation of the magnons by an external action.

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[1)]The neglect of the magnetic dipole interaction in the expression (2) for the energy is formally justified only if $\beta \gg 4\pi$.

[2)]A similar distribution of the magnetization recalls the "spherical domain." It must be borne in mind only that, in contrast to the usual theory of cylindrical domains in thin films, in our analysis we have neglected the magnetic dipole interaction.

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