# Pair production by a collapsing body as a quantum-geometry effect

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The process of black-hole production in gravitational collapse, accompanied by production of pairs from vacuum, is described as a quantum transition of space-time geometry from one quasistationary state into another. It is shown that the mass of the produced black hole is finite and depends on the spectrum of the collapsing matter.

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In the calculations made by Hawking[1] and others[2,3] of the evaporation of a black hole as a result of pair production from vacuum, the reaction of this production on the metric was calculated classically, as a result of which the details of the process have remained unclear. In the present paper we propose to regard the production of particles in a gravitational field and the influence of this process on the geometry as an effect of quantization of the geometry. The proposed formalism generalizes the approach developed by the author[4,5] to quantum theory of particles in a Minkowski space, and makes use of the definition formulated in[6] for particles in curved space-time. The application of this scheme to the case of a collapsing body confirms in the main Hawking's conclusions, but it predicts that as a result of the collapse, with the quantum radiation taken into account, a black hole of finite mass is left rather than a bare singularity.

The proposed calculation of the reaction of the particle production on the metric is based on two considerations:

1) Since pair production from the vacuum is a quantum effect, its action on the geometry should also be regarded as a quantum transition from one geometry to another. The probability amplitude of simultaneous pair production is the probability amplitude of the corresponding change of the geometry.

2) The production and annihilation of virtual states does not lead in general to a change of the geometry. The geometry is altered only by the production of real states that describe a stable particle and a stable antiparticle. In the formalism of quantum field theory, this corresponds to normal ordering of the energy-momentum tensor.

To define the concept of a real state and the production amplitude of a pair of real states, we use a causal propagator, i.e., the amplitude at $K(x,x')$ of the transition of a particle or antiparticle from one point of space-time to another.[4,5] It was proposed in[6] to define the causal propagator as the Green's function of the wave equation obtained by the de Witt—Schwinger proper time method or, equivalently, by a negative imaginary increment to the square of the mass. The states of the particles and antiparticles were defined in[6] as solutions of the wave equation, extended by this propagator to the future or to the past, respectively:

$$\psi^{\pm}(x) = (P^{\pm}\psi)(x) = \pm i \int_{\Sigma'} d\sigma^{\mu'} K(x, x') \overleftrightarrow{\partial}_{\mu'} \psi(x')$$

$$(x > \Sigma' \text{ for } P^{+};\ \Sigma' > x \text{ for } P^{-}) \quad .$$

We now postulate that it is precisely the states $\psi^{\pm}$ which are real in the sense that their production leads to a change of the geometry.

In[7] this definition was investigated in the particular case when the metric is real and positive-definite at pure imaginary values of the time parameter. In this case the propagator, defined as $m^2 - i0$, when analytically continued to the imaginary time axis (through the second and fourth quadrants of the complex plane) decreases at infinity and can be defined by means of this property (the analog of the Euclidean postulate of the ordinary quantum field theory). The real states of the particle and antiparticle are in this case orthogonal relative to the scalar product

$$(\psi, \psi') = i \int_{\Sigma} d\sigma^{\mu}\, \psi^{*}(x) \overleftrightarrow{\partial}_{\mu} \psi'(x),$$

so that we have the usual Fock space with a stable vacuum. We shall call this the quasistatic space-time. The nongravitational interactions of the particles in this space-time can be described with the aid of Feynman diagrams in which the internal lines correspond to propagators and the external lines to real states of particles and antiparticles or their complex conjugates. [6,7] In the case of a nonquasi-stationary space-time the definition of the particles must be refined by distinguishing between a particle that goes into the future and a particle that comes from the past. [8]

It is natural to assume that in quasistatic space the effects of pair productions do not influence the metric (real states are not produced), and consider the effect of production of real pairs with simultaneous conversion of one quasistatic space-time into another. To this end, two quasistatic spaces $\mathcal{X}_1$ and $\mathcal{X}_2$ would be joined together on a surface that is a symmetry surface of each of them (quasistatic spaces are symmetrical with respect to time reversal[7]). The geometrical initial conditions for the Einstein equation[9] are made to join continuously, and the energy-momentum transfer undergoes a discontinuity as a result of the pair production. For a geometry that is symmetric in time, the external curvature of the symmetry surface is equal to zero. The requirement that the initial conditions be compatible[9] causes the energy flux density also to be equal to zero on the joining surface.

