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Light localization signatures in backscattering from periodic disordered media

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The backscattering line shape is analytically predicted for thick disordered medium films where, remarkably, the medium configuration is periodic along the direction perpendicular to the incident light. A blunt triangular peak is found to emerge on the sharp top. The phenomenon roots in the coexistence of quasi-1D localization and 2D extended states.

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Introduction. Since the mid-eighties [1] the coherent backscattering (CBS) has been one of the pilots of studies of Anderson localization of light. Indeed, manifestations of weak localization (WL) in the CBS line shape have been well documented for various disordered dielectric media [2], while how strong localization (SL) affects CBS has been a long term fascinating subject [3]. The last decade has witnessed spectacular progress on CBS near [4, 5] or far below [6] the localization transition, which undoubtedly is an intellectual challenge both experimentally and theoretically. Indeed, to prepare strong scattering media and to extract localization from medium absorption are highly restrictive [4], while the failure of perturbation theory [7] – crucially mapping the pictorial reciprocal paths into (diagrammatical) one-loop approximation [1, 2]–enforces the invention of a nonperturbative theory to allow a microscopic analysis. Despite of these difficulties a common belief is that SL is responsible for rounding the CBS sharp top [3 – 6].

In studies of light localization much attention has been paid to fully disordered media. There have also been increasing interests on other medium structure such as disordered photonic crystals [8] where the Bloch symmetry is slightly destroyed by impurities, and systems with perfect Bloch symmetry [9] where WL is analytically found. Most interestingly, the recent invention of so-called planar random laser introduces a novel medium structure [10], which consists of a random gain layer sandwiched by two mirrors. It was then conjectured that SL in the layer plane might be responsible for the laser emission [10]. To prove it is yet a nontrivial task which may be traced back to the striking feature of the partially disordered structure. Indeed, (from the geometrical optics view) two perfect reflection mirrors map the medium to an extended periodic one but fully disordered inside the primitive cell (apart from the mirror symmetry). This immediately raises many important questions. For examples, *does Anderson localization exist in such structure? If so, can it be probed by CBS measurements?*

Unfortunately, the interplay between Anderson localization and the periodicity is a difficult issue. In general the common wisdom regarding localization may be drastically modified and very little has been known, among which are: a new scale essential to WL [9] may appear, and whether the constructive interference between reciprocal paths encompasses WL depends on periodic medium configuration [9, 11].

In this letter these two problems will be investigated for a simplified model–thick disordered medium film with a dielectric function *periodic* in the x-direction (Fig.1). The lattice constant is a and the primitive cell

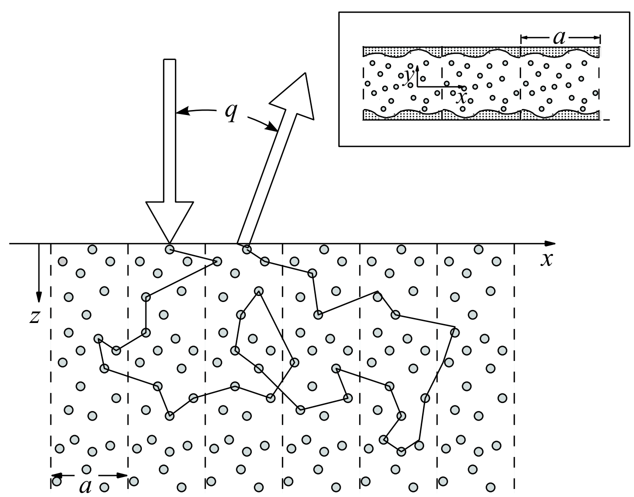

Fig.1. Light backscattering from a periodic thick disordered medium film. Inset: the film section

consists of randomly positioned point-like scatterers filling the half space $z > 0$. The film section is uniformly illuminated by a beam of stationary unpolarized light (with the wavelength λ and the frequency Ω) perpendicular to it. Analytical results are presented for the angular resolution of light intensity near the inverse incident direction for sufficiently large times.

