THRESHOLD PHENOMENA IN METALS WITH OPEN FERMI SURFACES

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Submitted 6 February 1973
ZhETF Pis. Red. 17, No. 6, 324 - 328 (20 March 1973)

A well-known conclusion, pertaining to electrodynamics of metals with open Fermi surfaces, is that undamped electromagnetic excitations with frequencies below the plasma frequency cannot exist on them in the presence of open orbits [1, 2]. Such a categoric prohibition is surprising, if it is recognized that the presence of open trajectories does not prevent clearly pronounced threshold phenomena such as acoustic cyclotron resonance [3] and magnetoacoustic resonance [4][1)].

The nonlocal conductivity singularities near the thresholds should lead to a number of resonant phenomena: the metal can become transparent in a definite frequency interval, and anomalies can occur in the dispersion and absorption of the sound. The present paper is devoted to a study of these questions. By way of a model we use a simple example of a Fermi surface of the "corrugated cylinder" type, which is realized in uniaxial metals or cubic metals with Fermi surface of the copper type, if the magnetic field is directed along [110]. In this case for narrow "neck" trajectories, the electron dispersion law takes the form [7]:

$$\epsilon_n(k_y, k_z) = \epsilon_n^0(k_z) + (-1)^{n+1}\frac{\hbar\Omega}{\pi}\arcsin\left(\frac{1}{a}\cos\frac{\pi k_y}{k_0}\right), \tag{1}$$

where $a = [1 + \exp(-\pi k_1{}^2 \ell_H{}^2\sqrt{\rho/k_1})]^{1/2}$, $\vec{H} \parallel z$, $\Omega(k_z)$ is the cyclotron frequency determined in accordance with the usual rules of quasiclassical quantization [8], ℓ_H is the magnetic length, $\hbar k_0 = \hbar a_x/2\ell_H{}^2$ is the limiting momentum of the magnetic zone, k_1 is the radius of the "neck," ρ is the radius of curvature of the trajectory on the boundary of the reciprocal-lattice neck (it is assumed that the trajectories in $\vec{k}$-space are open in the k_x direction), and a_x is the lattice period in the direction of the x axis; the centers of the magnetic zones (1) are obtained by solving the equation $S(\epsilon, k_z) = 2\pi(n + \gamma)\ell_H{}^2$, where S is the area bounded by the open trajectory and the boundaries of the reciprocal-lattice cell boundaries in the case of an open trajectory, or bounded only by the trajectory in the case of closed trajectories.

The regions of collisonless Landau damping, on the boundaries of which the susceptibilities (polarization operator, conductivity, etc.) have singularities, are obtained from the energy and momentum conservation laws and from the Pauli principle. Using (1) we can show that the boundaries of the Landau-damping

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1)A quantum theory of sound propagation in the presence of open trajectories was constructed in [5, 6].

regions, for arbitrary direction of wave propagation, are determined by the expression

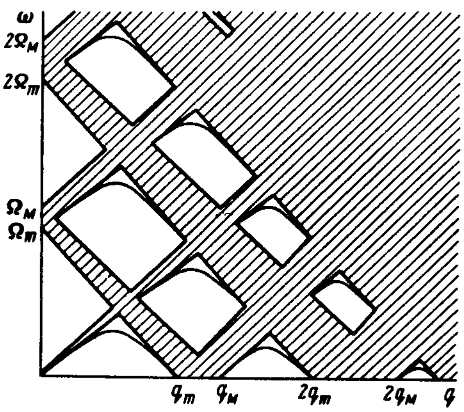
$$\omega = N\Omega_{extr} \pm v_z \cos\theta\, q \pm v_y \sin\theta \sin\phi\, q. \tag{2}$$

Here ω and q are the frequency and wave vector of the wave, θ is the angle between $\vec{q}$ and $\vec{H}$, ϕ is the angle between the projection $q \sin\theta$ of the vector $\vec{q}$ in the direction k_x of the opening, N is the number of the cyclotron-resonance harmonic, v_y is the component of the average velocity along the y axis, equal to $v_y = \Omega_{extr}/k_0 \sim v_F$ when $a - 1 \ll 1$, v_z is the maximum electron velocity along H, and Ω_{extr} are the extremal values of the cyclotron frequency; we shall assume that there exist three extrema, with $\Omega_M - \Omega_m < \Omega_m$, $\Omega_M = \Omega(0)$, $\Omega_m = \Omega(k_{zM})$, and the role of the saddle point, where $\Omega \to 0$ logarithmically [8], will be discussed later. The subscripts m and M denote respectively the minimum and maximum values. Generally speaking, v_y and v_z are complicated functions of x_z and x_y, and Ω depends on k_z. In the derivation of (2) we took into account, however, the fact that after averaging over the entire collective, the electrons singled out are those with extremal values of the parameters. The Landau-damping regions are shown in Fig. 1, where

$$q_m = \Omega_m/(v_{yM} \sin\theta \sin\phi + v_z \cos\theta),$$

$$q_M = \Omega_M/(v_{ym} \sin\theta \sin\phi - v_z \cos\theta).$$

As seen from the figure, transparency "windows" appear in the Landau-damping regions at $\tan\theta > v_z/v_{ym}\sin\phi$ and $\phi \neq 0$. The form and number of these windows depend on the function $\Omega(k_z)$ at $\theta = \pi/2$. When the angle ϕ varies from $\pi/2$ to zero, the damping regions degenerate into cyclotron-absorption bands. If the angle ϕ is varied from $\pi/2$ at $\phi \neq 0$, then the "windows" disappear at $\tan\theta_{cr} = v_z/v_{ym}\sin\phi$, and at $\theta < \theta_{cr}$ the absorption regions will be the same as in the case of a closed Fermi surface. Obviously, the existence of thresholds at $\omega \to 0$ and $\theta = \pi/2$ (see Fig. 1) causes magnetoacoustic oscillations [4], and at $\omega = N\Omega_{extr} \pm v_y q$ ($N = \pm 1, \pm 2 \ldots$) and $\theta > \theta_{cr}$ it causes acoustic cyclotron resonance [5].



