Trajectories of many-particle Regge poles

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We analyze the t-dependence of the trajectories of the many-particle Regge poles found in our preceding paper {Pis'ma Zh. Eksp. Teor. Fiz. **23**, 588 (1976) [JETP Lett. **23**, 538 (1976)]}. These poles realize the idea of Mandelstam (Comm. on Nuclear and Particle Physics. **3**, 3 (1969), and elsewhere) concerning the role of t-channel many-particle states in the formation of growing Regge trajectories.

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In a preceding paper we have shown[1] with the $g\phi^3$ theory as an example that there exist Regge poles generated by many-particle states in the t channel.

The intercept of these poles, in contrast to the case of Mandelstam branch points (which also result from many particle t-channel states), increases quadratically with the number of particles in the t channel. By the same token, we have a mechanism whereby the Regge poles are shifted to the right in perturbation theory even for zero-spin particles. On the other hand, it has been expected for a long time that a consistent allowance for t-channel multiparticle states should ensure linearity of the Regge trajectories in a rather large interval of t.

The most cogent arguments favoring this assumption were advanced by Mandelstam in 1966.[2]

We show in the present paper that the Regge poles obtained in [1] realize Mandelstam's idea concerning the role played by many-particle t-channel states in ensuring linearity of Regge trajectories.

We start with formula (1) of [1], which gives the leading logarithmic asymptotic diagram for elastic binary scattering with $n \equiv n_1 + n_2$ particles in the t channel (of which n_1 emit and n_2 absorb an arbitrary number of particles) with L horizontal cross lines (q is the transverse momentum in the scattering):

$$\frac{-i\,(s/m^2)^{-n+1}}{(L+m_1)!}\, g^{2(L+n)} \left[\ln \frac{s}{m^2}\right]^{L+m_1} \mathcal{K}(\mathbf{q}) \qquad (m_1 \equiv max(n_1, n_2)), \tag{1}$$

where $\mathcal{K}(\mathbf{q})$ is the so-called "transverse" integral corresponding to the contraction of all the horizontal lines of the diagram into points. $\mathcal{K}(\mathbf{q})$ was investigated in [1] only for the case $\mathbf{q}=0$.

For $\mathbf{q} \neq 0$, we can represent $\mathcal{K}(\mathbf{q})$ in the form

$$\mathcal{K}(\mathbf{q}) = \left(\frac{1}{2(2\pi)^2}\right)^{n-2} \int \prod_{i=1}^{L+1} \frac{d^2 x_i}{2(2\pi)^3} \prod_{i \neq j} K_0(m|x_i - x_j|)\, e^{i \mathbf{q}/n\,(x_i - x_j)}, \tag{2}$$

where the Macdonald function $K_0(m\,|x_i - x_j|)$ describes the "propagation" of a particle with mass m in two-dimensional space for the physical scattering region ($t=-\mathbf{q}^2>0$)[1]. For $t>0$ it is necessary to replace $K_0(m|x|)$ by $i\pi/H_0^{(1)}(m|x|)$, where $H_0^{(1)}(m|x|)$ is a Hankel function of the first kind.

We consider first in (2) the region of not very large $\mathbf{q}(|\mathbf{q}|<nm$. Proceeding in analogy with the estimate of $\mathcal{K}(0)$ in [1], we obtain after summing over all L and taking into account all possible intercepts of the vertical lines the following estimate for the trajectory of a n-particle Regge pole, which is a generalization of formula (3) of [1] to include the case $\mathbf{q}\neq 0$ ($|\mathbf{q}|<nm$):

$$\alpha^{(n_1,n_2)}(\mathbf{q}) = -n+1+\frac{g^2}{16\pi^2}\frac{\gamma^2}{m^2}K_o^2(2\gamma)n_1n_2\left[1-\beta\frac{\mathbf{q}^2}{n^2}\frac{\gamma^2}{m^2}\right], \tag{3}$$

where γ is an arbitrary constant introduced in [1], and β is the result of averaging over the angles in (2).