Qualitatively, as shown in (Fig.2), the traditional line shape [1, 2] develops at $|q_\perp| \geq 2\pi/a$ and is sharpened

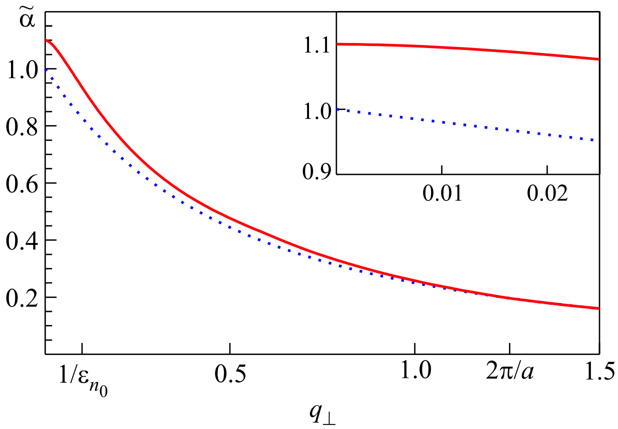

Fig.2. The predicted (solid) versus traditional (dotted) line shape (symmetric with respect to $q_\perp = 0$). The parameters are $a = 5l$, $\xi_0 = 10l$ (setting $l = 1$). Inset: the blunt triangular peak at $|q_\perp| \ll \xi_0^{-1} = 0.1$

at $\xi_0^{-1} \lesssim |q_\perp| < 2\pi/a$, eventually a blunt triangular peak emerges at $|q_\perp| \ll \xi_0^{-1}$. Here $q_\perp = (2\pi/\lambda)\sin\theta$ and $\xi_0 = \pi a\nu D$ with ν the photon density of states at Ω and $D = l/3$ the bare diffusion constant (l the transport mean free path and the velocity $c = 1$). Quantitatively, analytical predictions are made for $\lambda \ll l \ll a \ll l\,e^{l/\lambda}$, where the last inequality ensures photon states to be far from 2D SL [7]. In particular, the line shape $\tilde{\alpha}(\theta)$ is singular at $|q_\perp| = 0\,, 2\pi/a$, around which

$$\tilde{\alpha}(\theta) = \begin{cases} (1 + l|q_\perp|)^{-2} \equiv I_0(\theta), & |q_\perp| > \frac{2\pi}{a}; \\ \{1 + \gamma(\theta)\} I_0(\theta), & 0 < \frac{2\pi}{a} - |q_\perp| \ll \frac{\pi}{a}; \\ (1 + l/\xi_0) - a/(2\pi\xi_0) l|q_\perp|, & |q_\perp| \ll \xi_0^{-1}, \end{cases} \quad (1)$$

while a smooth interpolation between the last two lines is expected. Here the enhancement factor $\gamma(\theta) = (3l/2\xi_0)\,[1 - a|q_\perp|/(2\pi)]$.

Qualitative picture. The well known Bloch theorem allows us to reduce the photon motion into an effective one within a primitive cell dictated by the Bloch wave number k_ρ. The enhanced backscattering finds its origin in the constructive interference–described by low-energy hydrodynamic modes–between two counter-propagating photons (so-called cooperon) each of which carries a Bloch wave number $k_\rho\,, k'_\rho\,,$ respectively. $k_\rho + k'_\rho$ plays the role of "Aharonov-Bohm flux". The gauge invariance then requires that the transverse (x-direction) hydrodynamic wave number $2\pi N/a$ with N an integer satisfies $q_\perp = 2\pi N/a - (k_\rho + k'_\rho)$. Hydrodynamic modes with $N \neq 0$ ($N = 0$) describe 2D (quasi-1D) motion.

As $k_\rho\,, k'_\rho \in [-\pi/a, \pi/a)$ there are two contributions which correspond to two successive N responsible for the line shape. For $|q_\perp| > 2\pi/a$ both $N \neq 0$. Therefore, these two hydrodynamic modes are inhomogeneous in the transverse direction, and extended in the longitudinal (z-) direction because of $a \ll l\,e^{l/\lambda}$. As usual the diffusive 2D motion then leads to the traditional line shape.

For $|q_\perp| < 2\pi/a$ the line shape is contributed by both 2D diffusive and quasi-1D motion since the two successive integers are now 1 (or -1) and 0. The former (or latter) occupies a portion of $a|q_\perp|/2\pi$ (or $1 - a|q_\perp|/2\pi$). In the quasi-1D geometry there are two scales: l and ξ_0. An incident flux decays over the scale $\sim l$ then diffuses inside the medium and eventually exit at $z \sim l$. The penetration length is $\sim |q_\perp|^{-1} \gg l$.

