Manifestation in the reaction $ed \to enp$ of relativistic effects in the deuteron

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The description of the deuteron by means of a relativistic wave function at the light front increases the theoretical value of $d^2\sigma/d\Omega dE'$ for the reaction $ed \to enp$ in the region $1/2 < x < 1$ and eliminates the existing discrepancy with experiment [P. Bosted *et al.*, Phys. Rev. Lett. **49**, 1380 (1982)]. At $0 < x < 1/2$, on the other hand, the cross section is predicted to decrease. These two qualitative effects are manifestations of one extremely common property of a nonsimultaneous wave function.

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Bosted *et al.*[1] have shown that data on the electro-disintegration of the deuteron, $ed \to enp$, confirm the so-called y-scaling.[2,3] The y-scaling, like Bjorken scaling, is associated with the impulse approximation, which holds at high relative energies in the np system. The cross section for the reaction.

$$\frac{d^2\sigma}{d\Omega' dE'} = \left(\frac{d\sigma_p}{d\Omega} + \frac{d\sigma_n}{d\Omega}\right)\left(\frac{dE'}{dy}\right)^{-1} F(y) = \left(\frac{d\sigma_p}{d\Omega} + \frac{d\sigma_n}{d\Omega}\right)\frac{m^2 E}{2E' q_L}\, I \tag{1}$$

is proportional to the following integral, according to Bosted *et al.*[1]:

$$I(y) = \int_{|y|}^{|y+q_L|} \psi^2(k)\,\frac{k\,dk}{\epsilon(k)}\,. \tag{2}$$

The variable y is minimum momentum of the spectator nucleon in the laboratory frame, set by energy conservation, $E + 2m = E' + \epsilon(\mathbf{q}_L - y) + \epsilon(y)$ [E and E' are the energies of the electron before and after the scattering, $\mathbf{q}_L$ is the 3-momentum transfer in the laboratory frame, and $\epsilon(y) = \sqrt{m^2 + y^2}$], with $\cos \widehat{\mathbf{y}\,\mathbf{q}_L} = -1$ we thus find

$$y(Q^2, \nu) = -\frac{q_L}{2} + \left(\frac{\nu}{2} + m\right)\left[1 - \left(1 + \frac{\nu}{m}(1 - x)\right)^{-1}\right]^{1/2}, \tag{3}$$

where

$$q_L = \sqrt{\nu^2 + Q^2}, \quad Q^2 = -(k_e - k'_e)^2, \quad \nu = E - E', \quad x = Q^2/4m\nu\,.$$

The data[1] on the function $F(y)$ [see Eq. (1)] in the range $0.8 \leqslant Q^2 \leqslant 6$ (GeV/c)^2 conform to a single curve (the hatched region in Fig. 1). This curve was used along with Eqs. (1) and (2) by Bosted *et al.*[1] to extract a deuteron wave function which exceeds the theoretical predictions at $k > 0.2$ GeV/c; in Fig. 1, this would mean that the data for $y < -0.2$ GeV/c lie above the dashed curve, calculated from Eqs. (1) and (2) with the

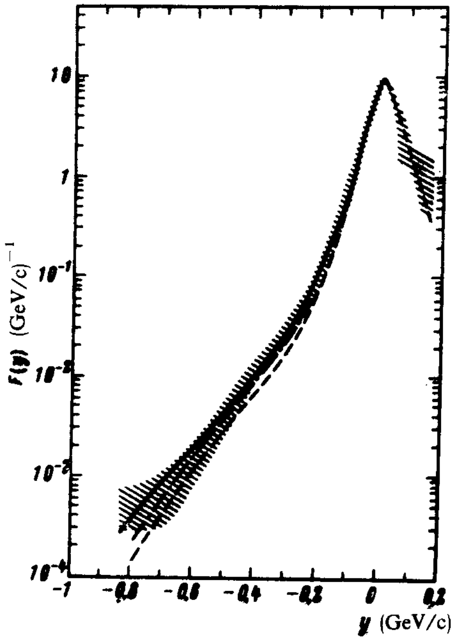

FIG. 1.

Paris wave function[4] and with the asymptotic value of the coefficient in (1) [Eq. (5) of Ref. 1]. Ableev *et al.*[5] have attempted to related this discrepancy to a six-quark structure of the deuteron.

In this paper we wish to make the point that in the relativistic region Eq. (2) does not correctly incorporate the contribution of the two-nucleon state in the deuteron, since it corresponds to a description of the deuteron by means of the Feynmann vertex $d \to NN$, which is not directly related to the two-nucleon component, and the correct equation has the cross section expressed in terms of the wave function at the light front. It is the wavefront at the light front which gives us the most systematic description of bound systems, as is confirmed by the parton model, for example. The data of Ref. 1 fall in an intermediate region where although relativistic effects are important, the values of Q^2 and ν are not yet so large that the asymptotic expresssions of the parton model can be used. The cross section for the disintegration $ed \to enp$ in light-front dynamics is discussed in Refs. 6 and 7, where the y-scaling problem is not taken up.

