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The running mass m_s at low scale from the heavy-light meson decay constants

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It is shown that a 25(20)% difference between the decay constants $f_{D_s}(f_{B_s})$ and $f_D(f_B)$ occurs due to large differences in the pole masses of the s and $d(u)$ quarks. The values $\eta_D = f_{D_s}/f_D \approx 1.23(15)$, recently observed in the CLEO experiment, and $\eta_B = f_{B_s}/f_B \approx 1.20$, obtained in unquenched lattice QCD, can be reached only if in the relativistic Hamiltonian the running mass m_s at low scale is $m_s(\sim 0.5\text{ GeV}) = 170 - 200$ MeV. Our results follow from the analytical expression for the pseudoscalar decay constant $f_{\rm P}$ based on the path-integral representation of the meson Green's function.

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Relativistic potential models (RPM) have been successful in their description of light-light and heavy-light (HL) meson spectra [1, 2]. Still there exists a fundamental problem, which remains partly unsolved up to now. It concerns the choice of the quark masses in the kinetic term of the relativistic Hamiltonian, which in different RPMs vary in a wide range. For example, HL mesons were studied with the use of the Dirac equation, taking for the light quark mass $m_n (n = u, d) = 7\,\text{MeV}$ in [3] and 72 MeV in [4], and in the Salpeter equation for the strange quark mass the values $m_s = 419$ MeV in [5] and $m_s = 180\,\text{MeV}$ in [6] have been used. However, in contrast to constituent quark models, where the constituent mass can be considered as a fitting parameter, a fundamental relativistic Hamiltonian has to contain only conventional quark masses–the pole masses. These masses are now well established for heavy and light quarks [7]. They have been used in the QCD string approach giving a good description of meson spectra [6, 8, 9]. However, the strange quark mass m_s is still not determined at low scale. At present, owing to the QCD sum rules calculations [10] and lattice QCD [11], m_s is well established at a rather large scale: $m_s(\mu = 2\,\text{GeV}) = 90 \pm 10\,\text{MeV}$, while in the Hamiltonian approach the mass m_s enters at a lower scale, which is evidently smaller that the scale $\mu_c \approx 1.2$ GeV for the c quark. Therefore it is very important to find physical quantities which are very sensitive to m_s at low scale: $\mu_s \le 1\,\text{GeV}$. In this letter we show that such information can be extracted from the analysis of the decay constants of HL mesons, namely, from the ratios f_{D_s}/f_D and f_{B_s}/f_B.

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Recently, direct measurements of the leptonic decay constants in the processes $D(D_s) \to \mu\nu_\mu$ [12, 13], and $B \to \tau\mu_\tau$ [14, 15] have been reported. In Refs. [12, 13] the CLEO collaboration gives $f_{D_s} = 274(20)\,\text{MeV}$ and $f_D = 222.6(20)\,\text{MeV}$ with $\eta_D = f_{D_s}/f_D = 1.23(15)$, having reached an accuracy much better than in previous experiments [16]. This central value for η_D appears to be larger than in many theoretical predictions which typically lie in the range 1.0 – 1.15 [17 – 20]. Therefore, one can expect that precise measurements of η_D and $\eta_{\rm B}$ in the future can become a very important criterium to distinguish different theoretical models and check their accuracy. In particular, relatively large values

$$\eta_D = 1.25(3), \quad \eta_B = 1.20(3), \tag{1}$$

have been obtained recently in lattice (unquenched) calculations [21, 22] and also in our paper [6]. In this letter we show that:

1. The running mass $m_s(\mu_1)$ at a low scale, $\mu_1 \approx 0.5$ GeV, can be extracted from the values η_D and η_B, if they are known with high accuracy, $\lesssim 5\%$.

2. The values η_D and η_B, as given in Eq. (1), can be obtained only if the running mass $m_s(\mu_1)$ lies inside the range 170–200 MeV. In particular, in the chiral limit, $m_d = m_u = 0$, as well as for $m_d = 8\,\text{MeV}$, and for $m_s(\mu_1) = 180$ MeV, the ratios η_D and η_B calculated here are

$$\eta_D = 1.25, \quad \eta_B = 1.19. \tag{2}$$

3. The value $m_s(0.5\text{ GeV})$ satisfies the relation

$$\frac{m_s(0.5\text{ GeV})}{m_s(2\text{ GeV})} \approx 1.97. \tag{3}$$

In our analysis we use the analytical expression for the leptonic decay constant in the pseudoscalar (P) channel, derived in Ref. [6] with the use of the path-integral representation for the correlator $G_{\rm P}$ of the currents $j_{\rm P}(x)$: $G_{\rm P}(x) = \langle j_{\rm P}(x) j_{\rm P}(0)\rangle_{\rm vac}$:

$$J_{\rm P} = \int G_{\rm P}(x) d^3x = 2N_c \sum_n \frac{\langle Y_{\rm P}\rangle_n |\varphi_n(0)|^2}{\omega_{qn}\omega_{Qn}} e^{-M_n T}, \tag{4}$$

where M_n and $\varphi_n(r)$ are the eigenvalues and eigenfunctions of the relativistic string Hamiltonian [23 – 25], while $\omega_{qn}(\omega_{Qn})$ is the average kinetic energy of a quark $q(Q)$ for a given nS state:

$$\omega_{qn} = \langle\sqrt{m_q^2 + \mathbf{p}^2}\rangle_{nS}, \quad \omega_{Qn} = \langle\sqrt{m_Q^2 + \mathbf{p}^2}\rangle_{nS}. \tag{5}$$

In Eq. (5), $m_q(m_Q)$ is the pole mass of the lighter (heavier) quark in a heavy-light meson. The matrix element $\langle Y_{\rm P}\rangle_n$ refers to the P channel (with exception of the π and K mesons where additional chiral terms occur) and was calculated in Ref. [6],

$$\langle Y_{\rm P}\rangle_n = m_q m_Q + \omega_{qn}\omega_{Qn} - \langle \mathbf{p}^2\rangle_{nS}. \tag{6}$$

On the other hand, for the integral $J_{\rm P}$ (4) one can use the conventional spectral decomposition:

$$J_{\rm P} = \int G_{\rm P}(x) d^3x = \sum_n \frac{1}{2M_n} (f_{\rm P}^n)^2 e^{-M_n T}. \tag{7}$$

Then from Eqs. (4) and (7) one obtains that

$$(f_{\rm P}^n)^2 = \frac{2N_c \langle Y_{\rm P}\rangle_n |\varphi_n(0)|^2}{\omega_{qn}\omega_{Qn} M_n}. \tag{8}$$

All necessary factors in Eq. (8) for the ground state ($n = 1$) and the first radial excitation ($n = 2$) are calculated in Ref. [6] but here we consider only ground states and omit the index n everywhere. Our calculations are performed with the static potential $V_0(r) = \sigma r - \frac{4}{3}\frac{\alpha_{\rm B}(r)}{r}$ [9], and the hyperfine and self-energy contributions are considered as a perturbation. It is important that our input parameters contain only fundamental values: the string tension σ, the QCD constant $\Lambda(n_f = 3)$ in $\alpha_{\rm B}(r)$, and the conventional pole quark masses. For the s-quark mass $m_s(\mu_1)$ one can expect that the scale μ_1 is close to the characteristic momentum $\mu_1 \approx \sqrt{\langle \mathbf{p}^2\rangle} \sim 0.5{-}0.6$ GeV. This scale also corresponds to the r.m.s. radii $R_M(1S)$ of the meson we consider. For HL mesons

$$R_D \approx R_{D_s} = 0.55(1)\ {\rm fm}, \quad R_B \approx R_{B_s} = 0.50(1)\ {\rm fm}, \tag{9}$$

so that $\mu_1 \sim R_M^{-1} \approx 0.4{-}0.5$ GeV. We show here that this mass $m_s(\mu_1)$ is strongly correlated with the values of η_D and η_B. For other quarks we take $m_c = 1.40$ GeV and $m_b = 4.78$ GeV [8].

It is of interest to notice that for HL mesons the ratios

$$\xi_D = \xi_{D_s} = \frac{|R_{1D}(0)|^2}{\omega_q \omega_c} = 0.347(3) \tag{10}$$

are equal for the D and D_s mesons with an accuracy better than 1%, and also that these fractions for B and B_s mesons coincide with $\lesssim 2\%$ accuracy ($\varphi_1^2(0) = R_1^2(0)/4\pi$):

$$\xi_B = \xi_{B_s} = \frac{|R_{1B}(0)|^2}{\omega_q \omega_b} = 0.146(2). \tag{11}$$

It is important that the equalities $\xi_D = \xi_{D_s}$ and $\xi_B = \xi_{B_s}$ practically do not depend on the details of the interaction in HL mesons. Therefore, in the ratios $\eta_D(\eta_B)$ the factors $\xi_D(\xi_B)$ cancel and one obtains

$$\eta^2_{D(B)} = \left(\frac{m_s m_{c(b)}}{\langle Y_{\rm P}\rangle_{D(B)}} + \frac{\omega_s \omega_{c(b)} - \langle \mathbf{p}^2\rangle_{D_s(B_s)}}{\langle Y_{\rm P}\rangle_{D(B)}} \right) \frac{M_{D(B)}}{M_{D_s(B_s)}}. \tag{12}$$