For $|q_\perp|$ closed to $2\pi/a$ photons penetrate into the medium of a distance $\ll \xi_0$ via quasi-1D diffusive motion. Upon penetration they may self-intersect then propagate around the formed loop along the same direction–so-called diffuson (Fig.3a). The probability

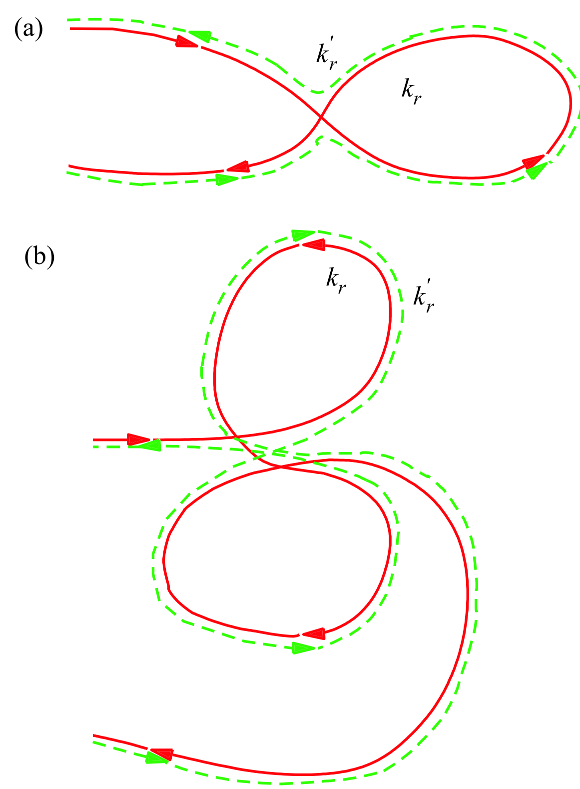

Fig.3. One-loop (a) and typical two-loop (b) interference picture underlying quasi-1D weak localization

is larger in quasi-1D than in 2D. As a result two initially closed but counter-propagating photons have a larger probability to be brought back to their starting point and the backscattered light intensity is thereby enhanced. At $z \sim l$ the probability of forming such a loop is $\sim l/D$. Consequently, the inverse diffusivity at $z \sim l$ increases with an amount of $l/a\nu D \sim l/\xi_0$. Noticing that the backscattered light intensity is proportional to the inverse diffusivity D^{-1} [1] and taking into account the weight of quasi-1D motion, we find that the traditional line shape is magnified by a factor of $1+\gamma(\theta)$.

Notice that the quasi-1D cooperon and diffuson have different masses: $D(k_\rho + k'_\rho)^2 = Dq_\perp^2$ and $D(k_\rho - k'_\rho)^2$, respectively. As $|q_\perp|$ decreases the diffuson tends to acquire a larger massive and be damped. Consequently, all the constructive interference involving diffuson-cooperon coupling (e.g., Fig.3a) tends to be suppressed. Opposed to this the cooperon becomes less massive. Consequently higher order loop-wise interference paths involving solely cooperon-cooperon coupling (e.g., Fig.3b) accumulate and eventually dominate the backscattered light intensity at $|q_\perp| \lesssim \xi_0^{-1}$, where photons penetrate deeply into the medium forming SL in the bulk.

In this region from one-parameter scaling hypothesis we expect that the diffusion coefficient exponentially decays from the interface, i.e., $-\ln D(z) \propto z/\xi_0$. Therefore, the average inverse diffusivity of the boundary layer increases also by an amount of $\sim l/\xi_0$. Thus for $|q_\perp| \ll \xi_0^{-1}$ the quasi-1D motion contributes to the line shape $(1+l/\xi_0)[1-a|q_\perp|/(2\pi)]$, together with the portion contributed by the 2D extended motion: $I_0(\theta)[a|q_\perp|/(2\pi)] \approx a|q_\perp|/(2\pi)$, and leads to a blunt triangular peak.

