

# Determination of Landau's Fermi-liquid parameters in Si-MOSFET systems

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We analyze experimental data in order to evaluate Landau's Fermi-liquid parameters. By using raw data of recent Shubnikov-de Haas measurements we derive, as function of the electron density  $n_s$ , results for the compressibility mass of the charged two-dimensional electron gas. The compressibility mass is nearly equal to the transport mass even in the density region where the transport mass has the tendency to diverge. We conclude that Landau's Fermi-liquid parameter  $F_0^s(n_s)$  is nearly independent of electron density and near to zero. This result is derived for silicon (100) and silicon (111) surfaces. We also obtain the dependence of  $F_1^s(n_s)$ , determining the transport mass, and of  $F_0^s(n_s)$ , determining the spin-susceptibility.

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Recently, it was found in experiment that in Si-MOSFETs with (100) or (111) orientation the electron effective mass (the transport mass or cyclotron mass)  $m^*$  drastically increases with decreasing electron density with a tendency to diverge at a definite electron concentration [1–3]. The critical electron density depends on the electron-electron interaction strength, so that the dependence  $m^*(r_s)/m_b$  is universal in both silicon systems [3]. Here  $m_b$  is the bulk effective mass and  $r_s$  is the dimensionless Wigner-Seitz radius. Simultaneously, in both systems a decrease of the Dingle temperature is observed by decreasing of the electron concentration [2, 3]. On the first glance the last observation seems to be very puzzling, because it means an increase of the single-particle relaxation time with decreasing density. In lowering the electron density disorder effects should become more important due to less screening and the single-particle relaxation time should decrease. We conclude that the observed behavior of the Dingle temperature is in contradiction with expectations obtained within the simplest theoretical consideration [4–6].

In a number of publications during last years [7–9] it was shown that despite of a strong increase of the effective mass the electron system can be described in the frame of the Fermi-liquid theory proposed by Landau [10]. In the two-dimensional case the effective transport mass in this theory is defined through the dimensionless parameter  $F_1^s$  (a Landau Fermi-liquid parameter)

$$\frac{m^*}{m_b} = \frac{p_F}{v_F m_b} = 1 + \frac{1}{2} F_1^s. \quad (1)$$

The possibility of a divergence of the effective mass can be easily seen if one uses in equation (1) the initial parameter of the theory  $f_1^s = \frac{m_b}{m^* \nu(0)} F_1^s$ . Here  $\nu(0)$  is the density of states defined with the mass  $m_b$ . With  $f_1^s$  the equation for the effective (transport) mass is written as

$$\frac{m^*}{m_b} = \frac{1}{1 - \frac{1}{2} \nu(0) f_1^s}. \quad (2)$$

The transport effective mass, corresponding to equations (1), (2), is not unique in Landau's Fermi-liquid theory. Another effective mass  $m_{com}$ , determining the thermodynamic density of states, the compressibility (com), and the screening behavior is defined by

$$m_{com} = \frac{\pi n_s^2 \hbar^2}{g_v} \kappa = \frac{m^*}{1 + F_0^s} = m_b \frac{1 + \frac{1}{2} F_1^s}{1 + F_0^s}, \quad (3)$$

where  $\kappa$  represents the compressibility,  $n_s$  is the electron density, and  $g_v$  is the number of valleys. The mass  $m_{com}$  is a function of two macroscopic parameters  $F_1^s$  and  $F_0^s$ , and can, in principle, have a very different dependence on the electron density in comparison to  $m^*$ . We stress that the density of states is proportional to the mass  $m_{com}$  and that this mass also is responsible for screening effects. It defines the screening length in the long-range limit.

The aim of the present paper is to explain qualitatively the unusual behavior of the single-particle relaxation time (or sometimes called quantum scattering time) as a function of electron density [2, 3] and to extract from the available experimental data the dependences of  $m_{com}$  and  $F_0^s$  on electron density.

