Three dimensional periodic solutions of the Higgs scalar equation

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Spherically-symmetrical periodic solutions (isochrones) were obtained for the scalar equation with spontaneous symmetry breaking. A number of proofs are presented that the periodic formations previously observed in the numerical experiment are isochrone excitations of the scalar field.

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The possibility of describing elementary particles by classical solutions of field theory is presently under extensive discussion (see, e.g., [1,2]). From our point of view particular interest attaches to the question of mesons of the gluon type, which contain no quark—antiquark pairs in first-order approximation.[3]

In the hadron model proposed by Vinciarelly and Drell, the gluon field is described by the equation

$$\Box u = 4u(1 - u^2). \tag{1}$$

The possible existence of gluon states described by (1) was discussed in[5]. Spherically-symmetrical periodic weakly-irradiating formations, called by the authors "single-scale clusters," were deduced numerically in[6]. The possible existence of solutions of this type was indicated earlier in[7].

Classical periodic solutions in the one-dimensional case for Eq. (1) were sought in analytic form in[2]. Since the series obtained for the solution in that paper was asymptotic, the calculation carried out there affords only an indication of the possible existence of a periodic solution. A numerical experiment for the one-dimensional equation (1) yielded long-lived states[8]; there are grounds for assuming that these states are described by solutions of the type of[2] (or solutions close to them).

If we assume the method of[2] to be correct,[1)] then it can be used to obtain periodic solutions also in the three-dimensional case. In the spherically-symmetrical equation (1), we carry out the following transformations

$$u = 1 + z; \qquad \tau = \frac{2\sqrt{2}t}{\sqrt{1+\epsilon^2}}; \qquad \rho = \frac{2\sqrt{2}\epsilon r}{\sqrt{1+\epsilon^2}}\ , \tag{2}$$

where ϵ is a certain arbitrary parameter. We seek the function r, following,[2] in the form of an expansion in the parameter ϵ

$$z = \epsilon^2 g_0(\rho) + \sum_{n=1}^{\infty} [\epsilon^{2n-1} f_{2n-1}(\rho)\sin(2n-1)\tau + \epsilon^{2n} g_{2n}(\rho)\cos 2n\tau], \tag{3}$$

where the functions g_i and f_i are also expanded in series in ϵ^2

$$g_i(\rho) = \sum_{j=0}^{\infty} g_i^{(j)}(\rho)\,\epsilon^{2j}; \quad f_i(\rho) = \sum_{j=0}^{\infty} f_i^{(j)}(\rho)\,\epsilon^{2j}. \tag{4}$$

Substituting (3) and (4) in (1), we obtain for the first terms of the expansion (4) the expressions

$$g_0^{(0)} = -\frac{3}{4}[f_1^{(0)}]^2; \quad g_2^{(0)} = -\frac{1}{4}[f_1^{(0)}]^2; \quad f_3^{(0)} = -\frac{1}{16}[f_1^{(0)}]^3 \text{ etc.}, \tag{5}$$

while for $f_1^{(0)}$ we have

$$\Delta_\rho f_1^{(0)} - f_1^{(0)} + \frac{3}{2}[f_1^{(0)}]^3 = 0 \tag{6}$$

with boundary conditions $f_1^{(0)}(\infty) = 0$; $f_1^{(0)}(0) < \infty$; $df_1^0/d\rho\,|_{\rho=0} = 0$.

In[10] there was proved for an equation of the type (6) the existence of an infinite set of solutions $\{u_n\}$, $n = 1, 2, \cdots$, satisfying the boundary conditions $u_n(0) < \infty$, $u_n(\infty) = 0$ and having $n-1$ zeroes in the interval $(0, \infty)$. The solution for $n = 1$ was obtained numerically by Synge[11] for an equation that reduces (6) by a scale transformation.

It can be shown that solutions of (6) that are bounded at zero have zero derivatives at zero. In fact, a bounded $f_1^{(0)}$ means that near zero we have

$$f_1^{(0)} = A\rho^a + 0(\rho^a), \tag{7}$$

with $a \geq 0$. Substituting (7) in (6) we see that the equality is possible only at $a = 0$. The next term of the expansion is automatically found to be $\sim \rho^2$, q.e.d.