Landau damping regions and schematic forms of dispersion relations for electromagnetic waves in metals with open Fermi surfaces.

It follows from our analysis that other threshold phenomena can also accompany the propagation of sound. For example, the phase relations are satisfied at the angle θ defined by the relation $v_{ym}\sin\theta\sin\phi - v_z\cos\theta = s$, and a phenomenon of the tilt-effect type [9] appears (s is the speed of sound, $\theta > \theta_{cr}$). In the angle interval $\Delta\phi = sk_0(1 - \Omega_m/\Omega_M)/\Omega_m \sim 1°$ ($\theta = \pi/2$), a sharp increase of the sound absorption coefficient takes place, proportional to the density of the electrons moving along the open orbits, and the tilt effect is produced at both limits of this interval.

Let us consider the region of higher frequencies. The conductivity has singularities on the boundaries of the numerous transparency "windows," and this causes the existence of electromagnetic excitations in metals with open Fermi surfaces. Let us demonstrate this analytically. If $\theta = \pi/2$ and $\phi = \pi/2$, then the equation for the ordinary wave with respect to the small parameter $\nu/\Omega \ll 1$ can be separated from the equation for the coupled longitudinal and extraordinary waves:

$$\frac{4\pi i}{\omega}\sigma_{xx} = \frac{c^2q^2}{\omega^2} \tag{3}$$

$$\frac{4\pi i}{\omega}(\sigma_{zz} - \sigma_{yz}^2/\sigma_{yy}) = \frac{c^2q^2}{\omega^2} \tag{4}$$

$$\sigma_{ik} = \frac{ie^2}{2\pi^2\hbar^2}\sum_{n=-\infty}^{+\infty}\sum_{\ell}\int_0^{\infty} d\epsilon\,\delta(\epsilon-\epsilon_F)\frac{1}{2k_0}\int_{-k_0}^{k_0}dk_y\int_{-\infty}^{+\infty}dk_z$$

$$\times\frac{V_{\ell ni}(k_z,k_y)V_{\ell nk}^{*}(k_z,k_y)}{\omega - n\Omega(k_z) + (-1)^{\ell}v_y(k_z,k_y)q + i\nu}$$

Here ν is the collision frequency, the asterisk denotes the complex conjugate, and $V_{n\ell i}(k_z, k_y)$ is the quasiclassical "matrix" element, equal, e.g., to

$$V_{\ell nx}(k_z,k_y) = \frac{1}{T_{cl}}\int_0^{T_{cl}}dt\,v_x\exp\left[iq\int_0^t(v_y-\bar{v}_y)dt_1 - in\Omega t\right] \tag{5}$$

where $\bar{v}$ is the average velocity.

Analysis shows that when the conditions

$$\nu \ll n\Omega_m - v_{yM}q - \omega \ll \omega \qquad \nu \ll n\Omega_m + v_{ym}q - \omega \ll \omega \tag{6}$$

are satisfied and $q \gg k_0$ ($\Omega_M - \Omega_m \ll \Omega_m$, k_0^{-1} replaces the Larmor radius in such a geometry), the asymptotic solutions of (3) take the form

$$\omega = N\Omega_m\left[1 - \frac{q}{k_0} - \alpha\left(\frac{\delta}{k_0}\right)^4\left(\frac{k_0}{q}\right)^6\right] - i\nu \tag{7}$$

$$\omega = N\Omega_m\left[\frac{q}{k_0} - \beta\left(\frac{\delta}{k_0}\right)^4\left(\frac{k_0}{q}\right)^6\right] - i\nu, \tag{8}$$

where $\alpha \sim 1$, $\beta \sim 1$, $\delta = \omega_p/c$, and ω_p is the plasma frequency. The solutions of (4) depend on the locations of the extrema of the function $\Omega(k_z)$ and on the sign of the difference $V^2_{\ell nz} - (V_{\ell ny}V_{\ell nz})^2/V^2_{\ell ny}$, and cannot be obtained in general form. The schematic form of the dispersion curves is shown in Fig. 1.

A feature of the obtained solutions is their degeneracy - one value of ω corresponds to several values of q. The degree of degeneracy at $\theta = \pi/2$ depends on the form of the function $\Omega(k_z)$. The most interesting among the obtained solutions is apparently the one near the thresholds $\omega = \Omega_m - v_{yM}q$ and $\omega = v_{yM}q$ at $q \lesssim k_0$. It is the analog of the doppleron [10], which exists in metals with closed Fermi surfaces.

To observe weakly-damped electromagnetic waves it is also necessary to satisfy the conditions (6) in addition to having a sufficiently low temperature $kT \ll h\Omega_m$. The contribution made to the collisionless damping by self-intersecting trajectories will be small if $k_1 \sim \ell_H^{-1}$ or, if $1 \ll k_1\ell_H \ll \sqrt{\ell_H/a_x}$, will be smaller the better the right-hand side of conditions (6) is satisfied.

The authors thank A.V. Gaponov, V.Ya. Demikhovskii, A.A. Andronov, V.M. Genkin, and G.M. Genkin for a discussion of the results.

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