In Eq. (12) the second term is close to 1.05, while the first term, proportional to m_s, is not small, changing by 30–60% for different m_s (below we take m_s from the range $140 \pm 60\,{\rm MeV}/c^2$). With an accuracy of $\lesssim 2\%$

$$\eta_D^2 = 2.708 \times m_s({\rm GeV}) + 1.07(1), \ \text{if}\ m_d = m_u = 0,$$
$$\eta_{D^+}^2 = 2.648 \times m_s({\rm GeV}) + 1.05(1), \ \text{if}\ m_d = 8\ {\rm MeV}, \tag{13}$$

i.e., in the chiral limit

$$\eta_D = 1.14\ (m_s = 85\ {\rm MeV}),$$
$$1.25\ (m_s = 180\ {\rm MeV}), \quad 1.27\ (m_s = 200\ {\rm MeV}), \tag{14}$$

and for $m_d = 8$ MeV, $\eta_D = 1.13$, 1.24, and 1.26, respectively, for the same values of m_s, so decreasing only by $\sim 1\%$.

For the B and B_s mesons

$$\eta_B^2 = 1.90 \times m_s + 1.07(1) \quad (m_d = m = 0);$$
$$\eta_{B^0}^2 = 1.871 \times m_s + 1.07(1) \quad (m_d = 8\ {\rm MeV}), \tag{15}$$

which practically coincide, and in the chiral limit ($m_d = m_u = 0$)

$$\eta_{\rm B} = 1.11\ (m_s = 85\ {\rm MeV}),$$
$$1.19\ (m_s = 180\ {\rm MeV}), \quad 1.21\ (m_s = 200\ {\rm MeV}). \tag{16}$$

These values of η_B appear to be only by $3-5\%$ smaller than η_D.

Thus for $m_s = 180$ MeV and $m_d = 8$ MeV we have obtained

$$\eta_{D^+} = 1.25(2), \quad \eta_B = 1.19(1), \tag{17}$$

in good agreement with the CLEO data: $\eta_D(\exp) = 1.23(15)$ [13].

To check our choice of $m_s = 180\,\text{MeV}$, we estimate the ratio $m_s(0.5\text{ GeV})/m_s(2\,\text{GeV})$ using the conventional perturbative (one-loop) formula for the running mass [26]

$$m(\mu^2) = m_0 \left(\frac{1}{2}\ln\frac{\mu^2}{\Lambda^2}\right)^{-d_m}\left[1 - d_1\frac{\ln\ln\frac{\mu^2}{\Lambda^2}}{\ln\frac{\mu^2}{\Lambda^2}}\right]. \quad (18)$$

Here m_0 is an integration constant and the other constants are

$$d_1 = \frac{8}{\beta_0^3}\left(51 - \frac{19}{3}n_f\right),\ \beta_0 = 11 - \frac{2}{3}n_f,\ d_m = \frac{4}{\beta_0}. \quad (19)$$

To calculate $m_s(2\text{ GeV})$ we take $n_f = 4$, $\Lambda(n_f = 4) = 250\,\text{MeV}$ ($\beta_0 = 25/3$, $d_m = 12/25$, $d_1 = 0.3548$). We take $n_f = 3$, $\Lambda(n_f = 3) = 280\,\text{MeV}$ to estimate $m_s(1\,\text{GeV})$ and $m_s(0.5\,\text{GeV})$ (for $n_f = 3$, $\beta_0 = 9$, $d_m = 4/9$, $d_1 = 0.3512$). Then, from Eqs. (18,19) $m_s(2\,\text{GeV}) = 0.618\ m_0$, $m_s(1\,\text{GeV}) = 0.7825\ m_0$, and $m_s(0.5\,\text{GeV}) = 1.217\ m_0$ and therefore our estimates are

$$\frac{m_s(1\text{ GeV})}{m_s(2\text{ GeV})} = 1.27, \quad \frac{m_s(0.5\text{ GeV})}{m_s(2\text{ GeV})} = 1.97. \quad (20)$$

These estimates can be done because the QCD costant $\Lambda(n_f = 3) = 0.28\,\text{GeV}$ is relatively small, so that the scale $\mu_1 = 1\,\text{GeV}$, for which $\ln(\mu_1^2/\Lambda^2) = 2.55$, still lies far from the Landau singularity.

The ratio (20) means that $m_s(0.5\text{ GeV}) = 180$ MeV, which we have used in our calculations, corresponds to $m_s(2\text{ GeV}) = 91$ MeV which is in agreement with the conventional value of $m_s(2\text{ GeV}) = 90\pm15$ MeV [7]. Thus our estimate of $m_s(0.5\text{ GeV}) = 180$ MeV supports our choice of this mass in the relativistic string Hamiltonian, which provides a good description of the HL meson spectra and decay constants, and gives rise to the relatively large values of η_D and η_B in Eq. (1).

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