The calculation of the next terms of the series (4) cannot be carried out in explicit form, since we do not know the solution of (6). We note that in the planar case the terms of the series (4) increase very rapidly with j for a fixed point near $x = 0$ (apparently more rapidly than $j!$).

The solution (2)—(4), if it does exist, has one interesting property. Let us return in (3) to the old variables r and t. Then z is periodic with frequency

$$\omega = \frac{2\sqrt{2}}{\sqrt{1+\epsilon^2}}, \tag{8}$$

where ϵ characterizes the amplitude of (3). At small amplitude, i.e., at small ϵ, the period of the solution is independent of the amplitude, accurate to ϵ^2, and we therefore propose to call the solutions (2)—(4) isochronous excitations of the scalar field or isochrones. It is clear that this property does not depend on the number of spatial dimensions of Eq. (1).

The isochronism of the oscillations at small amplitude is apparently observed in the numerical experiment. According to (8), the period of the oscillations is equal to

$$T = \frac{\pi}{\sqrt{2}}\sqrt{1+\epsilon^2} \approx 2.2 + 0(\epsilon^2).$$

In the experiment we had isochrones with periods $T = 2.5$—2.7. The values of ϵ obtained for them agree sufficiently well with the experimental amplitudes. A similar picture is observed also in the one-dimensional case.

Comparison with the results of numerical experiments leads to the conclusion that the periodic formations obtained in[6] are described by solutions of the type (2)—(4) or solutions close to them, and this means that the method of[2] is correct. We note that in the employed approximation we do not obtain damping of the isochronous oscillations, which should take place by virtue of the radiation.

The answer to a question of whether the isochronous excitations considered here correspond to real gluon mesons depends on the applicability of the Vinciarelly—Drell model for the description of real hadrons, and also on whether classical isochronous solutions correspond to quantum states of the meson type.

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[1)]This method was first used in[9] for a different equation.

[1]L. D. Fadeev, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 148 (1975) [JETP Lett. **22**, 64 (1975)]; A. M. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. **20**, 430 (1974) [JETP Lett. **20**, 194 (1974)]; I. S. Shapiro, Pis'ma Zh. Eksp. Teor. Fiz. **21**, 624 (1975) [JETP Lett. **21**, 293 (1975)]; Zh. Eksp. Teor. Fiz. **70**, 2050 (1976) [Sov. Phys. JETP **43**, 1069 (1976)].
[2]R. Dashen, B. Hasslacher, and A. Neveu, Phys. Rev. D **11**, 3424 (1975).
[3]M. Fitsch and M. Gell-Mann, Proc. Sixteenth Conf. on High Energy Physics, 1972.
[4]P. Vinciarelly, Nucl. Phys. B **89**, 463 (1975); W. A. Bardeen, M. Chanowitz, S. D. Drell, M. Weinstein, and T.-M. Yan, Phys. Rev. D **11**, 1094 (1975).
[5]I. Yu. Kobzarev, Materially III Shkoly ITEF, (Proc. of 3rd School of Inst. Theor. Expt. Phys.) No. 1, p. 27, 1975.
[6]I. L. Bogolyubskiĭ and V. G. Makhan'kov, Pis'ma Zh. Eksp. Teor. Fiz. **24**, 15 (1976) [JETP Lett. **24**, 12 (1976)].
[7]N. A. Voronov, I. Yu. Kobzarev, and N. B. Konyukhova, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 590 (1975) [JETP Lett. **22**, 290 (1975)].
[8]A. E. Kudryavtsev, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 178 (1975) [JETP Lett. **22**, 82 (1975)].
[9]A. M. Kosevich and A. S. Kovalev, Zh. Eksp. Teor. Fiz. **67**, 1793 (1974) [Sov. Phys. JETP **40**, 891 (1975)].
[10]G. Ryder, Pacific J. Math. **22**, 477 (1967).
[11]J. L. Synge, Proc. Royal Irish Acad. **62** (1961).