Nature of "high-temperature" oscillations of the magnetoresistance of Bi

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A connection is established between the "high-temperature" oscillations of the transverse (j⊥H) magnetoresistance of bismuth and the Fermi energy. This connection makes it possible to attribute these oscillations to resonant scattering of carriers on the Fermi surface by optical phonons.

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The purpose of the present study was to investigate the "high-temperature" oscillations (HTO) of the magnetoresistance (MR) of bismuth under conditions when the Fermi level changes strongly when the external magnetic field and temperature are varied. These experiments, as indicated in[1], can help determine unambiguously which of the parameters of the energy spectrum of Bi is connected with the HTO.

A magnetic field up to 56 kOe was produced in a superconducting solenoid of inside diameter 40 mm. The experiments were performed on samples Bi-11 under conditions with $\mathbf{j} \parallel z \parallel C_3$, $\mathbf{H} \perp C_3$ ($C_1 \parallel \mathbf{X}$, $C_2 \parallel \mathbf{Y}$ and C_3 are respectively the bisector, binary, and trigonal crystallographic axes, $\mathbf{j}$ is the current-density vector, and H is the magnetic field intensity), and also on sample Bi-6a at $\mathbf{j} \parallel C_1$ and $\mathbf{H} \parallel C_3$. The characteristics of the single crystals together with a detailed description of the distinguishing features of the method are contained in[1]. We indicate only that during the course of the experiment the temperature was maintained constant within 10^{-2} deg, and the oscillations of ρ_{yz} and ρ_{xy}, which were investigated by us, were registered with an x–y recorder as functions of $1/H$.

Figure 1 shows the reciprocal magnetic field corresponding to the extrema of ρ_{yz} as a function of the number N of the extremum (the zero value of N is chosen from considerations that will be explained later on; both the minima and the maxima of ρ_{yz} are shown). It follows from the figures that: a) the period of the HTO does not remain constant at a given orientation of the vector **H** in the investigated interval of magnetic fields; b) in a sufficiently narrow field interval, in which it is possible to separate a constant period of the oscillations, the period decreases with increasing temperature. At the same time, in the vicinity of $\mathbf{H} \parallel C_3$ the period of the HTO does not depend on the magnetic field (Fig. 2).

We shall make the customary assumption concerning the Fermi surface of Bi, i.e., that it does not contain new groups of carriers, and that the effective mass of the holes does not depend on the energy (the latter statement is assumed valid accurate tc $\sim 1\%$).

If a certain characteristic energy ϵ_0 is present in the crystal, such that a resonance condition of the type

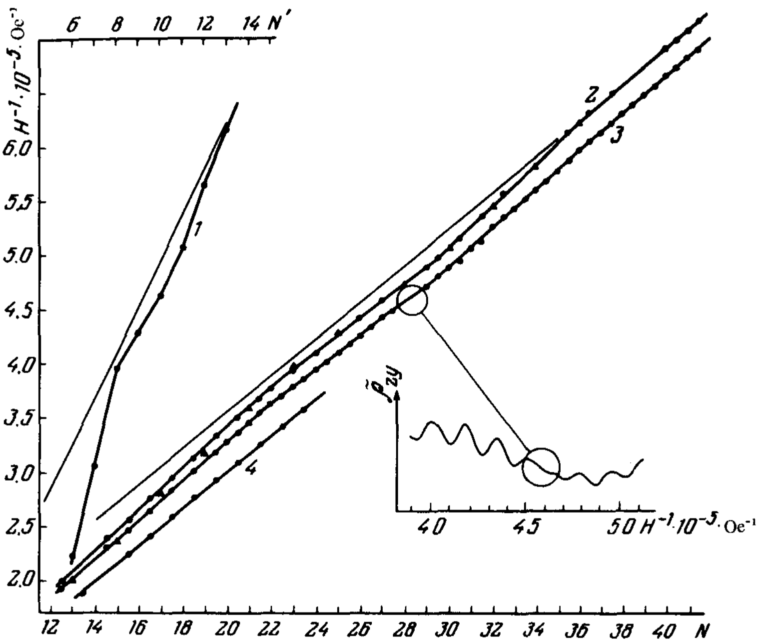

FIG. 1. $\mathbf{H} \parallel C_1$. Curves 2, 3, and 4 correspond to HTO at $T=18$, 36, and 61 °K, while curve 1 corresponds to the Shubnikov—de Haas effect in accordance with the data of[2] (scale N'). The straight lines are drawn to illustrate the deviations from periodicity of the HTO in the reciprocal field. ▲—calculation by formula (3).

$$\epsilon_0 = M\hbar\Omega, \qquad M = 1, 2, 3, \ldots, \tag{1}$$

connected with the transitions of the carriers between the Landau subbands is satisfied in the magnetic field ($\Omega = eH/m^*c$ is the cyclotron frequency), then conductivity oscillations with a constant period $\Delta(1/H) = e\hbar/\epsilon_0 m^*c$ can arise. The HTO in bismuth at $\mathbf{H} \parallel C_1$, are connected with the Fermi surface of the holes.[1] The cyclotron mass of the holes, as already mentioned, remains constant when the Fermi level ϵ_F is altered by the magnetic field. The HTO period, however, depends substantially on H (Fig. 1), and consequently a resonance of the type (1) does not describe the phenomenon.

