Concerning the difference between the normal and anomalous skin effects

K. Saermark

Physics Laboratory of the Technical University (Denmark)
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A criterion is discussed for the difference between the normal and anomalous skin effects. The significance of the critical frequency $(\omega/\omega_p)_{cr} = (v_F/c)$ is emphasized.

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It was indicated in[1] that the difference between the normal and anomalous skin effects can be formulated with the aid of the inequality $|8k\nu^4| \gtrless 1$, where the upper and lower signs pertain to the normal and anomalous skin effects, respectively.

Denoting by $h = \omega_c/\omega$ the ratio of the cyclotron frequency to the signal frequency, we have $\nu = (1 + i/\omega\tau)/h$ and $8k = (c\omega/v_F\omega_p)^2 h^4$, where τ is the relaxation time, v_F is the Fermi velocity, and ω_p is the plasma frequency.

We consider only weak fields $|\nu| \gg 1$. This condition is obtained in the following manner. Let the metal have a cylindrical Fermi surface, the axis of which is parallel to the surface of the sample and to the applied magnetic field. Using the notation of Kaner and Skobov,[2] the surface impedance can be expressed in terms of the quantity $T(0)$ (see[1,2]), which can be approximately calculated in terms of the magnetic-conductivity tensor given in[3]. For $|8k\nu^4| > 1$, $|\nu| \gg 1$ and $\omega\tau \ll 1$ we obtain the continuous expression $Z = (2\pi\omega\delta/c^2)(1 - i)$. This case thus corresponds to the normal skin effect, whereas for the case $|8k\nu^4| < 1$, generally speaking, the Reuter-Sondheimer expression is not obtained. The present article deals with the condition $|8k\nu^4| \gtrless 1$ in greater detail.

Generally speaking, $8k\nu^4$ is a complex quantity

$$8k\nu^4 = |8k\nu^4|\, e^{i\theta}, \qquad \theta = 3\phi; \quad \mathrm{tg}\,\phi = 1/\omega\tau, \tag{1}$$

with the dependence of the argument θ on the value of $\omega\tau$ illustrated in Table 1. Only at $\omega\tau = 1/\sqrt{3}$ does (1) become real.

TABLE I. Argument θ of the quantity $(8k\nu^4)$ for different values of $\omega\tau$.

$\omega\tau$	θ
$]0;\ 3\sqrt{3}[$	$]3\pi/2;\ \pi[$
$1/\sqrt{3}$	π
$]1/\sqrt{3};\ \sqrt{3}[$	$]\pi;\ \pi/2[$
$\sqrt{3}$	$\pi/2$
$]\sqrt{3};\ \infty[$	$]\pi/2;\ 0[$

The condition $|8k\nu^4| = 1$ can be written in the form

$$\left[1+\left(\frac{1}{\omega\tau}\right)^2\right]^{3/2}\left(\frac{c\,\omega}{v_F\,\omega_p}\right)^2 = 1 \tag{2}$$

or, introducing the relative velocity

$$v_r = (\omega/\omega_p)\,c \tag{3}$$

in the form

$$\omega\tau = [(v_F/v_r)^{4/3} - 1]^{-1/2} \tag{4}$$

If $v_r \to v_F$ from below, it follows from (4) that $\omega\tau \to \infty$, and for frequencies above critical $(\omega/\omega_p)_{cr} = (v_F/c)$ it is necessary to use in the inequality $|8k\nu^4| \gtrless 1$ the upper sign for any (positive) value of $\omega\tau$. It follows also from (4) that the plot of $\omega\tau$ against (ω/ω_p) has a vertical tangent as $(\omega/\omega_p) \to 0$. This is shown sche-