General formalism. We then outline the proof [12]. Let us start from the retarded (advanced) Green's function: $G^{R,A}_{\Omega^2}(\mathbf{R},\mathbf{R}') = \langle \mathbf{R}|\{\nabla^2 + \Omega^2[1+\epsilon(\mathbf{R})] \pm i0^+\}^{-1}|\mathbf{R}'\rangle$ describing the propagation of the electric field. The fluctuating dielectric field $\epsilon(\mathbf{R})$ has zero mean and vanishes for $z<0$, while for $z>0$ is periodic in x and satisfies $\Omega^4\langle\epsilon(\mathbf{R})\epsilon(\mathbf{R}')\rangle = \Delta\,\delta(y-y')\,\delta(z-z')\sum_{N\in\mathbb{Z}}\delta(\rho-\rho'-Na)$, where $(\exists m\in\mathbb{Z})\;\rho = x - ma \in [0,a)$ stands for the relative x-coordinate in the primitive cell. The scattering is elastic with $l = 4\pi/\Delta$. Using the effective medium approximation the incident field is $\bar{E}(\mathbf{R}) = E_0 e^{i\Omega\hat{\mathbf{z}}\cdot\mathbf{R} - z/2l}$ [13]. Alternatively, one may introduce the Green's functions: $\mathcal{G}^{R,A}_{\Omega^2}(\mathbf{r},\mathbf{r}';k_\rho) = \langle \mathbf{r}|(\Omega^2 - \hat{H}(k_\rho) \pm i0^+)^{-1}|\mathbf{r}'\rangle$ for the effective motion within the primitive cell with $\hat{H}(k_\rho) = -[(\partial_\rho + ik_\rho)^2 + \partial_y^2 + \partial_z^2] - \Omega^2\,\epsilon(\mathbf{r})\;(\mathbf{r}\equiv(\rho,y,z))$. The two sets of Green's functions are related through

$$G^{R,A}_{\Omega^2}(\mathbf{R},\mathbf{R}') = \sum_{k_\rho} e^{ik_\rho(x-x')}\mathcal{G}^{R,A}_{\Omega^2}(\mathbf{r},\mathbf{r}';k_\rho). \quad (2)$$

The albedo characterizing the radiation intensity in the direction $\mathbf{s} = (\sin\theta, 0, -\cos\theta)$ [13] generally depends on the time t. With the single scattering event ignored the albedo at $t\to\infty$, denoted as $\alpha(\theta)$, can be shown to be

$$\alpha(\theta) = \iint d\mathbf{R}_1 d\mathbf{R}_2\, e^{-\frac{z_1+z_2}{l}}[\overline{G^R_{\Omega^2}(\mathbf{R}_1,\mathbf{R}_2)\,G^A_{\Omega^2}(\mathbf{R}_2,\mathbf{R}_1)} + e^{i\Omega\mathbf{s}\cdot(\mathbf{R}_2-\mathbf{R}_1)}\,\overline{G^R_{\Omega^2}(\mathbf{R}_1,\mathbf{R}_2)\,G^A_{\Omega^2}(\mathbf{R}_1,\mathbf{R}_2)}] \quad (3)$$

in the unit of $\Delta^2 E_0^2/16\pi$ with $\overline{(\cdots)}$ the average over the fluctuating dielectric field $\epsilon(\mathbf{R})$. The first (second) term gives the background intensity α_0 (line shape $\tilde{\alpha}(\theta)$). Notice that the y-dependence of Green's functions is irrelevant and will be ignored from now on. Substituting Eq. (2) into Eq. (3) gives (up to an irrelevant overall normalization factor)

$$\alpha_0 = \iint_0^\infty dz dz' e^{-\frac{z+z'}{l}} \sum_{k_\rho} \mathcal{Y}^{\rm D}_0(z,z';k_\rho,k_\rho), \quad (4)$$

$$\tilde{\alpha}(\theta) = \iint_0^\infty dz dz' e^{-\frac{z+z'}{l}} \sum_N {\sum_{k_\rho,k'_\rho}}'^N \mathcal{Y}^{\rm C}_N(z,z';k_\rho,k'_\rho), \quad (5)$$

where we have quantified the propagators: diffuson and cooperon introduced above to be $\mathcal{Y}^{\rm D}(\mathbf{r},\mathbf{r}';k_\rho,k'_\rho) \equiv \overline{\mathcal{G}^R_{\Omega^2}(\mathbf{r},\mathbf{r}';k_\rho)\,\mathcal{G}^A_{\Omega^2}(\mathbf{r}',\mathbf{r};k'_\rho)}$ and $\mathcal{Y}^{\rm C}(\mathbf{r},\mathbf{r}';k_\rho,k'_\rho) \equiv \overline{\mathcal{G}^R_{\Omega^2}(\mathbf{r},\mathbf{r}';k_\rho)\,\mathcal{G}^A_{\Omega^2}(\mathbf{r},\mathbf{r}';k'_\rho)}$, and introduced their Fourier transformations, i.e., $\mathcal{Y}^{\rm D,C}(\mathbf{r},\mathbf{r}';k_\rho,k'_\rho) = a^{-1}\sum_{N\in\mathbb{Z}} e^{i(\rho-\rho')2\pi N/a}\,\mathcal{Y}^{\rm D,C}_N(z,z';k_\rho,k'_\rho)$. The partial summation ${\sum'}^N_{k_\rho,k'_\rho} \equiv \sum_{k_\rho,k'_\rho}\delta_{k_\rho+k'_\rho,2\pi N/a - q_\perp}$.