By comparing at $\mathbf{H} \parallel C_1$ the plots of $1/H=f(H)$ for the HTO and for the Shubnikov—de Haas (SdH) oscillations connected with the Fermi surface,[2] we have found the corresponding curves to be perfectly analogous. Thus, the change of the SdH oscillations period in the magnetic field, which is a result of the $\epsilon_F(H)$ dependence, is reflected in the behavior of the HTO (Fig. 1). In addition, in the regions where the slope of the $1/H=f(N)$ curves changes, the oscillations of $\rho_{yz}(H)$ exhibit a characteristic collapse (insert in Fig. 1), which can be attributed to the influence of the magnetic field on ϵ_F. Thus, the HTO seem to be connected with the Fermi energy. This is also evidenced by the constancy of the period near $\mathbf{H} \parallel C_3$ (Fig. 2), where ϵ_F changes in fields up to 50 kOe by approximately one order of magnitude less than at $\mathbf{H} \parallel C_1$.[2] On the other hand, by estimating the ratio $kT/\hbar\Omega$ for $T=61$ °K and $H=30$ kOe, it can be concluded

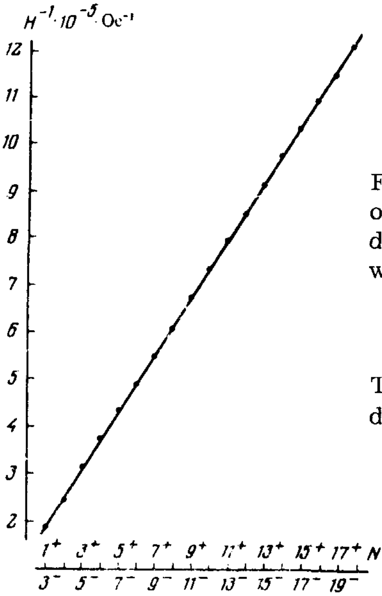

FIG. 2. $\mathbf{H} \parallel C_3$, $T = 18$ °K. In the calculation of N it is assumed that the spin splitting is double the orbital splitting, in accordance with the formula

$$(1/H)^{\pm} = \frac{e\hbar}{m^* c\{\epsilon_F + \epsilon_0(\hbar\omega_0)\}} \begin{cases} (N+\frac{3}{2}) \\ (N-\frac{1}{2}) \end{cases}.$$

The ± signs correspond to the different spin directions.

that the HTO are more readily not connected with the SdH oscillations, since they are observed at $kT/\hbar\Omega \approx 3$. Taking all the foregoing into account, we have attempted to formulate the condition for the resonance scattering of the carriers, capable of causing the oscillations of the magnetic conductivity with the characteristic distinguishing features of the HTO, such as the periodicity in the reciprocal magnetic field (with a period smaller than that given by the SdH effect), the dependence of the period on the Fermi energy, and the slow damping of the amplitude with temperature. Account was taken of the fact that the connection between the HTO and ϵ_F should lead to a temperature damping of the amplitude in the form $\exp\{-(2\pi^2 kT/\hbar\Omega)\}$, but the damping can be decrease with the aid of the Planck distribution function. In other words, the characteristic internal energy should be a function of the end-point frequency of the optical phonons $\hbar\omega_0$ at resonance. As a result, the resonance condition takes the form

$$\epsilon_F(H, T) + \epsilon_0(\hbar\omega_0) = (N + \tfrac{1}{2})\hbar\Omega, \qquad (2)$$

and describes transitions from the Fermi level to states with quantum numbers $N, k_y, k_z = 0$. The values of the magnetic field at resonance are determined by the relation

$$\frac{1}{H} = \frac{(N+\tfrac{1}{2})e\hbar}{\{\epsilon_F(H,T) + \epsilon_0(\hbar\omega_0)\}m^* c}, \qquad (3)$$

and at a constant Fermi energy their difference is

$$\Delta\left(\frac{1}{H}\right) = \frac{e\hbar}{m^* c\{\epsilon_F + \epsilon_0(\hbar\omega_0)\}}. \qquad (4)$$

Assuming $(\epsilon_F)_{T=18K} \approx (\epsilon_F)_{T=4K}$ and neglecting at $H < 15$ kOe the $\epsilon_F(H)$ dependence, we can determine $\epsilon_0(\hbar\omega_0)$ with the aid of formula (4) from the period of the HTO at $\mathbf{H} \parallel C_1$, and then, identifying N by means of the extremal point of the magnetoresistance, we can calculate with the aid of (3) the positions of all the remaining extrema in the direction $\mathbf{H} \parallel C_1$.[1)] The result of this procedure is shown in Fig. 1. The value of N was obtained at $H = 16$ kOe, and the function $\epsilon_F(H)$ and the cyclotron masses were taken from[2]. The discrepancy between the experiment and the calculation did not exceed 5%. The calculation errors due to the inaccuracy in the determination of $\epsilon_0(\hbar\omega_0)$ and N is $\pm$ 15%. In the case $\mathbf{H} \parallel C_1$ we have $\epsilon_0(\hbar\omega_0) = 20.6^{+1.9}_{-1.0}$ meV and at $\mathbf{H} \parallel C_3$ we have $\epsilon_0(\hbar\omega_0) = 18.3 \pm 0.8$ meV. Comparison of the results with the known data for the phonon spectrum of bismuth[3] show that $\epsilon_0(\hbar\omega_0)$ differs by a factor of 1.5 from the end-point energy of the optical phonons. This figure, however, may be inaccurate since not all the branches of the spectrum were determined in[3].

