Critical charge for anomalous nuclei

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The dependence of the critical charge on the density of nuclear matter and on the ratio Z/A is calculated.

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The critical charge of the nucleus Z_{cr} and the problem of spontaneous production of positrons at $Z > Z_{cr}$ were considered recently in a number of papers (a discussion of various aspects of this problem and references to the literature can be found in the reviews [1,2]). The customarily cited values of Z_{cr} pertain to normal density of nuclear matter $n_0 = 3/4\pi r_0^3$, $r_0 = 1.1$ F. Yet there are theoretical indications that anomalous nuclei can exist with a density substantially different from n_0. Migdal [3] has investigated the problem of the stability of vacuum and pair production in critical fields for Bose particles. The possibility of a phase transition with production of a π condensate in nuclear matter with $N = Z$ and in a neutron medium ($N \gg Z$) was demonstrated; the possible existence of superdense, neutron, and supercharged ($Z \gtrsim 137^{3/2}$) nuclei has also been discussed.[4] Other arguments favoring the existence of stable superdense ($n = (2-5)n_0$) nuclei were advanced by Lee and Wick.[5,6] It appears that superdense states of nuclear matter can be obtained in experiments with collisions of two heavy nuclei.

In connection with these studies, we have calculated the critical charge Z_{cr} for anomalous nuclei, i.e., we calculated the dependence of Z_{cr} on the density of nuclear matter and on the ratio Z/A.

Let us describe briefly the method of calculating Z_{cr}. At $\zeta > 1$ it is necessary to take into account the finite dimensions of the nucleus [7], i.e., to solve the Dirac equation with a Coulomb potential cutoff at short distances:

$$V(r) = \begin{cases} -\dfrac{\zeta}{R} f(r/R), & 0 < r < R \\ -\zeta/r, & r > R \end{cases} \tag{1}$$

$$(\hbar = c = m_e = 1, \quad e^2 = 1/137, \quad \zeta = Ze^2).$$

The Dirac equation at an energy $\epsilon = -1$ (the boundary of the lower continuum) and at $V(r) = -\zeta/r$ has an exact solution that can be expressed in terms of Bessel functions with imaginary index. From this follows an equation for the critical charge of the nucleus [8]

$$zK'_{i\nu}(z)/K_{i\nu}(z) = 2\xi, \tag{2}$$

where $z = (8\zeta_{cr}R)^{1/2}$, $\zeta_{cr} = Z_{cr}e^2$, R is the radius of the nucleus in units of $\hbar/m_ec = 386$ F, $\nu = 2[\zeta_{cr}^2 - (j+\frac{1}{2})^2]^{1/2}$, $K_{i\nu}(z)$ is a Macdonald function, and ξ is the logarithmic derivative of the internal ($r < R$) wave function at the edge of the nucleus.

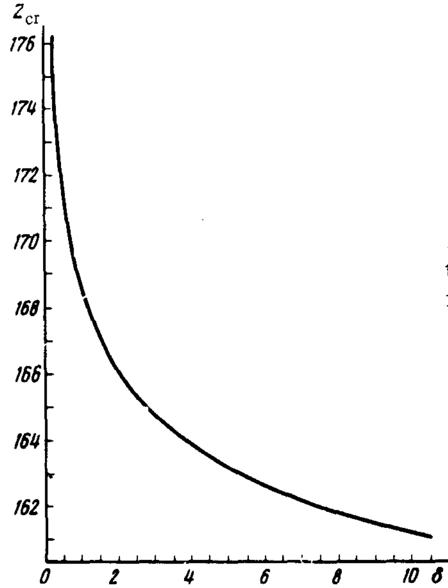

FIG. 1. Critical charge of the nucleus for the ground level $1s_{1/2}$. The abscissas represent the density ratio $\delta = n_p/n_p^{(0)}$.