One may proceed to sum up over all the (maximally crossing) ladder diagrams (e.g., Refs. [7, 13]) producing a bare diffuson (cooperon) $\mathcal{Y}^{\rm D(C)}$. Nevertheless such a diagrammatic expansion fails in the nonperturbative analysis–intrinsic to localization. Instead, for interactionless systems such as photons to fulfill this task the supersymmetric method–to which we switch below–turns out to be perfectly suitable [7]. Conceptually, the introduced slow varying Q-field interprets the bare diffuson and cooperon as Goldstone modes of the spontaneous supersymmetry breaking, and encapsulates their mutual interactions underlying localization through the nonlinear constraint: $Q^2 = 1$. Technically, it can be shown

$$\mathcal{Y}^{\rm D,C}(\mathbf{r},\mathbf{r}';k_\rho,k'_\rho) = \frac{1}{2^7}\langle {\rm str}[k\Lambda^+\tau^- Q(\mathbf{r})\Lambda^-\tau^\mp kQ(\mathbf{r}')]\rangle \quad (6)$$

following standard derivations [7], where $Q(\mathbf{r}) = T^{-1}(\mathbf{r})\Lambda T(\mathbf{r})$ is a matrix field with full orthogonal symmetry, i.e., $T \in U(2,2/4)/U(2/2)\times U(2/2)$. The average is defined as $\langle\ldots\rangle \equiv \int D[Q](\cdots)e^{-F[Q]}$ with $Q|_{z=-z_0} = \Lambda, Q|_{\rho=0} = Q|_{\rho=a}$, where the action

$$F[Q] = \frac{\pi\nu D}{8}\int_0^\infty dz \int_0^a d\rho\, \mathrm{str}[(\partial_z Q)^2 + (D_\rho Q)^2]$$

with $D_\rho \equiv \partial_\rho + i[\bar{\bar{k}}\tau_3,\cdot]$. str is the supertrace and all the matrices follow the definitions of Ref. [7]. In addition, $\Lambda^\pm = (1\pm\Lambda)/2$, $\tau^\pm = (1\pm\tau_3)/2$, and the matrix $\bar{\bar{k}} = \mathrm{diag}(k_\rho, k'_\rho)$ is diagonal in the retarded-advanced sector. The boundary condition $Q(-z_0) = \Lambda$ accounts for the fact that the albedo is contributed by optical paths not crossing the trapping plane located at $z = -z_0 \approx -0.7\,l \approx 0$ [1], which coincides with the medium boundary.

Eqs. (4)–(6) constitute the general formalism of calculating the albedo. By making an appropriate global rotation for the Q-field reflecting the symmetry: $\mathcal{Y}^{\mathrm{D}}(\mathbf{r},\mathbf{r}';k_\rho,k'_\rho) = \mathcal{Y}^{\mathrm{C}}(\mathbf{r},\mathbf{r}';k_\rho,-k'_\rho)$, it can be shown that, similar to the fully disordered medium with conserved reciprocity [1], the background intensity $\alpha_0 = \tilde{\alpha}(0)$. We then turn to analyze the line shape.

Line shape. Three regions: (i) $|q_\perp| \geq 2\pi/a$, (ii) $\xi_0^{-1} \lesssim |q_\perp| < 2\pi/a$ and (iii) $|q_\perp| \lesssim \xi_0^{-1}$ will be studied separately. It can be shown that in (i) and (ii) $Q(\mathbf{r})$ mildly fluctuates around Λ. A perturbation theory near it generates leading diffusive motion–described by the bare diffuson (cooperon)–and loop-wise interference (Fig.3) encompassing WL. Opposed to this, in (iii) though locked at Λ at the boundary $Q(\mathbf{r})$ may strongly fluctuate along the z-direction driving photons into SL states, while remains homogeneous in the ρ-direction. Consequently the perturbation theory breaks down.

