Anomalous resistance in high-frequency heating of tokamak plasma

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(Submitted June 18, 1976)
Pis'ma Zh. Eksp. Teor. Fiz. **24**, No. 4, 199–202 (20 August 1976)

It is shown that induced scattering of electromagnetic waves by ions in a plasma with a longitudinal current can lead to the appearance of an appreciable anomalous resistance.

PACS numbers: 52.40.Db, 52.50.Gj, 52.55.Gb

It was shown [1] in experiments on additional heating of a tokamak plasma by electromagnetic waves in the frequency range $\omega_{pi} = (\sqrt{4\pi e^2 n/M_i} \lesssim \Omega \ll \omega_{pe}$ in the TM-3 installation that the high-frequency heating occurs mainly on the periphery of the plasma pinch and is accompanied by turbulization of the heating region and by the rowding out of the longitudinal current from this region. It was demonstrated by the procedure described in [2] that the resistance anomaly A produced by the RF waves depends on the plasma density and reaches $A \sim 10$ at $\bar{n} \sim 10^{12}$ cm^{-3}, decreasing to unity at $\bar{n} \gtrsim 3 \times 10^{13}$ cm^{-3}. It must be emphasized that under the conditions of the experiment of [1] the longitudinal momentum of the electromagnetic waves, even for RF energy asymmetrically introduced into the plasma, was small in comparison with the momentum acquired by the electrons from the solenoidal field, and could not cause an appreciable anomaly. The experimentally observed large value of A can therefore be explained only by resorting to some linear mechanism whereby the electromagnetic waves are transformed into short-wave plasma oscillations that have a large longitudinal momentum.

In the case of the isothermal plasma considered here, this mechanism can be the induced scattering of electromagnetic waves by ions, [3] which leads to transformation of the electromagnetic quanta into Langmuir quanta with $\omega_l = \omega_{pe} k_{\parallel l}/k_l \approx \Omega$ and $k_{\parallel l} \sim \Omega / v_{Te} \gg K_{\parallel}^{\Omega} \approx \Omega / c$ (here $\mathbf{k}^{\Omega}$ and $\mathbf{k}_{\lambda}$ are the wave vectors of the electromagnetic and Langmuir waves, respectively, and $v_{Te} = \sqrt{2T_e/m}$ is the thermal velocity of the electrons). The recoil momentum is transferred in this case to the resonant ions and has practically no effect on the value of the longitudinal current.

The system of equations describing the induced scattering takes the form[3]

$$\frac{\partial w_o}{\partial t} = -a w_o \int d\mathbf{k}_l \frac{w_{\mathbf{k}l}}{nT} + \frac{P}{V}; \qquad \frac{\partial w_{\mathbf{k}l}}{dt} = a\frac{w_o}{nt} w_{\mathbf{k}l} - (\gamma_{\mathbf{k}l} + \nu_{ei}) w_{\mathbf{k}l}. \tag{1}$$

Here P/V is the power of the electromagnetic-wave source per unit volume, $w_0 \approx (\omega_{pe}^2/\Omega^2)(|E_{\parallel}^0|^2/4\pi)$ is the energy density of the electromagnetic waves, $w_{\mathbf{k}l} \approx (\omega_{pe}^2/\Omega^2)(|E_{\parallel l}|^2_{\mathbf{k}l}/4\pi)$ is the spectral energy density of the Langmuir waves,

ν_{ei} is the frequency of the electron-ion collisions, and

$$\gamma_{\mathbf{k}\,l} = \frac{\sqrt{\pi}}{2} \frac{\omega_l^3}{|k_{\parallel l}| k_{\parallel l}} \frac{\partial f_l}{\partial v_\parallel} \Big/ \omega_l / k_{\parallel l} \tag{2}$$

is the decrement of the Langmuir-wave damping by the resonant electrons. The nonlinear-interaction parameter α depends on the principal characteristics of the plasma.

It follows from (2) that at a given level of the electromagnetic-wave energy Langmuir quanta with $k_{\min} \le k_{\parallel l} \lesssim k_{\max}$ ($k_{\min} \lesssim 0, k_{\max} \gtrsim 0$) are produced in the course of the scattering. The quantities $K_{\min}$ and $k_{\max}$ are governed by the damping decrement $\gamma_{\mathbf{k}l}$, which increases exponentially with increasing $|k_{\parallel l}|$. It follows from (2) that in the absence of a solenoidal electric field E_0, i.e., if the electron distribution function is symmetrical about $v_\parallel$, we have $k_{\min} = -k_{\max}$. In this case the combined longitudinal momentum of the produced Langmuir waves is therefore equal to zero. In the case $E_0 \neq 0$, however, it follows from (2) that the unstable oscillations are those with $k_{\max} < -k_{\min}$ if $\int v_\parallel f_l \, d\mathbf{v} > 0$ and conversely with $k_{\max} > -k_{\min}$ if $\int v_\parallel f_l \, d\mathbf{v} < 0$. Thus, if a longitudinal current is present in the plasma, the induced scattering leads to production of Langmuir oscillations with an average longitudinal momentum directed opposite to the electron momentum. When such waves interact with electrons, they transfer to the latter their energy and momentum and decrease the magnitude of the longitudinal current, and this should experimentally manifest itself by the appearance of an anomalous resistance in the system.