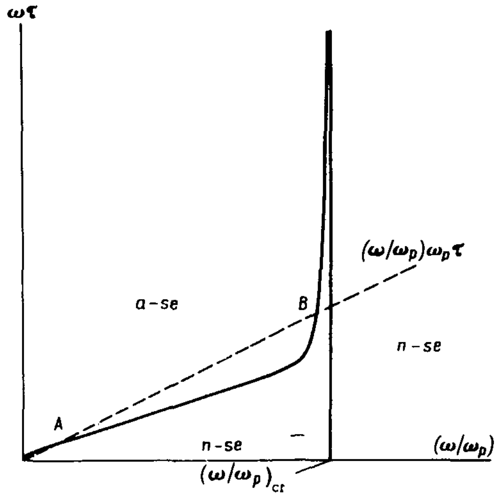

FIG. 1. Arrangement of the regions of the normal skin effect $n-se$ and the anomalous skin effect $a-se$. The solid curve is the plot of against (ω/ω_p) and is the boundary between the regions $n-$se and $a-se$; it has a vertical tangent at the origin.

matically by the solid curve in Fig. 1; the region $n-se$ corresponds to the inequality $|8k\nu^4|>1$, and the region $a-se$ to the inequality $|8k\nu^4|<1$.

For a given sample, i.e., for a fixed value of τ, the straight line with the slope $\omega_p\tau$ in the figure gives the value of $\omega\tau$ as a function of the frequency. It is seen that at very low frequencies the normal skin effect takes place in the plasma, the anomalous skin effect sets in at the point A, and at the point B the skin effect again becomes normal. We note that the plot in the figure is drawn excessively not to scale, since the change in the frequency is by many orders of magnitude.

Assuming that the solid curve in the figure is the boundary between the regions of the normal and anomalous skin effects, it is of interest to calculate the surface impedance under the condition $|8k\nu^4|=1$. This can be done by the method indicated in[1]. As already mentioned, in the discussion of (1), the quantity $8k\nu^4$ is real and is equal to -1 only if $\omega\tau=1/\sqrt{3}$. For this particular case we obtain the surface impedance

$$Z = 1.3(8/9)(\sqrt{3}\pi\omega^2 l/c^4\sigma)^{1/3}(1-i\sqrt{3}) = 1.3A_{RS}(1-i\sqrt{3}) = 1.3Z_{RS}, \qquad (5)$$

where A_{RS} is the amplitude of the Reuter-Sondheimer expression for the surface impedance Z_{RS}.[4] We note that the expression for Z_{RS} was derived for a spherical Fermi surface, whereas we are considering a cylindrical Fermi surface. In the same way we obtain for any value of θ in formula (1), under the condition $|8k\nu^4|=1$,

$$Z = C[1+(1/\omega\tau)^2]^{-1/2} e^{i\theta} A_{RS}(1-i/\omega\tau), \qquad (6)$$

where C is a constant that depends on θ. In the derivation of (5) and (6) we used the case of the equality sign in formula (2). It turns out therefore that the Reuter-Sondheimer equation for the surface impedance Z_{RS} pertains to a certain particular case; nonetheless, it can yield a good approximation at $|8k\nu^4|\lesssim 1$ and $\omega\tau\sim 1/\sqrt{3}$.

For frequencies higher than critical, i.e., higher than $(\omega/\omega_p)_{cr}$, we can have either $\omega\tau<1$ or $\omega\tau>1$, depending on the value of $\omega_p\tau$. In this region, the surface impedance Z is given by the formula (see[1]):

$$Z = \frac{-4\pi i}{c}\left(\frac{\omega}{\omega_p}\right)\left[\left(1+(\omega\tau)^{-2}\right)^{1/4} e^{i\phi/2} - \frac{\sqrt{3}}{2}\left(\frac{v_F}{v_r}\right)^3\left(1+(\omega\tau)^{-2}e^{-i\phi/2}\right)\right].$$

Considering the limiting cases $\omega\tau\ll 1$ and $\omega\tau\gg 1$, we can determine from this formula the reflection coefficient of the electromagnetic wave. At $\omega\tau\ll 1$ we obtain the Hagen-Rubens relation, and at $\omega\tau\gg 1$ we obtain the expression for the relaxation region.[5] In both cases, however, correction terms appear and depend on (l/δ) and (v_F/v_r), respectively. These conditions are not always insignificant.

A detailed communication will follow.

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