The single-particle relaxation time describes the amplitude of Shubnikov – de Haas (SdH) oscillations and the Dingle temperature via  $k_B T_D = \hbar/2\pi\tau_s$ . Note that the effective transport mass  $m^*$  is given by the energy difference of the Landau ladder seen in SdH oscillations. Therefore, we argue that one can determine two Landau Fermi-liquid parameters by measuring SdH oscillations as function of temperature.

First of all we would like to explain why we use the single particle relaxation time  $\tau_s$ , and not the transport scattering time  $\tau_t$ , where the compressibility mass also is an important parameter. The transport scattering time [11] is very sensitive to multiple scattering processes. Experimental data in both silicon systems are taken in the vicinity of the metal-insulator transition [12], where multiple scattering processes play a crucial role. The single-particle relaxation time is defined by density of state modulations and is much less sensitive to such multiple scattering processes [6]. For  $\varepsilon_F \tau_s > \hbar$  one can write  $\tau_s$  as [6]

$$\frac{\hbar}{\tau_s} = \frac{2m^*}{\pi\hbar^2} \int_0^{2k_F} dq \frac{\langle |U(q)|^2 \rangle}{\varepsilon(q)^2} \frac{1}{\sqrt{4k_F^2 - q^2}}, \quad (4)$$

with  $\varepsilon(q < 2k_F) = 1 + q_s(q)F_c(q)/q$  as the screening function, the  $q$ -dependent screening number  $q_s(q)/q_s = g_v(m_{com}/m_b)(1 - G(q))$  and  $q_s = g_s/a_B^*$ , where  $a_B^*$  is the effective Bohr radius.  $F_c(q)$  is the form factor for the Coulomb interaction due to the finite extension of the electron gas [4].  $\langle |U(q)|^2 \rangle$  is the random potential due to disorder, and  $G(q)$  describes the local field corrections.  $G(q)$  takes into account exchange and correlation effects beyond the mean-field theory (the random-phase approximation) and reduces screening effects [10]. Impurity scattering with impurities at the Si/SiO<sub>2</sub> interface and interface-roughness scattering are important for the two-dimensional electron gas on silicon surfaces [4]. The analyzed Si(100) and Si(111) data for  $\tau_s$  are in the validity range of equation (4), see also figure 10 in [6].

There are different contributions to the single-particle relaxation time in Eq.(4). The increase of the effective mass  $m^*$  with decreasing electron density in the numerator decreases  $\tau_s$  for decreasing electron concentration. Similar influence has the effective quasi-particle interaction, because  $G(2k_F)$  [13, 14] tends to unity as the electron density decreases. About the compressibility mass  $m_{com}$  nothing is really known from experiment. From numerical calculations of Ref. ([9]) we conclude that the mass  $m_{com}$  should strongly increase near the critical electron density, the density where the transport mass diverges.

In the integral of Eq.(4) at  $q_s \gg 2k_F$  we can neglect the unity in  $\varepsilon(q)$ , because the second term exceeds the

unity significantly, and deduce that  $m^*/\tau_s \propto m^*/m_{com}^2$ . Hence, the single-particle relaxation time is roughly proportional to the square of  $m_{com}$  and the effect of the mass increase, according to the experiment, prevails both other effects described above.

In Fig.1 the ratio  $m^*/m_{com}$  is shown versus electron

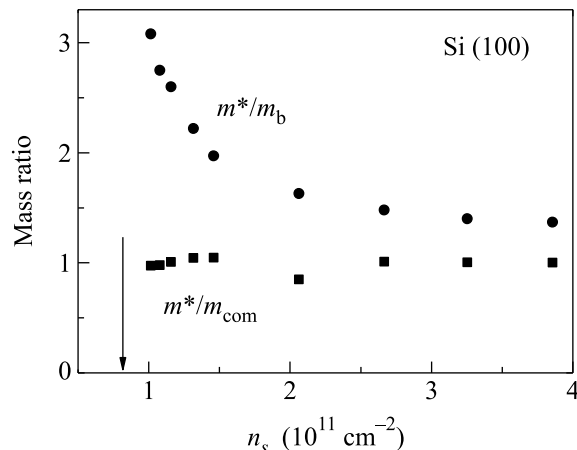


Fig.1. Different mass ratios versus electron density for silicon (100): the ratios  $m^*/m_{com}$  and  $m^*/m_b$  extracted from experiment [2] are shown. The long arrow indicates the value of the MIT

