Electron-hole liquid in strongly anisotropic semiconductors and semimetals

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It is shown that in semiconductors and semimetals with strongly anisotropic electron spectra the electron-hole liquid has a minimal energy at densities corresponding to strong compression. The binding energy of the liquid greatly exceeds in this case the binding energy of the excitons.

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It was shown in a preceding paper[1] that in sufficiently strong magnetic fields, $H \gg 1$, the energy of the electron-hole liquid, as a function the density n of the electron-hole pairs, has a minimum at $n \sim H^{8/7} \gg 1$. We use here the "Coulomb" system of units $e^2/\epsilon = \hbar = m = 1$ and measure H in units of $e^3m^2c/\epsilon^2\hbar^2$, where ϵ is the dielectric constant of the crystal (without allowance for the contribution of the free electrons and holes), and m is one of the effective masses, the choice of which will be specified concretely in each individual case.

Thus, the electron-hole plasma turns out to be strongly compressed by the joint action of the magnetic field and the Coulomb interaction. It will be shown below that this tendency to "self-compression" of the electron-hole liquid, i.e., the fact that its energy has a minimum at $n \gg 1$, is a characteristic fea-

ture of a large number of model systems even without a magnetic field. A common feature of these models is a strong anisotropy of the electron spectrum. From a more formal point of view, a criterion for the selection of such systems is the existence for them of a region of concentrations n in which the inequalities

$$1 << p_F << n^{1/4}, \tag{1}$$

are satisfied (p_F is the Fermi momentum). In the well-known formula for the correlation energy per particle pair

$$E_{corr} = \frac{1}{2n} \int_0^1 \frac{d\lambda}{\lambda} \int \frac{d^3k d\omega}{(2\pi)^4} \left[\frac{4\pi\chi(\mathbf{k}, i\omega; \lambda)}{1 + 4\pi\chi(\mathbf{k}, i\omega; \lambda)} - 4\pi\chi^{(0)}(\mathbf{k}, i\omega; \lambda) \right], \tag{2}$$

where $\chi(\mathbf{k}, i\omega; \lambda)$ is the polarizability of the system of electrons and holes with concentration n and charge $\pm\sqrt{\lambda}$ for each particle (+for the holes), if (1) is satisfied, then the principal contribution to χ in terms of the parameter p_F^{-1} is made by the diagram $\chi^0(k, i\omega; \lambda)$ of lowest order in λ, while the main contribution to the integral with respect to the momentum transfer k is made by the region $k \propto n^{1/4} \gg p_F$. It is important that the effective radius of this interaction is $k^{-1} \propto n^{-1/4} \gg n^{-1/3}$, where $n^{-1/3}$ is the average distance between particles. In this momentum range we have

$$\chi^{(0)}(\mathbf{k}, i\omega; \lambda) \approx \lambda \sum_i \frac{2n_i \epsilon_i(\mathbf{k})}{k^2[\epsilon_i^2(\mathbf{k}) + \omega^2]}, \tag{3}$$

where the subscript i labels different types of charged particles, and $\epsilon_i(\mathbf{k})$ are their energies as functions of the momentum. Assuming all the dispersion laws of ϵ_i to be quadratic, we easily obtain from (2) and (3)

$$E_{corr} = -An^{1/4}, \tag{4}$$

where A is a coefficient that depends on the electron and hole mass ratio, on their anisotropy, on the number of equivalent minima in the electron and hole bands, and on other details of the electron spectrum. An explicit form of this coefficient is given below for serveral typical systems satisfying the criterion (1). For all these systems, the exchange energy in the considered concentration range is small in comparison with E_{corr}, and the total energy $E(n)$ per pair of particles, which is equal to the sum of the Fermi energy and E_{corr}, reaches a minimum value E_{min} at a concentration n_{min} satisfying the criterion (1).