In Eq. (5) the summation over N picks up two terms with successive integers $N = N', N'+1$. In (i) they both do not vanish corresponding to 2D low-energy extended motion. The leading order perturbation then gives

$$\left\{-\partial_z D\partial_z + D\left[\frac{2\pi N}{a} + (k_\rho \mp k'_\rho)\right]^2\right\}\mathcal{Y}_N^{\mathrm{D,C}}(z,z';k_\rho,k'_\rho) = \frac{1}{\pi\nu}\delta(z-z'), \qquad \mathcal{Y}_N^{\mathrm{D,C}}|_{z=0} = 0. \quad (7)$$

Substituting its solution into Eq. (5) recovers the first line of Eq. (1) [14, 15]. In (ii) and (iii) $|N| = 0, 1$. Taking into account the weight of 2D motion, i.e., $a|q_\perp|/(2\pi)$ the term with $|N| = 1$ is found to be $\frac{a|q_\perp|}{2\pi} I_0(\theta)$. The term with $N = 0$ arises from the quasi–1D motion suffering from localization effects, and below will be calculated separately for (ii) and (iii) with the simplified action

$$F[Q] = \frac{\pi a\nu D}{8}\int_0^\infty dz\, \mathrm{str}\{(\partial_z Q)^2 + [i\bar{\bar{k}}\tau_3, Q]^2\}$$

(since Q does not depend on ρ).

For (ii) one-loop expansion show that Eq. (7) ($N = 0$) still holds reflecting the flux conservation. However, the diffusion coefficient acquires the position-dependence and is found to be $D(z) = D\{1 - (\xi_0|k_\rho \pm k_{\rho'}|)^{-1}[1 - e^{-2z|k_\rho \pm k_{\rho'}|}]\}$ with the $\pm$ sign corresponding to $\mathcal{Y}_0^{\mathrm{D}}$ and $\mathcal{Y}_0^{\mathrm{C}}$, respectively. Therefore, we justify microscopically the crucial conjecture–position-dependent diffusion coefficient–made in Ref. [5] at the one-loop level. Nevertheless the difference should be stressed that at the medium boundary $z = 0$ the bare diffusion constant is protected against one-loop WL. Such an important property persists up to higher order loop corrections enforcing

$$D(z=0) = D. \quad (8)$$

In the bulk: $z \gg |k_\rho \pm k'_\rho|^{-1}$ the diffusion coefficients become homogeneous: $D(z) \to D[1 - (\xi_0|k_\rho \pm k'_\rho|)^{-1}]$ but strongly depend on their infrared cutoffs. By contrast, since Eq. (5) suggests that all the interfering optical paths contributing to the line shape reside in a boundary layer of size $\sim |q_\perp|^{-1}$ ($\gg l$), for $0 < 2\pi/a - |q_\perp| \ll \pi/a$ the local diffusion coefficient of $\mathcal{Y}_0^{\mathrm{C}}$ is simplified as $D(z) \approx D(1 - 2z/\xi_0)$, and well approximated by $D(z) \approx D/(1 + 2z/\xi_0)$ as the leading order l/ξ_0 correction concerned. Replacing D in Eq. (7) with the latter we obtain:

$$\mathcal{Y}_0^{\mathrm{C}} = \frac{1}{D\xi_0|q_\perp|^2}\left[f_-(x_>)f_+(x_<) - Cf_-(x_>)f_-(x_<)\right], \quad (9)$$

where $x_{>(<)} = \xi_0|q_\perp|[1 + \xi_0^{-1}\max(\min)\{z, z'\}]$, $C = f_+(\xi_0|q_\perp|)/f_-(\xi_0|q_\perp|)$ and $f_+(x) = xI_1(x)$ and $f_-(x) = xK_1(x)$ with $I_1(x), K_1(x)$ the modified Bessel functions. Eq. (9) is then inserted into Eq. (5). For $\xi_0|q_\perp| \gg 1$ one may use the asymptotic expressions of the Bessel functions and eventually find the line shape given by the second line of Eq. (1) [15]. At $|q_\perp| \sim \pi/a$ interfering optical paths significantly penetrate into the bulk and suffer from stronger WL, resulting in a larger enhancement factor.

For (ii) higher order expansion of Eq. (5) shows that the line shape, indeed, is contributed by interfering optical paths forming the loop-wise structure classified into: diffuson-cooperon coupling where two paths may trace some loops in the same direction (e.g, Fig.3a), and cooperon-cooperon coupling where two paths trace all the loops in the opposite direction (e.g, Fig.3b). As shown below it is the latter leading to SL in the bulk at $|q_\perp| \sim \xi_0^{-1}$, and the line shape at $|q_\perp| \lesssim \xi_0^{-1}$ is mainly responsible for by (radiative) SL states with lower symmetric T.