When $\epsilon_0(\hbar\omega_0) < \hbar\Omega$, the HTO should vanish, i.e., it is possible in principle to verify directly the hypothesis concerning the origin of the HTO. To do so in experiment it is necessary to register reliably resistance changes of $\sim 10^{-2}$% in magnetic fields >200 kOe. For carriers with low effective mass, however, the inequality $\hbar\Omega > \epsilon_0(\hbar\omega_0)$ is already satisfied under our experimental conditions. This may be precisely why we do not observe the corresponding periods of the HTO.

We shall not discuss here the temperature dependence of the HTO amplitude. We note only that the product $\exp(-2\pi^2 kT/\hbar\Omega) \times [\exp(\hbar\omega_0/kT) - 1]^{-1}$ can shift the maximum of the amplitude towards sufficiently low temperatures. In addition, it is easy to show that even in "pure" magnetophonon resonance[4] the maximum of the amplitude shifts in compensated conductors towards lower temperatures in such a way that its position on the temperature scale is determined, in particular, by the ratio of the probabilities of scattering from optical and acoustic phonons.

A magnetophonon effect with a resonance condition

$$(\epsilon_F \pm \hbar\omega_0) = (N + \tfrac{1}{2})\,\hbar\Omega \qquad (5)$$

analogous to formula (2) was predicted in[5], but for the longitudinal ($\mathbf{j} \parallel \mathbf{H}$) magnetic conductivity. The physical nature of the effect is connected with the fact that at $\mathbf{j} \parallel \mathbf{H}$ the contribution to the current is made by electrons situated

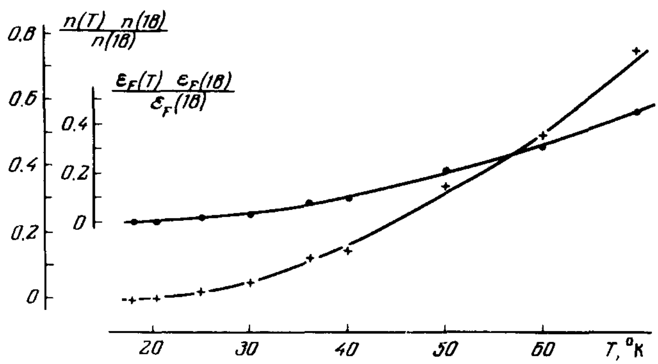

FIG. 3. Relative change of the Fermi energy (•) and of the carrier density (×) with increasing temperature ($N = 18$).

near the Fermi surface.[2)] For anisotropic crystals, the analysis of[5] can be formally generalized to include the case of transverse magnetoresistance, inasmuch as each element of the matrix $|\rho_{ik}|$ contains, generally speaking, all the elements of the matrix $|\sigma_{ik}|$.[3)] This does not contradict the observation of HTO in the vicinity of $\mathbf{H} \parallel C_3$, since the magnetoresistance tensor is substantially anisotropic near the given direction, and it is impossible to align the magnetic fields strictly parallel to the threefold axis.[7] Within the framework of the aforegoing generalization, it is not clear why no "pure" magnetophonon resonance of type (1) is observed in the transverse magnetoresistance. The still unanswered questions are the cause of the doubling of the fundamental HTO frequency[1] and the contribution made to the resonance of the off-diagonal components of the magnetoresistance tensor. It seems to us, however, that on the whole the proposed explanation of the HTO of the transverse magnetoresistance as being due to resonant electron-phonon interaction, with account taken of the degeneracy of the electron spectrum, is quite convincing. In conclusion, we present the temperature dependence of the Fermi energy of bismuth, calculated with the aid of relation (3) from the shift of the extrema in the interval 18—61 °K, and the dependence of the carrier density n on the temperature (Fig. 3).

[1)]The spin splitting of the hole levels tends to zero at $\mathbf{H} \parallel C_1$ and is approximately double the orbital splitting at $\mathbf{H} \parallel C_3$.[2] It has therefore practically no effect on the positions of the magnetoresistance extrema.

[2)]The final formula (6) given in[5] for the magnetoresistance is incorrect.

[3)]Attempts to observe HTO for longitudinal magnetoresistance of bismuth were unsuccessful. One cannot exclude the possibility that the reason is the insufficient sensitivity of the installation, since the longitudinal effect should be weaker by several orders of magnitude than the transverse one.

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