NONLINEAR INTERACTION OF ELECTROMAGNETIC AND ACOUSTIC WAVES IN III-V SEMICONDUCTORS AND THE POSSIBILITY OF HYPERSOUND GENERATION

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The previously uninvestigated nonlinear interaction between electromagnetic and acoustic waves in III-V piezosemiconductors is considered. Under suitable conditions, the investigated nonlinearity can be used to develop parametric hypersonic generators up to frequencies on the order of 10^{11} sec^{-1}.

1. The interaction of electromagnetic and acoustic waves of comparable frequencies in solids is at present a very timely problem, particularly in connection with the possibility of the developing parametric acoustic amplifiers and generators.

This communication deals with the previously uninvestigated nonlinear interaction of electromagnetic and acoustic waves in III-V piezosemiconductors; this interaction is described by a nonlinear polarization proportional to the electric field and quadratic in the acoustic wave. Under certain conditions, this nonlinearity can be used to construct parametric hypersonic generators up to frequencies on the order of 10^{11} sec^{-1}. As will be shown below, in the case of n-InSb this nonlinearity, at rather low values of the acoustic flux, can predominate over the nonlinearity determined by the photoelastic constant.

2. We consider the nonlinear effect determined by the nonlinear polarization[1])

$$P_a^{NL}(\mathbf{r},t) = \chi_{ab}^{cd,fg}\begin{pmatrix}\omega, & \omega_1, & \omega_2\\ \mathbf{k}, & \mathbf{q}_1, & \mathbf{q}_2\end{pmatrix} E_b(\omega,\mathbf{k}) S_{cd}(\omega_1,\mathbf{q}_1) S_{fg}(\omega_2,\mathbf{q}_2)$$

$$\times \exp\{ i[(\omega+\omega_1+\omega_2)t - (\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2)\mathbf{r}]\}, \qquad (1)$$

E is the electric field, S_{cd} is the strain tensor. The complete system of equations consists of the equations of elasticity theory, the Poisson equation, the

[1])Formula (1) implies summation over all repeated indices, frequencies, and wave vectors.

continuity equation, and the kinetic equation for the electron distribution function, which determines the current (and accordingly the polarization). In the linear approximation, we have the conduction current and the diffusion current. In the kinetic equation, the field term includes the given electric field and the field generated by the sound wave. Since in a piezosemiconductor the sound wave is accompanied by an electric field, the considered nonlinear mixing of acoustic and electric fields to produce nonlinear polarization is in essence a nonlinear effect of mixing three electric fields. On the other hand, it is well known [1] that in the low-frequency limit $\hbar\omega \ll E_g$ the cubic electronic nonlinearity differs from zero and is determined by the fourth derivative of $\varepsilon(k)$. Here E_g is the width of the forbidden band and $\varepsilon(k)$ is the energy spectrum of the electrons. Accordingly, the cubic electronic nonlinearity for III-V semiconductors with a Kane dispersion law is quite large [2] also in the microwave band. Integrating accurate to cubic terms with respect to the fields we can easily obtain the following approximate formula for the mechanism of the nonlinearity due to the non-parabolicity[2)] of the conduction band in the semiconductors

$$\chi_{ab}^{cd,fg} \approx \left(\frac{2\pi}{\epsilon_0}\right)^2 \frac{n_0 e^4}{m^2 E_g (\omega + \omega_1 + \omega_2)(\nu_p + i\omega)\omega_1\omega_2} \left(\frac{v_s}{v_T}\right)^2 \sum_{\ell, t} \beta_{\ell cd}\beta_{tfg}$$

$$\times (\delta_{b\ell} \; \delta_{\ell t} + \delta_{\ell t}). \qquad (2)$$

Here v_s is the speed of sound, v_T is the characteristic electron velocity, n_0 is the equilibrium electron concentration, $\beta_{ik\ell}$ is the piezotensor, ν_p is the momentum relaxation frequency, δ_{ik} is the Kronecker symbol. In (2) we have $\kappa < q < 6v_T\kappa^2/\nu_p$ and $q\ell > 1$, where κ is the reciprocal radius and ℓ is the electron mean free path.

The nonlinearity under consideration determines the parametric interaction of two acoustic waves in the presence of an electric pump field E_H of frequency ω_H. Indeed, the nonlinear polarization P_{NL} determines the increment to the free energy of the crystal (see, e.g., [4]), which in turn introduces nonlinear terms into the equations of motion for S_1 and S_2. From this we readily obtain [4] the following abbreviated equations of first order for the amplitude[3)] S_1 and S_2 of the acoustic waves $S_1(x)\exp[i(\omega_1 t - q_1 x)]$ and $S_2(x)\exp[i(\omega_2 t - q_2 x)]$

$$\frac{\partial S_1}{\partial x} + \alpha S_1 = i\frac{3q_2^2\chi}{cq_1}[|E_H|^2 S_1 + E_H^2 S_2^* \exp(i\Delta q x)],$$

$$\frac{\partial S_2}{\partial x} + \sigma S_2 = i\frac{3q_1^2\chi}{Cq_2}[|E_H|^2 S_2 + E_H^2 S_1^* \exp(i\Delta q x)], \qquad (3)$$

where α is the nonlinear loss, C is the modulus of elasticity, and $\Delta q = q_1 - q_2$. Such equations describe the gain of one signal at the expense of the other, and

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2) Of course, there exists also a nonlinearity due to heating and distortion of the equilibrium distribution function [3]. These nonlinearity sources are not analyzed in this communication and we plan to do this in the future. The purpose of the present communication is to call attention to the nonlinearity in question and to its possible uses.