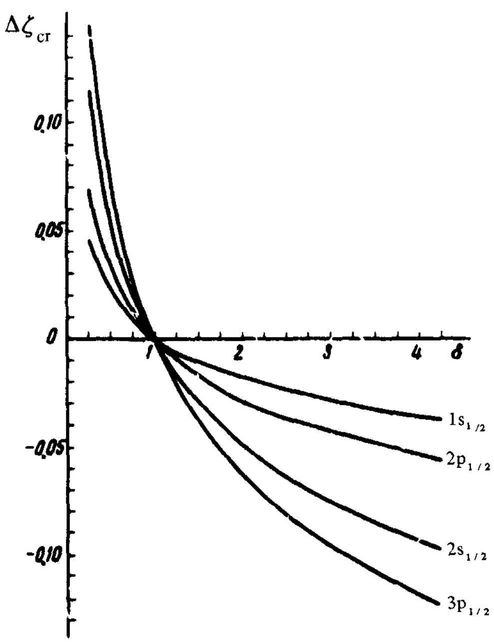

FIG. 2. Plots of $\zeta_{cr} = Z_{cr}/137$ against the density of nuclear matter. The ordinates represent the quantity $\Delta\zeta_{cr} = \zeta_{cr}(\delta) - \zeta_{cr}(1)$; the values of $\zeta_{cr}(1)$ for normal nuclear density are listed in the table.

Level	ζ_{cr}	Z_{cr}
$1s_{1/2}$	1.232	168.8
$2p_{1/2}$	1.323	161.3
$2s_{1/2}$	1.691	231.7
$3p_{1/2}$	1.858	254.5

Equation (2) was solved for the first four levels of the discrete spectrum at $f(r/R) = \frac{1}{2}(3 - r^2/R^2)$; this choice of the cutoff function in (1) corresponds to a uniform volume density of the nuclear charge: $n_p(r) = n_p\theta(R - r)$. As a result we obtain the curve $\zeta_{cr} = \zeta_{cr}(R)$ for the corresponding level. On the other hand $\zeta = aR^3$, where $a = 4\pi e^2 n_p/3$. The intersection of these two curves determines the critical charge of the nucleus at a given proton density n_p.

The results of the numerical calculations are shown in Figs. 1 and 2. The value of Z_{cr} decreases with increasing n_p and in the limit $n_p \gg n_p^{(0)}$ it approaches 137 (the critical charge for a pointlike nucleus); here $n_p^{(0)} = 0.385$, where n_0 is the density of the protons in ordinary heavy nuclei). However, this approximation is quite slow, as is seen from the asymptotic form:

$$\zeta_{cr} \approx 1 + \frac{c_1}{(\ln\delta + c_2)^2}, \quad \delta \to \infty \tag{3}$$

where $\delta = n_p/n_p^{(0)}$, $c_1 = 9\pi^2/2$, and $c_2 = 14.7$. For excited levels the dependence of Z_{cr} on n_p is much stronger (see Fig. 2). The values of Z_{cr} for the normal nuclear density $n_p^{(0)}$ are listed in the table.

We now estimate the extent to which Z_{cr} varies when account is taken of the diffuseness of the edge of the nucleus. Choosing the density of the protons in the form

$$n_p(r) = C\left\{1 + \exp\left(\frac{r - R}{a}\right)\right\}^{-1}, \tag{4}$$

where the constant $C = [1 - (\pi a/(R)^2] n_p^{(0)}$ at $R \gg a$, and calculating the level shift $\Delta\epsilon$ by perturbation theory, we arrive at the formula

$$\Delta\epsilon = \frac{Ze^2}{R}\left(\frac{\pi a}{R}\right)^2 \int_0^R \left(1 - \frac{r^2}{2R^2}\right)(G^2 + F^2)\,dr, \tag{5}$$

Here $G(r)$ and $F(r)$ are the radial functions for the upper and lower component of the Dirac bispinar at $\epsilon = -1$, normalized by the condition

$$\int_0^\infty (G^2 + F^2)\,dr = 1.$$

This normalization condition, which is usual for states of the discrete spectrum, is preserved also at the edge of the lower continuum. [8] Calculation by formula (5) shows that the diffuseness of the edge of the nucleus raises Z_{cr} by approximately 7% (for the ground level and at $n_p = n_p^{(0)}$).

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