For (iii) to calculate the lineshape namely $\mathcal{Y}_0^{\mathrm{C}}(z,z';k_\rho,k'_\rho)$ exactly is a very hard task. Due to the broken translational symmetry the technique of Ref. [7] breaks down and the solution there is not applicable. Instead, our method combines the microscopic yet advanced mathematical theory namely performing super-Fourier analysis [16] for Eqs. (5) and (6) (This is far beyond the scope of this letter and the details are to be reported elsewhere [17].) and (phenomenological) hydrodynamic methods [18]. Observing that $\tilde{\alpha}(\theta) \approx l^2 \sum_N \sum_{k_\rho,k'_\rho}{}'^N \mathcal{Y}_N^{\mathrm{C}}(l,l;k_\rho,k'_\rho)$ we need to consider only the returning probability-like propagator $\mathcal{Y}_0^{\mathrm{C}}(l,l;k_\rho,k'_\rho)$ in (iii).

Applying the heat kernel method [16] to Eq. (6) we succeed to calculate $\mathcal{Y}_0^{\mathrm{C}}(z,z;0,0)$ exactly provided that T is lowered down to the $\mathbf{GMat}(3,2|\Lambda)$ symmetry [17]. The most important feature of the solution is the exponential divergence [17]: (All the numerical factors are unimportant and not given here.)

$$\ln \mathcal{Y}_0^{\mathrm{C}}(z,z;0,0) \sim z/\xi_0 + o(e^{-z/\xi_0}) \tag{10}$$

for $z \gtrsim \xi_0$. On the other hand, because it was shown that bulk SL states display hydrodynamic behavior [18] we expect in the presence of vacuum-medium interface the exponential divergence Eq. (10) to be reflected at the same macroscopic level. To achieve this we notice that the boundary leakage introduces the level broadening $\Gamma(z)$ scaling as $-\ln\Gamma(z) \sim z/\xi_0$ [19]. On the physical ground the contribution of local currents to the restoring force–leading to SL–exponentially decays in time, modifying the Vollhardt-Wölfle model [18] to be

$$\partial_t \mathcal{Y}_0^{\mathrm{C}} + \nabla\cdot\mathbf{j} = \frac{1}{\pi\nu}\,\delta(z-z'), \tag{11}$$
$$\partial_t \mathbf{j} + \frac{D}{l}\,\nabla\mathcal{Y}_0^{\mathrm{C}} = -\frac{\mathbf{j}}{l} - \frac{D}{\xi\, l}\int_0^t dt'\, e^{-\Gamma(z)(t-t')}\,\mathbf{j}(t')$$

with $\mathcal{Y}_0^{\mathrm{C}}|_{z=0} = 0$ and the covariant derivative $\nabla \equiv \equiv (\partial_z, \partial_\rho + iq_\perp)$, where the overall coefficient of the restoring force term is fixed by Eq. (8). Solving Eq. (11) indeed confirms Eq. (10) ($q_\perp = 0$ and the localization length $\xi = \xi_0$).

The presence of nonvanishing $k_\rho \approx -k'_\rho$ alters the microscopic symmetry and therefore the localization class. Indeed, (for $|q_\perp| \lesssim \xi_0^{-1}$) one may follow Ref. [11], average Eq. (6) over $|k_\rho - k'_\rho|$ and subsequently obtain an effective action:

$$F[Q] = \frac{\pi a\nu D}{8}\int_0^\infty dz\,\mathrm{str}\{(\partial_z Q)^2 + [iq_\perp\Lambda/2, Q]^2\}.$$

The T-field symmetry is lowered down to $U(1,1/2)/U(1/1)\times U(1/1)$ giving $\xi = 2\xi_0$ [7].

On the other hand, the common belief of one-parameter scaling hypothesis (e.g. Ref. [20]) implies that the microscopic symmetry enters only through the localization length leaving the hydrodynamic model unaffected. To find $\mathcal{Y}_0^{\mathrm{C}}(z,z';k_\rho,k'_\rho)$ we insert the z-dependent level broadening: $\Gamma(z) = D/(2\xi_0)^2\, e^{-z/2\xi_0}$ [19] and $\xi = 2\xi_0$ into Eq. (11). For $t \gg \xi_0^2/D$ the steady distribution is approached solving: $-D\{\partial_z e^{-z/2\xi_0}\partial_z - |q_\perp|^2 e^{-\frac{z}{2\xi_0}}\}\mathcal{Y}_0^{\mathrm{C}} = (\pi\nu)^{-1}\delta(z-z')$, which coincides with the diffusive model for the single channel SL [5]. Substituting the solution and the bare propagator, Eq. (7) into Eq. (5) one may find

$$\tilde{\alpha}(\theta) = \left(1-\frac{a|q_\perp|}{2\pi}\right)\left[1+\frac{3-\sqrt{1+(4\xi_0 q_\perp)^2}}{2\xi_0/l}\right] + \frac{a|q_\perp|}{2\pi}I_0(\theta)$$
$$\to (1+l/\xi_0) - a/(2\pi\xi_0)\,l|q_\perp|, \quad |q_\perp| \ll \xi_0^{-1}. \tag{12}$$