3) We do not consider here the one-dimensional case. For simplicity, we have omitted the tensor indices throughout.

if the coefficients of the nonlinear terms exceed the losses, generation of oscillations takes place. The frequencies of the amplified or generated oscillations satisfy the conditions

$$\omega_1(q_1) + \omega_2(q_2) = 2\omega_H . \tag{4}$$

Equations (3) coincide with the equations for coupled Stokes and anti-Stokes components in stimulated Raman scattering [4]. The amplification (generation) occurs at $\Delta q \neq 0$, as determined by the self-action terms proportional to $|E_H|^2$ in (3). Since $\Delta q \ll q_1, q_2$, we can put approximately $\omega_1 \sim \omega_2 \sim \omega_H$ in (4). The minimum length L_m at which generation takes place satisfies the condition (at $q_1 \sim q_2$)

$$\left[\frac{3q\chi E_H^2}{C}\exp(i\Delta q L_m) - \alpha\right] L_m \gtrsim 1 . \tag{5}$$

3. Let us make some estimates and comparisons. For frequencies $\omega \sim \omega_1 \sim \omega_2 \sim 10^{10}$ sec^{-1} for n-InSb at $n_0 \sim 10^{15}$ cm^{-3}, T $\sim$ 77°K, $\nu_p \sim 2 \times 10^{11}$ sec^{-1} we obtain $\chi \sim 5 \times 10^9$. All the estimates given below pertain to n-InSb at $n_0 \sim 10^{15}$ cm^{-3} and T $\sim$ 77°K. There exists in the crystals a nonlinear polarization proportional to the product of the electric field and the strain tensor and determined by the photoelastic constant p. For III-V semiconductors we have $p < 1$, so that for n-InSb at $\omega \sim 10^{10}$ sec^{-1} and at $S \gtrsim 5 \times 10^{-9}$ (the acoustic power flux $\Pi_{ac} \gtrsim 10^{-10}$ W/cm^2) the nonlinearity considered in the present paper exceeds the photoelastic nonlinearity. This is due to the fact that this nonlinearity corresponds in essence to the quadratic nonlinearity determined by the interband electronic transitions, and therefore is independent of the frequency at $\hbar\omega < E_g$, whereas the nonlinearity considered by us is cubic and is determined by intraband motion, and therefore increases rapidly with decreasing field frequencies.

a) We consider the mixing of acoustic and electric fields in a resonator with figure of merit Q, tuned to a nonlinear polarization frequency $\omega_\Sigma = \omega + \omega_1 + \omega_2 \sim 3\omega$ at $\omega \sim \omega_1 \sim \omega_2$. For a radiation power P in a waveguide coupled to the resonator with a coupling coefficient k, it is easy to obtain the following estimates

$$P \sim k Q \omega_\Sigma \chi^2 \frac{E^2 \Pi_{ac}^2}{(\rho_0 v_s^3)^2} a V_0 , \tag{6}$$

where ρ_0 is the crystal density, a is the filling factor of the resonator, V_0 the volume of the working medium. For $\omega \sim 10^{10}$ sec^{-1} we obtain for a piezo-active sound wave with $v_s \simeq 2.2 \times 10^5$ cm/sec, for $Q \sim 10^3$, $k \sim 0.5$, and $aV_0 \sim 10^{-3}$ cm^{-3} (with allowance for the skin effect) we obtain a radiation power in the waveguide $P \sim 3 \times 10^{-4}$ W at $\Pi_{ac} \sim 10^{-3}$ W/cm^2 and E $\sim$ 1 cgs esu.

b) Let us estimate the possibility of parametric sound generation. For frequencies $\omega \sim 5 \times 10^{10}$ sec^{-1} we have $\chi \sim 8 \times 10^7$ and then $3qE_H{}^2\chi/C \sim 180$ cm^{-1} at E $\sim$ 1 cgs esu; for $\omega \sim 10^{11}$ sec^{-1} we have $\chi \sim 10^7$ and $3qE_H{}^2\chi/C \sim 100$ cm^{-1} at E $\sim$ 1.5 cgs esu. Therefore the generation condition can be satisfied at such frequencies if $\alpha < 10^2$ cm^{-1}.

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