I. *Quasi-one-dimensional systems*, i.e., systems of parallel conducting filaments. The transitions of electrons and holes from one filament to another are assumed to have negligibly low probability, and the density of the number of filaments per unit of the employed scale for the surface perpendicular to them is $N \gg 1$. This case is closest to the case of a strong magnetic field, considered in [1], and the results are practically the same, apart from the substitutions $N \rightleftharpoons H$. We present them here, however, in a somewhat more general form, taking into account the difference between the electron and hole masses m_e and m_h, and assuming $m = m_e m_h/(m_e + m_h) = 1$ and $N^{-1/2} \lesssim \sigma \equiv m_e/m_h \leq 1$. The proportionality coefficient in (4) for this case is

$$A_{\mathrm{I}} = \left(\frac{4}{\pi}\right)^{1/4} \frac{64\pi}{5[\Gamma(1/4)]^2} f_{\mathrm{I}}(\sigma),$$

where

$$f_{\mathrm{I}}(\sigma) = \frac{[\Gamma(1/4)]^2}{8 \cdot 2^{1/4} \pi^{3/2}} \int_0^\infty dx \left[\frac{1}{(1+\sigma)x^2 + \frac{1}{1+\sigma}} + \frac{1}{\frac{1+\sigma}{\sigma} x^2 + \frac{\sigma}{1+\sigma}} \right]^{5/4}, \tag{5}$$

$$f_{\mathrm{I}}(1) = 1 \qquad f_{\mathrm{I}}(\sigma << 1) = (1/2)^{3/2} \frac{1}{\sigma^{1/4}}, \tag{6}$$

$$p_F = \frac{\pi}{2} Nn; \quad n_{min} = \left[\frac{3}{2\pi^2} A_{\mathrm{I}}\right]^{4/7} N^{8/7}; \quad E_{min} = -\frac{7}{8}\left(\frac{3}{2}\right)^{1/7} \frac{1}{\pi^{2/7}} A_{\mathrm{I}}^{8/7} N^{2/7}. \tag{7}$$

II. *Quasi-two-dimensional (layered) systems*, i.e., systems of parallel conducting planes with distances $c \ll 1$ between them and with negligibly low probability of carrier transfer from one plane to another.

Assuming $2m_e m_h/(m_e+m_h) = 1$ and $c \lesssim \sigma \leq 1$, we obtain for the case of spectra that are isotropic in the plane of the layer

$$A_{\mathrm{II}} = \left(\frac{\pi}{2}\right)^{1/4} \frac{256\pi^2}{5[\Gamma(1/4)]^2} f_{\mathrm{II}}(\sigma) \approx 3.27 f_{\mathrm{II}}(\sigma), \tag{8}$$

where

$$f_{\mathrm{II}}(\sigma) = \frac{5[\Gamma(1/4)]^2 \left(\frac{1}{\sqrt{\sigma}} + \sqrt{\sigma}\right)^{3/2}}{32\sqrt{2}\pi} \int_0^1 \frac{dx}{x^{1/4}} \times \sqrt{1 - \left(\sigma + \frac{1}{\sigma}\right)x + \sqrt{1 - 2x\left(\frac{1}{\sqrt{\sigma}} - \sqrt{\sigma}\right)^2 + x^2 \frac{1}{\sigma} - \sigma^2}}$$

$$= \begin{cases} 1, & \sigma = 1 \\ \dfrac{1}{4(4\sigma)^{1/4}}, & \sigma << 1 \end{cases}, \tag{9}$$

$$p_F = \sqrt{2\pi n c}; \quad n_{min} = \left(\frac{A_{\mathrm{II}}}{4\pi}\right)^{1/3} c^{-4/3}, \quad E_{min} = -\frac{3}{4}\left(\frac{A_{\mathrm{II}}}{4\pi}\right)^{1/3} c^{-1/3} \approx -\frac{1.57}{c^{1/3}} f_{\mathrm{II}}^{4/3}(\sigma). \tag{10}$$