density for experimental results obtained for Si(100) [2]. We found this ratio within the following procedure. In SdH experiment  $m^*$  is measured together with the single-particle relaxation time. At low electron density  $(m^*/\tau_s)^{1/2}$  is proportional to  $m^*/m_{com}$  if the coefficient defined from Eq. (4) is slowly dependent on electron concentration. We tested this assumption and found that the variation of this coefficient is not more than 10%. Taking  $(m^*/\tau_s)^{1/2} \propto m^*/m_{com}$  equal to 1 at relatively large density, where the density dependence of  $m^*$  is negligible, we can conclude that  $m^* = m_{com}$ . Our numerical results indicate that  $m^* = m_{com}$  holds for all densities. This would mean that the parameter  $F_0^s$  is independent of density and near to zero. Note the strong density dependence of  $m^*/m_b$  [1], which is also shown in Fig.1.

Experimental results for Si(111) are more complicated to analyze. MOSFET with (111) orientation are more disordered than the Si(100) samples and the effective mass increase is shifted to higher electron concentration. It means that the contribution from surface roughness scattering is more important. Nevertheless the same procedure as described above gives very similar result. In Fig.2 we show the mass ratio  $m^*/m_{com}$  versus electron density for experimental results obtained for Si(111) [3]. At large density the ratio was fixed to 1. A weak density

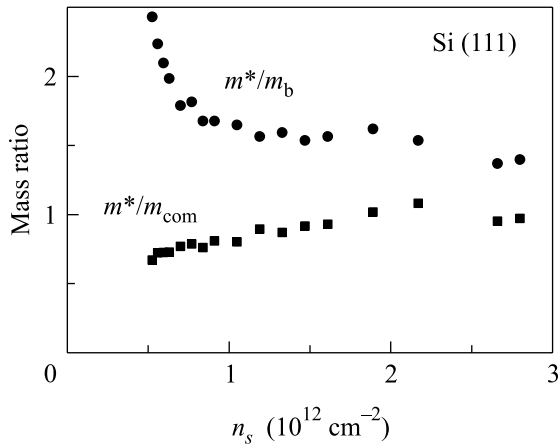


Fig.2. Different mass ratios versus electron density for silicon(111): the ratios  $m^*/m_{com}$  and  $m^*/m_b$  extracted from experiment [3] are shown

dependence is found. But again we can conclude that the essential density dependence of the mass  $m_{com}$  can be expressed by  $m^*$ . We conclude that  $0 < F_0^s < 0.3$ . It is not clear whether this is due to the higher disorder in the (111) sample compared to the (100) sample or related to degeneracy effects in Si(111).

From existing experimental data one can extract two other Landau liquid parameters. Using Eqs. (1), (2) and mass values from SdH measurements we find  $F_1^s$  versus electron density. Our results are shown in Fig.3. For

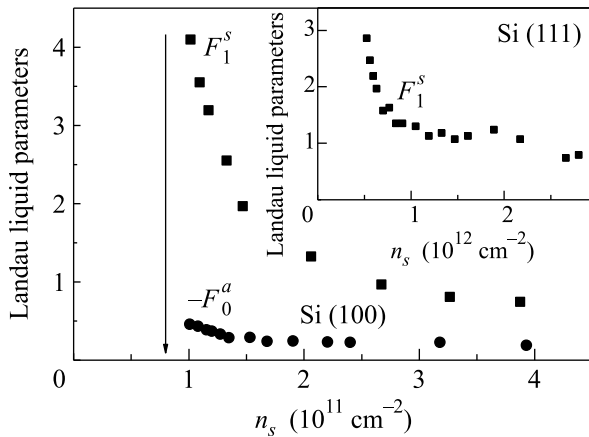


Fig.3. Landau's Fermi-liquid parameter  $F_1^s$  for the effective transport mass and  $F_0^a$  for the spin susceptibility for Si(100) as deduced from experimental data [1] and [15], respectively. In the inset we show  $F_1^s$  for the effective transport mass for Si(111) as deduced from experimental data [3]. The long arrow indicates the value of the MIT

both silicon surfaces  $F_1^s$  exhibits a strong tendency to diverge at low electron density.