We wish to show that calculations in light-front dynamics increase the cross section at $y<0$ and eliminate the discrepancy between theory and experiment, while at $y>0$, in contrast, they reduce the cross section. Both of these effects are direct manifestations of a single, extremely common property of the wave function at the light front, as has been discussed previously.[8,9]

The property, paradoxical at first glance, can be described by saying that in a nonsimultaneous wave function the rest frame of the deuteron is not the same as the rest frame of the center of mass of the nucleons making up the deuteron. We will use a method developed in Refs. 8 and 9. in which the wave functions are determined on the

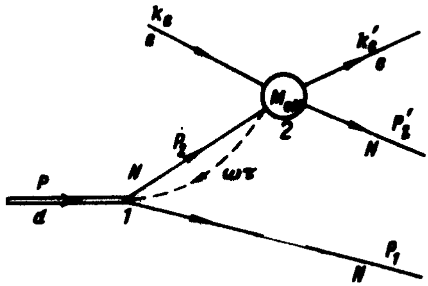

FIG. 2.

invariant light-front surface: $\omega x = 0$, $\omega = (\omega_0, \vec{\omega})$, $\omega^2 = 0$. Figure 2 shows a diagram of the disintegration $ed \to enp$; the dashed arc corresponds to a "shpurion."[10] According to Ref. 8, the wave function is $\psi = \psi(p_1, p_2, p, \omega\tau)$, where p, p_1, and p_2 are the 4-momenta of the deuteron and the nucleons; $p^2 = M_d^2$; $p_1^2 = p_2^2 = m^2$ $(\omega\tau)^2 = 0$, $\tau > 0$; and the arguments of the wave functions are related by (see vertex 1 in Fig. 2)

$$p_1 + p_2 = p + \omega\tau. \tag{4}$$

We see from (4) that if $\mathbf{p} = 0$, then $\mathbf{p}_1 + \mathbf{p}_2 \neq 0$. Introducing the variable $\mathbf{q}$, which is momentum of nucleon 1 in the c.m. frame of the np pair (the rest frame of the intermediate nucleons in Fig. 2), and the vector $\mathbf{n}$, which is the direction of $\vec{\omega}$ in this frame,[8] we can write the wave function as $\psi = \psi(q, n)$.

Our calculation of the $ed \to enp$ cross section consists primarily of transforming an integral over a phase volume to the variable $\mathbf{q}$, the argument of the wave function. We impose the condition $\omega Q = \omega(k_e - k_e') = 0$. This condition suppresses the production of $N\bar{N}$ pairs by a virtual γ ray (cf. Refs. 6 and 7). The elementary cross sections are taken through the integral sign, as in Refs. 1–3. The unknown cross section is given by expression (1), where

$$I = q_L \int_{|q_1|}^{|q_2|} \int_0^{2\pi} \frac{|\psi(\mathbf{q}, \mathbf{n})|^2}{1 + \mathbf{nq}/\epsilon(\mathbf{q})} \frac{d\phi}{2\pi} \frac{q\,dq}{\epsilon(\mathbf{q}) D(\mathbf{q})}, \tag{5}$$

Here $D(q) = [Q^2 + (m\nu - q^2)^2/\epsilon^2(q)]^{1/2}$, and the integration over $d\phi$ is carried out with the help of the formulas

$$\cos \widehat{\mathbf{nq}} = \cos\theta_n \cos\theta_q + \sin\theta_n \sin\theta_q \cos\phi,$$

$$\cos\theta_n = \frac{m\nu - \mathbf{q}^2}{\epsilon(\mathbf{q}) D(\mathbf{q})}, \quad \cos\theta_q = \frac{Q^2/2 + \mathbf{q}^2 - m\nu}{qD(\mathbf{q})}.$$

The limits on the integration in (5) are determined by

$$q_{1,2}^2 = 2\nu^2 x(1-x)\left[1 \mp \sqrt{1 + \frac{m(m+\nu)}{\nu^2 x(1-x)}}\right] + m\nu. \tag{6}$$

