Possibility of a cold cosmological singularity in the spectrum of primordial black holes

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We consider entropy generation accompanying evaporation of primordial black holes. Bounds are obtained on the number of black holes with mass $M < 10^{13}$ g. It is shown that the initial cold matter, when transformed halfway into black holes with $M \sim 10^4$ g, acquires the presently observable entropy after the black-hole evaporation.

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Besides the black holes that are produced during the final stage of the evolution of massive stars and clusters (these black holes have masses $M > M_{\odot} = 2\times 10^{33}$ g), there can exist in nature primordial black holes (PBH) produced from large-amplitude density perturbations during an earlier superdense stage of the evolution of the universe.[1,2] The PBH can have either a large or a small mass, the only lower bound being the quantum Planck mass $M_{Pl} = \sqrt{\hbar c/G} \approx 2\times 10^{-5}$ g. A very important role in investigations of the existence of PBH and their number is played by the effect of spontaneous evaporation of the black holes,[3] wherein PBH with mass $M < 10^{15}$ g have decayed completely have been transformed into radiation in the present stage of the evolution of the universe, $t \sim 10^{18}$ sec.

For PBH with $M > 10^{15}$ g, the evaporation effect is inessential, and therefore the upper bounds of their number, obtained in[1], remain in force (see also Carr's paper[4]). On the other hand, if $M < 10^{15}$ g, then these estimates cannot be used, since the energy contained in such PBH has time to be converted into radiation energy. As a result, the specific entropy S of the universe per baryon increases. Carr[5] and Chapline[6] have recently pointed out the possibility that the matter of the universe was cold during the initial stage of expansion ($S_0 \ll 1$), and that the entire entropy observed at the present time ($S_1 \sim 10^9$) was the result of formation and subsequent evaporation of PBH.

We calculate in this paper the specific entropy of the universe, generated during the course of the evaporation of PBH and subsequent relaxation of the produced radiation. Assume that at the instant t_1 the fraction β ($0 < \beta < 1$) of matter with an equation of state $p = \epsilon/3$ has been converted into PBH. We assume further that $\hbar = c = 1$ and disregard all the numerical factors of order unity, including $(1-\beta)$. The characteristic mass of these PBH is $M = G^{-1}t_1$ (the gravitational radius of the PBH is of the order of the horizon). Then at $t > t_1$ the expansion of the isotropic universe follows the law $a(t) \sim t^{1/2}$ if $t_1 < t < t_2$, where $t_2 = t_1\beta^{-2}$, and $a(t) \sim t^{2/3}$ if $t > t_2$. During the stage $t > t_2$, the energy density of the PBH, which are practically at rest and effectively constitute a medium with an equation of state $p = 0$, is much higher than the energy density of the matter remaining outside the black hole. Let $\beta M/M_{Pl} > 1$ (otherwise little entropy is generated). Then the time of evaporation of the PBH is $t_3 = G^2M^3 = G^{-1}t_1^3$

$> t_2$. At $t_2 < t < t_3$ we have $\epsilon_{PBH} = (Gt^2)^{-1}$, and the energy density of the matter outside the PBH is

$$\epsilon_m = G^{-1} t_2^{-2}\left(\frac{t_2}{t}\right)^{8/3} + t_1^{-3} t^{-1},$$

where the second term describes the particles radiated by the PBH during the evaporation process. At $t = t_3$, the PBH vanish completely, leaving after them neutral matter[1)] with $\epsilon_1 = (Gt_3^2)^{-1} = Gt_1^{-6}$, a particle-number density $n_1 = Gt_1^{-5}$, and a characteristic particle energy t_1^{-1}. This matter, however, is very far from equilibrium. Indeed, the radiation of each PBH is close to that of the black body with temperature $T_1 = (GM)^{-1} = t_1^{-1}$ only in a solid angle on the order of $(r_g/r)^2$, which decreases with increasing distance r from the PBH. In the space between the PBH, the radiation spectrum, averaged over the angles, is much harder than the equilibrium spectrum. After the relaxation, an equilibrium temperature $T_2 = \epsilon_1^{1/4}$ is established, and the number of particles increases very strongly and reaches the value $n_2 = T_2^3 = G^{3/4} t_1^{-9/2}$.

The baryon-charge density at that instant is

$$n_B = (1 + S_0)^{-1} (Gt_2^2)^{-3/4} \left(\frac{t_2}{t_3}\right)^2 = (1 + S_0)^{-1} \beta^{-1} G^{5/4} t_1^{-11/2} ,$$

where S_0 is the initial specific entropy at the instant t_1. The specific entropy after relaxation[2)] is

$$S = \frac{n_2}{n_B} = (1 + S_0)\beta \frac{M}{M_{Pl}} \quad . \tag{1}$$

Comparing with the contemporary value of the entropy, we obtain for β the upper bound

$$\beta \frac{M}{M_{Pl}} \lesssim 10^9 \quad . \tag{2}$$

From the estimate (2) it follows that a noticeable fraction of the matter can be converted only into PBH with a mass $M \lesssim 10^4$ g. The cold cosmological singularity hypothesis ($S_1 \ll 1$) is valid if $\beta \sim 1/2$ at $M \sim 10^4$ g. Such PBH are produced at $t \sim 10^{-34}$ sec and are evaporated at $t \sim 10^{-16}$ sec, i.e., long before nucleosynthesis.

An estimate of the relaxation time shows that the PBH radiation has time to become completely thermalized if $T_2 \gtrsim m_e$, i.e., at $M < 5 \times 10^9$ g ($t_3 \lesssim 10$ sec). At $T_2 < m_e$, this radiation is still not in equilibrium, because the bremsstrahlung cannot ensure a sufficient growth in the number of photons in the Rayleigh-Jeans region, owing to the low density of the remaining baryons and electrons (for details see[9]), where an arbitrary source of energy is considered). In order for the spectrum of the primordial radiation not to deviate noticeably from equilibrium in this case, ϵ_{PBH} must not exceed ϵ_m. It is therefore necessary to have $t_2 > t_3$, which leads to the estimate

$$\beta \frac{M}{M_{Pl}} < 1 \tag{3}$$

in the mass interval $10^{11} \lesssim M \lesssim 10^{13}$ g. At $5 \times 10^9 \lesssim M \lesssim 10^{11}$ g, a transition from the bound (2) to the stronger bound (3) takes place. The upper bounds on the number of black holes with $M \sim 10^{14}$–10^{15} g, which evaporate already after recombination, were obtained earlier in[10,4,5].

[1)] We do not consider here possible effects of CP violation.[7,8]

[2)] Were we to disregard the relaxation process, we would obtain a much smaller value $S \sim \beta(M/M_{\mathrm{Pl}})^{1/2}$ (this result is cited by Chapline without proof[6]). Formula (1) differs also from the result of Carr,[5] who did not consider the stage $t_2 < t < t_3$, where $\epsilon_{\mathrm{PBH}} \gg \epsilon_m$.

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