The passage of matter through the joining surface will be regarded as a quantum process, assuming the amplitude of the transition of the particles to be equal to $(\psi_1^{+}, \psi_2^{-})$, and amplitude of pair production to be equal to $(\psi_2^{+}, \psi_1^{-})$. The subscripts show that the real states are defined with respect to the propagators $K_1$ and $K_2$ that act in the space $\mathcal{X}_1$ and $\mathcal{X}_2$. The calculation should start with a specification of the joining surface, the geometrical initial conditions on this surface, and a statement concerning which particles in which space pass through the surface and which pairs are produced and are annihilated. (This can be specified on the surface.) By the same token we define the energy-momentum tensor on the boundary, and consequently the geometries on both sides of the surface. Knowing the geometry, we can obtain the propagators and calculate the amplitudes of all the quantum transitions that occur on the boundary. The total amplitude plays the role of the amplitude of the transition probability of the matter and of the geometries from one state to another.

To calculate the collapse we assume as the model of the past half-space $\mathcal{X}_1$ the external Schwarzschild geometry with mass $M$, and as a model of the future half-space of $\mathcal{X}_2$ the total Schwarzschild—Kruskal space with mass $M'$. The propagator in $\mathcal{X}_2$, obtained from the Euclidean postulate, coincides[7] with the Hartle—Hawking propagator,[10] while the propagator in $\mathcal{X}_1$ coincides with the Bulware operator.[11] Both spaces are joined together on the surface $t=\text{const}$, where $t$ is the Schwarzschild time for both geometries and must be made to tend to $+\infty$ after the calculation. The Bulware propagator $K_1$ separates as real states of the particle the state with definite and positive Schwarzschild energy $\psi_E=\exp(-iEt)\Psi(\mathbf{x})=P_1^+\psi_E$. The Hartle—Hawking propagator $K_2$, according to the results of[12,13], coincides in the external region of the Kruskal space with the temperature Green's function corresponding to the temperature $T'=(\hbar c^3/8\pi kG)M'^{-1}=10^{26}\ M'^{-1}$. This means that $P_2^+\psi_E=(1+n_E)\psi_E$ and $P_2^-\psi_E=-n_E\psi_E$, where $n_E=[\exp(E/kT')-1]^{-1}$.

We consider a real state $\psi_E$ in the space $\mathcal{X}_1$. The amplitude of its transition to a real state $\psi_i^+$ in the space $\mathcal{X}_2$ is equal to $(\psi_i^+,\psi_E)$, while the probability of the transition to any of the real states is obtained by summing over $i$ the square of the modulus of the amplitude, and is equal to $(\psi_E,\ P_2^+\psi_E)=(1+n_E)(\psi_E,\psi_E)$. Analogously the amplitude of production of a pair with a particle in a state $\psi_E$ in $\mathcal{X}_1$ and an antiparticle in the state $\psi_i^-$ in $\mathcal{X}_2$ is equal to $(\psi_E,\psi_i^-)$, and the total probability of the production of the state $\psi_E$ is equal to $-(\psi_E,P_2^-,\psi_E)=n_E(\psi_E,\psi_E)$. The ratio $n_E/(1+n_E)=\exp(-E/kT')$ of the probability of the production of a particle to the probability of its absorption determines, by virtue of the detailed balancing principle,[10] the thermal character of the emission spectrum and its temperature.

The energy fluxes of the collapsing body and of the radiation should cancel each other, by virtue of the joining conditions, on the joining surface. To this end, the radiation energy must equal to the mass of the body (i.e., the mass of the black hole is much less than the mass of the body), and both fluxes should have a thermal character with one and the same temperature. If the collapsing matter consists from the very outset of particles whose total energy is distributed over the energies in accordance with the thermal spectrum, the temperature of the spectrum $T$ is one that determines the temperature (and by the same token the mass) of the black hole produced after the collapse: $T'=T$. If this is not the case, then the thermal character of the spectrum is established as the instant of collapse is approached (the black hole acts as a thermostat). The assumption that the geometry $\mathcal{X}_1$ is quasi-static then becomes incorrect, and the considered method makes it possible to estimate only the order of magnitude. The collapse of matter consisting of particles with mass $m$ of small kinetic energy leads to a black hole with temperature $T'\sim mc^2/k$ and a gravitational radius $R'\sim\hbar/4\pi mc$. If $m$ is the proton mass, then $T'\sim10^{13}$, $M'\sim10^{13}$ g, and $R'\sim10^{-15}$ cm.

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