From (3) we obtain for the slope of the trajectory of the multiparticle Regge pole

$$\alpha'^{(n_1,n_2)} = \frac{4n_1n_2}{(n_1+n_2)^2}\alpha'^{(1,1)}, \tag{4}$$

where $\alpha'^{(1,1)}$ is the slope of the Regge-pole trajectory for a simple ladder with two particles in the t channel.

We note that formulas (3) and (4) are approximate, since it is impossible to obtain exact factors of the type γ and β (i.e., it is impossible to solve the "n-body problem" exactly), but they are accurate in the sense of the dependence of $a^{(n_1,n_2)}$ and $a'^{(n_1,n_2)}$ on n_1 and n_2, which is the most important aspect of the present problem.

It is seen from (4) that the slope of the trajectory of a many-particle Regge pole is determined essentially by the e configurations of the particles in the t channel.

For example, at the most optimal configuration from the point of view of the asymptotic behavior ($n_1=n_2$) we have ($n_1=n_2$)$a'^{(n_1,n_2)}=a'^{(1,1)}$ but $a'^{(n_1,n_2)}\approx n_2/n_1 a'^{(1,1)}\ll a'^{(1,1)}$ at $n_1\gg n_2$. The last case recalls the branch points for which $a_c'(N,N)=1/Na'^{(1,1)}$.

We consider now large $\mathbf{q}(|\mathbf{q}|\gg nm)$. In this case the essential region of integration in (2) is determined by values $|x_i-x_j|$ of the order of $n/|\mathbf{q}|$, and for the trajectory of the (n_1,n_2) pole we obtain

$$\mathrm{Re}\,a^{(n_1,n_2)}(\mathbf{q}) \approx -n+1+n_1n_2n^2\frac{g^2}{16\pi^2}\frac{1}{\mathbf{q}^2}\ln^2\left(\frac{2|\mathbf{q}|}{nm}\right). \tag{5}$$

We present for the sake of completeness an expression similar to (5) for the Mandelstam branch point with $n_1=n_2=N$:

$$\mathrm{Re}\,a_c^{(N,N)}(\mathbf{q}) \approx -2N+1+N^3\frac{g^2}{16\pi^2}\frac{1}{|\mathbf{q}|^2}\ln^2\left(\frac{2|\mathbf{q}|}{Nm}\right).$$

Thus, the trajectories of multiparticle Regge poles with n particles in the t channel have straight-line sections up to the t-channel threshold $\mathbf{q}^2=n^2m^2$, near this point they begin to bend, and with further increase of $|\mathbf{q}|$ they fall off like $\approx n^4/\mathbf{q}^2\ln^2 2|\mathbf{q}|/nm$, approaching asymptotically the value $-n+1$.

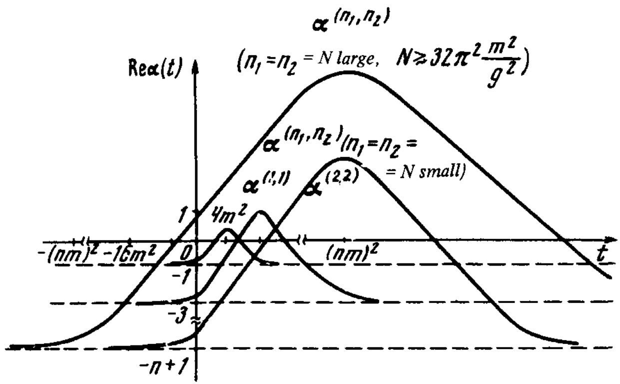

FIG. 1.

The conclusions agree with the picture expected in [2]. This is illustrated in the figure.

[1)]It is important to bear in mind that only four such propagators enter each point x_i.

[1]S. G. Matinyan and A. G. Sedrakyan, Pis'ma Zh. Eksp. Teor. Fiz. 23, 588 (1976) [JETP Lett. 23, 538 (1976)].

[2]S. Mandelstam, 1966 Tokyo Summer Lectures in Theoretical Physics, Syokabo, Tokyo and Benjamin, NY, 1967; Comm. on Nucl. and Part. Phys. 3, 3 (1969).