The first line holds for $|q_\perp| \lesssim \xi_0^{-1}$ and, apart from the factor: $[1-a|q_\perp|/2\pi]$ accounting for the weight of quasi-1D motion, the first term resembles the rounded line shape of fully disordered media below localization transition [5]. Remarkably, the second line suggests that the line shape displays a blunt triangular peak in a very narrow region: $|q_\perp| \ll \xi_0^{-1}$ (inset of Fig.2).

Conclusions. Analytical studies of the coherent backscattering line shape have been presented for periodic thick disordered medium films which arrest both extended and quasi-1D localization states. The result is expected to be qualitatively correct for $\lambda \lesssim l \lesssim a\,(\lesssim le^{l/\lambda})$ which, together with the realization of the perfect periodicity along one direction, may be well within the reach of up-to-date experimental conditions [10]. However, to study realistic media the present theory still needs to be extended so that the large size (namely the film thickness much larger than the lattice constant) effects and the parity (mirror) symmetry are taken into account, which is left for future work.

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1. A. A. Golubentsev, Zh. Eksp. Teor. Fiz. **86**, 47 (1984) [Sov. Phys.-JETP **59**, 26 (1984)]; M. P. van Albada and

A. Lagendijk, Phys. Rev. Lett. **55**, 2692 (1985); P. E. Wolf and G. Maret, *ibid.* **55**, 2696 (1985).
2. M. C. W. van Rosum and Th. M. Niuwenhuizen, Rev. Mod. Phys. **71**, 313 (1999).
3. R. Berkovits and M. Kaveh, Phys. Rev. B **36**, 9322 (1987); I. Edrei and M. J. Stephen, *ibid.* **42**, 110 (1990).
4. D. S. Wiersma et al., Nature (London) **390**, 671 (1997); *ibid.* **398**, 207 (1999); F. Scheffold et al., *ibid.* **398**, 206 (1999); M. Stözer et al., Phys. Rev. Lett. **96**, 063904 (2006).
5. B. A. van Tiggelen et al., Phys. Rev. Lett. **84**, 4333 (2000).
6. X. D. Zhang and Z. Q. Zhang, Phys. Rev. B **65**, 155208 (2002).
7. K. B. Efetov, *Supersymmetry in disorder and chaos*, Cambridge, UK, 1997.
8. A. F. Koenderink et al., Phys. Lett. A **268**, 104 (2000); A. Yu. Sivachenko, et al., Phys. Rev. B **63**, 245103 (2001).
9. C. Tian and A. Larkin, Phys. Rev. Lett. **95**, 246601 (2005).
10. Q. Song *et al.*, Phys. Rev. Lett. **96**, 033902 (2006).
11. N. Taniguchi and B. L. Altshuler, Phys. Rev. Lett. **24**, 4031 (1993).
12. Medium gain/loss and internal reflections are ignored.
13. M. J. Stephen and G. Cwilich, Phys. Rev. B **34**, 7564 (1986).
14. The overall factor: $l^3/(2D)\,(\sum'^{N}_{k_\rho,k'_\rho} 1 + \sum'^{N+1}_{k_\rho,k'_\rho} 1) = l^3/(2D)(\sum_{k_\rho} 1)$ is dropped out hereafter.
15. In fact, 2D WL renormalizes the bare diffusion constant of $F[Q]$ (and subsequently the overall factor of $\alpha(\theta)$) and slightly enhances the numerical prefactor of $\gamma(\theta)$.
16. M. R. Zirnbauer, Commun. Math. Phys. **141**, 503 (1991).
17. C. Tian, to be published.
18. D. Vollhardt and P. Wölfle, Phys. Rev. Lett. **45**, 842 (1980).
19. V. Ya. Chernyak et al., Solid. State. Commun. **84**, 209 (1992).
20. A. Garcia-Martin, M. Governale, and P. Wölfle, Phys. Rev. B **66**, 233307 (2002); S. Kettemann and R. Mazzarello, *ibid.* **65**, 085318 (2002); H. Schomerus and C. W. J. Beenakker, Phys. Rev. Lett. **84**, 3927 (2000).