III. *Multivalley semiconductors and semimetals*, i.e., having several equivalent (by virtue of the crystal symmetry) minima in the electron and (or) hole spectra. The fact that the presence of many valleys increases the role of E_{corr} and the binding energy of the electron-hole liquid has already been noted in a number of papers.[2-5] From the formal point of view, the use of a number of valleys in the electron and hole bands, ν_e and ν_h, as large parameters is analogous to the expansion, known from the theory of phase transition, in terms of the reciprocal of the number of components of the order parameter (the $1/n$ expansion). It justifies the replacement of χ by $\chi^{(0)}$ in (2), and on the other hand it ensures at $n \ll \nu^4$ satisfaction of the condition $p_F \sim (n/\nu)^{1/3} \ll n^{1/4}$ and hence the validity of the asymptotic form (3). It is convenient to define the system of units in this case by the relation

$$m^{1/4} = \frac{[\Gamma(1/4)]^2}{8\pi 2^{1/4}(2\pi)^{3/2}} \int_0^\infty dx \left[\frac{1}{\nu_e} \sum_{k=1}^{\nu_e} \frac{(m_e^{-1})_{ij}^{(k)} n_i n_j}{x^2 + [(m_e^{-1})_{ij}^{(k)} n_i n_j]^2} \cdot \right.$$

$$\left. + \frac{1}{\nu_h} \sum_{k=1}^{\nu_h} \frac{(m_h^{-1})_{ij}^{(k)} n_i n_j}{x^2 + [(m_h^{-1})_{ij}^{(k)} n_i n_j]^2} \right]^{5/4} = 1 , \qquad (11)$$

where $\overline{[\cdots]}$ denotes averaging over the direction of the unit vector $\mathbf{n}$, and $(m_{e,h}^{-1})_{ij}^{(k)}$ is the reciprocal-mass tensor of thek-th valley in terms of the axes of the reciprocal lattice of the crystal. Then

$$A_{III} = \frac{32\sqrt{2}(2\pi)^{3/4}}{5[\Gamma(1/4)]^2} ; \quad n_{min} = \left[\frac{5A_{III} M}{4\pi^{2/3}(3\pi)^{2/3}} \right]^{12/5} ;$$

$$E_{min} = -\frac{5}{8}\left(\frac{5}{4 \cdot 3^{2/3}\pi^{4/3}}\right)^{3/5} A_{III}^{8/5} M^{3/5} , \qquad (12)$$

where

$$M^{-1} = M_e^{-1} + M_h^{-1} ; \quad M_{e,h} = [\det(m_{e,h}^{-1})]^{-1/3} \nu_{e,h}^{2/3} .$$

IV. We present also the result for a semimetal with ν electron valleys and one hole valley in a strong magnetic field directed in equivalent fashion relative to all the electron ellipsoids. Let $m_{e,h\parallel}$ be the effective masses of motion in the field direction, $\sigma = m_{e\parallel}/m_{h\parallel}$, $m^{-1} = m_{e\parallel}^{-1} + m_{h\parallel}^{-1} = 1$. Then

$$A_{IV}(\sigma) = \left(\frac{4}{\pi}\right)^{1/4} \frac{64\pi}{5[\Gamma(1/4)]^2} f_{IV}(\sigma), \qquad f_{IV}(\sigma) = f_I(\sigma) ,$$

$$n_{min} = \left[\frac{A_{IV}(\sigma)(1+\sigma)\nu^2}{1+\nu^2\sigma} \right]^{4/7} \left(\frac{eH}{c}\right)^{8/7} \left(\frac{3}{16\pi^4}\right)^{4/7} ,$$

$$E_{min} = -\frac{7}{8}\left[\frac{3}{16\pi^4} A_{IV}(\sigma) \frac{(1+\sigma)\nu^2}{1+\sigma\nu^2} \right]^{1/7} \left(\frac{eH}{c}\right)^{2/7} .$$

We note in conclusion, as one of the consequences of our results, that semiconductors with a narrow forbidden band E_g pertaining to one of the classes considered above become unstable relative to a first-order phase transition into the semimetallic state as E_{min} approaches E_g, i.e., long before the exciton instability sets in.

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