For a self-consistent description of the slowing down of the electrons, the system (2) must be supplemented by a quasi-linear equation for the electron distribution function [4]:

$$\frac{\partial f_\parallel}{\partial t} + \frac{eE_0}{m} \frac{\partial f_\parallel}{\partial v_\parallel} = \frac{\partial}{\partial v_\parallel} \frac{e^2}{m^2} \nu_{ei} \int d\mathbf{k}_l \frac{|E_{\parallel l}|^2_{\mathbf{k}l}}{(\omega_l - k_{\parallel l} v_\parallel)^2} \frac{\partial f_\parallel}{\partial v_\parallel} - \nu_{ei} f_\parallel; \quad f_\parallel = \int f_e \, d\mathbf{v}_\perp. \tag{3}$$

In the derivation of (3), only the nonresonant interaction of the Langmuir waves with the electrons was taken into account, since the bulk of the Langmuir-wave energy is lost to collisions in the course of the induced scattering.

To estimate the maximum of the anomaly introduced by the induced scattering, we consider the following limiting situation: We assume that the longitudinal current produced by the field E_0 is so large that the scattering process results only in the production of Langmuir waves with $k_\parallel \sim k_{\min} < 0$. The value of $k_{\min}$ can be chosen with good accuracy equal to $|k_{\min}| = k_1 = \Omega / 5 v_{Te}$. We multiply (3) by $mv_\parallel$ and integrate it over velocity space:

$$\frac{\partial}{\partial t} mnu = eE_0 n - \frac{\omega_{pl}^2}{\omega_l^3} \nu_{ei} k_1 \frac{|E_{\parallel l}|^2}{4\pi} - \nu_{ei} mnu; \quad u = \int v_\parallel f_\parallel dv_\parallel. \tag{4}$$

In the stationary state the system (2) takes the form

$$w_o = \frac{P}{V}\frac{nT}{a w_l} \quad ; \quad a w_o / nT = \nu_{ei} \ . \tag{5}$$

Expressing w_l in terms of P by means of (5) and substituting in (4), we obtain

$$eE_o n = \frac{k_1}{\omega_l}\frac{P}{V} + \nu_{ei}\, mnu \ . \tag{6}$$

Relation (6) has a simple physical meaning. P is the energy fed to the plasma by the electromagnetic wave per unit time. It can be written in the form $P = \hbar\omega_l N_0$, where N_0 is the number of quanta entering the plasma per unit time. As a result of the induced scattering the quanta acquire a longitudinal momentum $\hbar k_\parallel$ with practically no change in their energy. Thus the interaction with the Langmuir waves imparts to the electrons a momentum $\hbar k_1 N_0/V = Pk_1/V\omega_l$ per unit time and per unit volume. The remaining terms of (6) are universally known. A numerical estimate based on the experimental data of [1] shows that the terms $eE_0 n$ and $(P/V)(k_1/\omega_l)$ become comparable at $n_{cr} \sim 3\times 10^{13}$ cm^{-3}. At plasma densities $n \ll n_{cr}$ the momentum acquired by the electrons from the Langmuir waves greatly exceeds that acquired from the solenoidal field, and this should lead to an appreciable anomaly. The following remark is in order here. We have considered above the limiting case of large solenoidal currents, when the scattering process is strongly asymmetrical. It is clear that when the strong inequality $eE_0 n \ll (P/V)(k_1/\omega_1)$ is satisfied the current in the plasma is greatly decrease, and this should lead to symmetrization of the scattered waves with respect to the longitudinal momentum and therefore lead to a self-consistent decrease of the term $(P/V)(k_1/\omega_l)$, so as to satisfy the natural inequality $eE_0 n \gtrsim (P/V)(k_1/\omega_l)$.

We note in conclusion that the mechanism described above can be used to prevent skinning of the solenoidal current in large-scale tokamak installations.

The author thanks V. V. Alikaev, Yu. N. Dnestrovskii, O. P. Pogutse, and K. A. Razumova for useful discussions.

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