The most important distinction between (5) and (2) arises from the difference between the integration at the lower limits $|q_1|$ *and* $|y|$. *At* $y<0 (1/2<x\leqslant 1)$, we find $|q_1|<|y|$ from a comparison of (3) and (6); the result is to make the value of I calculated from (5) larger than that calculated from (2). At $y>0$ $(0\leqslant x<1/2)$ we have the opposite condition, $q_1>y$, so that I and the cross section are decreased. The effects can be seen particularly clearly in the ultrarelativistic limit, Q^2, $\nu\rightarrow\infty$, where we have, as $x\rightarrow 1$, $|q_1|=m/2\sqrt{1-x}<|y|=m/4(1-x)$, while as $x\rightarrow 0$ we have $q_1=m/2\sqrt{x}>y=3m/4$. These relationships between q_1 and y are a consequence of the same property of the wave function at the light front which we were discussing above. It can be seen from the equation $\omega Q=0$ that $\vec{\omega}$ and $\mathbf{q}_L$ are nearly parallel ($\cos\vec{\omega}\,\hat{q}_L \approx 1-2mx/\upsilon\rightarrow 1$ as $\upsilon\rightarrow\infty$). From (4) with $\mathbf{p}=0$ we find $\vec{\omega}\tau=\mathbf{p}_1+\mathbf{p}_2$. It follows that the c.m. frame of the np pair is moving along $\vec{\omega}$, i.e., nearly along $\mathbf{q}_L$. The transformation to this frame reduces the momentum of the spectator nucleon, which is emitted along $\mathbf{q}_L$ in the case $y<0$; i.e., $|q_1|<|y|$. At $y>0$ this nucleon is emitted in the opposite direction, and we find $q_1>y$.

Figure 1 shows the results of numerical calculations of the function F [see (1)]. The solid curve is calculated from Eqs. (1) and (5) with $Q^2=6(\mathrm{GeV}/c)^2$ and the Paris wave function.[4] We are at this point ignoring the $\mathbf{n}$ dependence of the wave function. The dot-dashed curve shows the results calculated from Eqs. (1) and (2) with the Reid soft-core wave function.[11] The results calculated with the Reid wave function[11] from Eqs. (1) and (5) (not shown in Fig. 1) run along the upper boundary of the hatched region.

In summary, our calculations (solid curve) agree with the experimental data of Ref. 1, lending support to the use of the wave function at the light front to describe the

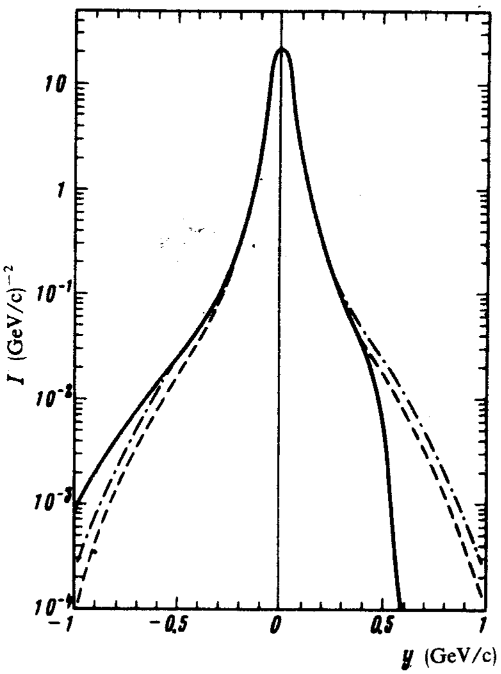

FIG. 3.

deuteron. Comparison of the solid and dot-dashed curves shows that at $y < 0$ the effect in question may be masked by an effect of the structure of the wave function (if we do not insist that the Paris wave function be preferable). At $y > 0$, however, these effects work in opposite directions, as can be seen in Fig. 3, which shows the values of the integrals in (2) and (5) [for $Q^2 = 6\ (\mathrm{GeV/c})^2$]. The notation for the curves is the same as in Fig. 1. There is a qualitative discrepancy between the results calculated from Eqs. (2) and (5); this discrepancy reaches a factor of ten and more at $y > 0.5$ GeV/c. The curves calculated from Eq. (2) are symmetric with respect to $y = 0$, while those calculated from Eq. (5) are asymmetric. We might note that expression (5) approximately follows y-scaling [within 10% in the interval $Q^2 = 2 - 6\ (\mathrm{GeV/c})^2$]. Incorporating the n dependence of the wave function (in accordance with Ref. 9) changes the result slightly at $y < 0$ and intensifies the effect at $y > 0$.

It would be exceedingly interesting to see an experimental study of the reaction $ed \to enp$ at $y > 0$ $(0 < x < 1/2)$ (unfortunately, such experiments are severely complicated by the pion background).

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Erratum: Interaction between localized magnetic moments in disordered conductors [JETP Lett. 38, No. 3, 153–157 (10 August 1983)]

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In the paper of B. L. Al'tshuler and A. G. Aronov, published in Vol. 38, No. 3 the author's name "A. Yu. Zyuzin" was omitted.

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