Another Fermi liquid parameter  $F_0^a$  exists, defining the spin susceptibility  $\chi_p$  in terms of the spin-susceptibility  $\chi_p^0$  of the free electron gas without interaction effects.  $\chi_p$  is given by [10]

$$\frac{\chi_p^0}{\chi_p} = \frac{m_b}{m^*} (1 + F_0^a) = \frac{m_b}{m^*} g_b / g^*, \quad (5)$$

where  $g^*$  is the effective Lande  $g$ -factor of the electron gas with interaction and  $g_b = 2$  the Lande  $g$ -factor of the electron gas in silicon without interaction effects. Experimental results for the Lande  $g$ -factor of Si(100) have been published in Refs. [1, 15]. For our analysis we will use the data of Ref. [1], because this data correspond to scaling in a weak magnetic fields and are more close to the case of a spin-unpolarized electron gas. We tested that the other data do agree with conclusions made below. From the original experimental data one can see that the product  $g^* m^*$  increases more rapidly than  $m^*$ . The authors of the experimental publication [2] never discussed this fact due to the believe that differences were connected with differences in methods and with possible inaccuracy in experiment. But we think that the difference has a more fundamental origin, because this fact is good reproducible in experiments with different experimental methods. Our result for the parameter  $F_0^a$  is shown in Fig.3. Near the point of a possible transport mass divergency, we observe an increase of  $-F_0^a$ . Nevertheless, the behavior of this parameter seems to be far from being critical. We remember that critical behavior of  $F_0^a$  corresponds to  $F_0^a \rightarrow -1$ .

With a magnetic field oriented parallel to the sample surface one can spin-polarize a two-dimensional electron gas [16]. The Fermi-liquid parameters discussed in this paper are for a magnetic field approaching zero. It was found in experiment that the effective mass does not depend on a parallel magnetic field [2]. Therefore we conclude that  $F_1^s$  is independent on the spin degeneracy. However, we cannot rule out that  $F_0^s$  depends on the spin degeneracy (or the spin-polarization). In fact, experimental results obtained for Si(100) indicate that  $F_0^s$  may become negative with increasing parallel magnetic field [17]. We believe that more experimental results are necessary to quantify this conclusion in more detail.

Let us finally discuss some published experimental results on Landau's Fermi-liquid parameters  $F_1^s$  and  $F_0^a$  of the two-dimensional electron gas. Various attempts to extract from experiment Landau parameters have been made before [18–20]. However, the results of the fitting procedure of the experimental data with the proposed theory [21, 22] are rather controversial. The authors [18–20] concluded that the main effect is connected with the increase of the transport mass, but the results obtained

for the parameter  $F_0^a$  only have the same order of magnitude. Moreover, in these publications one never takes into account a possible difference between the transport mass and the compressibility mass. Another method is used in Ref. [23]. Here the data for SdH were analyzed. The raw data for  $m^*g^*$  in the corresponding interval of electron densities are in excellent agreement with data of Ref. [1]. In the attempt to find the transport mass and the effective  $g$  factor separately the authors supposed that the Dingle temperature is temperature dependent in the same manner as the inverse transport scattering time. We believe that this assumption may overestimate a possible temperature dependence of the Dingle temperature, because very different microscopic processes define the temperature dependence in these two cases.

In conclusion we argue that we obtained quantitative and qualitative results for Landau Fermi-liquid parameters of the two-dimensional electron gas as realized in silicon (100) and silicon (111). We believe that the obtained numbers need confirmation by additional experimental results, for instance from other systems, like GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterostructures and AlAs or AlP quantum wells.

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