Pis'ma v ZhETF, vol. 87, iss. 2, pp. 75 – 80

Classical gravitational radiation from quasi-elastic particle scattering in models with extra dimensions

B. Koch[+*1)], *M. Bleicher*[+]

[+]*Institut für Theoretische Physik, J.W. Goethe Universität, 60438 Frankfurt am Main, Germany*

[*]*Frankfurt Institute for Advanced Studies (FIAS), 60438 Frankfurt am Main, Germany*

Submitted 29 November 2007

We derive an equation for the emission of classical gravitational waves due to quasi-elastic $N \to N$ particle scattering in a model with compactified extra dimensions. In addition to previous classical studies we also calculate additional terms that are suppressed by factors of one over frequency times compactification radius. From this a single formula is given which predicts the energy loss into gravitational radiation from elastic collisions as well in the low as in the high energy limit.

PACS: 04.50.–h, 12.60.–i

1. Motivation. The search for a unified theory of gravity and the standard model of particle physics has been an elusive goal in quantum field theory over the last decades. Several approaches to study quantum gravity (e.g. loop quantum gravity, SUGRA or string theory) are currently discussed. Especially string theory seems to be one of the most promising and widely studied approaches to reach this goal. A crucial ingredient of super string theory is that it needs more than three spatial dimensions for its consistency. Also, supergravity which is supposed to be the low energy effective description of an $3+d$ dimensional M-theory [1, 2] needs the feature of additional space like extra dimensions. Recently several simplified approaches to incorporate extra dimensions into low energy field theory have been proposed, see e.g. Refs. [3 – 7]. Most of these models (except the universal extra dimension scenario 6) have in common, that only gravity is allowed to propagate into the extra dimensions. In the ADD model 8, the extra dimensions are compactified which leads to a reduced Planck mass $\bar{M}_D$, which can be as low as a few TeV. Therefore such models the strong hierarchy between the standard Planck mass $\bar{M}_P$ and the electro-weak scale M_Z is softened for the reduced Planck mass $\bar{M}_D$. The gravitational coupling G_d is related to this reduced Planck mass by $G_d = 8\pi \bar{M}_D^{-(2+d)}$ and the reduced Planck mass is related to the standard Planck mass by

$$\bar{M}_P^2 = (2\pi)^d \bar{M}_D^{2+d} R^d \quad , \tag{1}$$

where R is the compactification radius of the d extra spatial dimensions [9]. Due to the reduced Planck mass all short ranging gravitational interactions will be amplified in such a scenario with large extra dimensions. Even the production probability for microscopical black holes in colliders would be strongly enhanced, so that black holes might be observable in future colliders, as it was widely discussed in the literature [10 – 22]. Although mini black holes in colliders or from high energetic cosmic rays would probably be the most prominent observable of large extra dimensions, their production might be suppressed [23 – 25] and the shape of their detector signals is still under investigation [26, 27]. Therefore it is expedient to look for additional signatures of large extra dimensions. In this paper, we discuss the emission of gravitational waves from the quasi-elastic interaction of two high energetic particles. Especially in the interaction of ultra high energetic particles the increased coupling strength of gravity might lead to observable effects already at energies accessible in next generation particle accelerators, e.g. LHC or ILC. Even more interesting are modifications of cosmic ray air shower properties which are induced by primary particles with energies exceeding 10^{20} eV. Especially on this field there is a lot of new experimental research made, which might give more access to particle phenomenology and large extra dimensions [28 – 37].

2. Einstein's equations with more dimensions. Einstein's field equations with $3+d$ spatial dimensions are a straight forward generalisation of the three dimensional case [38]. However all the indices N,M run from $0\ldots 3+d$ instead of $0\ldots 3$, i.e.

$$R_{MN} - \frac{1}{2} g_{MN} R = -8\pi G T_{MN}. \tag{2}$$

Or with the use of the trace of the Ricci tensor $R = R_M^M$

1)e-mail: koch@th.physik.uni-frankfurt.de

$$R_{MN} = -8\pi G(T_{MN} - \frac{1}{2+d} g_{MN} T^L_L), \quad (3)$$

and therefore the $3+d$ dimensional gravitational source term $\mathcal{S}_{MN}$ can be defined as

$$\mathcal{S}_{NM} := (T_{NM} - \frac{1}{2+d} g_{MN} T^L_L). \quad (4)$$

Assuming small perturbations h_{MN} from the $3+d$ dimensional Minkowski metric η_{NM} with the signature $(+,-,-,-,-,\dots)$ we choose the following ansatz for the metric tensor

$$g_{NM} = \eta_{MN} + h_{MN}. \quad (5)$$

Inserting this ansatz into Eq. (3), yields Einstein's field equations to first order in the perturbation h

$$\partial_L \partial^L h_{MN} - \partial_L \partial_N h^L_M - \partial_L \partial_M h^L_N + \partial_M \partial_N h^L_L = \\ = -8\pi G \mathcal{S}_{MN}. \quad (6)$$

By exploitig the gauge invariance of Einstein's field equations, the harmonic gauge is chosen, which reads in lowest order in h

$$\partial_L h^L_N = \frac{1}{2} \partial_L h^N_N. \quad (7)$$

Inserting Eq. (7) in Eq. (6) we find

$$\partial^L \partial_L h_{MN} = -16\pi G \mathcal{S}_{MN}. \quad (8)$$

3. The Greens function and polarization tensor in infinite extra dimensions. The retarded ($\tau = t - t_0 - |x-y| > 0$) solution of Eq. (8) can be obtained with the help of the $3+d$ dimensional Greens function $G^{(3+d)}(|x-y|)$. This gives

$$h_{MN} = N \int dt_0 \int d^{3+d}\underline{y} G^{(3+d)}_{ret}(t-t_0, |\underline{x}-\underline{y}|) \mathcal{S}_{MN}(t_0, \underline{y}), \quad (9)$$

with a normalisation constant $N = -16\pi G$. Let us examine the $3+d$-dimensional retarded Greens function [39–41] closer:

$$G^{3+d}_{ret} = -\frac{1}{(2\pi)^{4+d}} \int d^{3+d}\underline{k} e^{i\underline{k}\underline{x}} \int dk_0 \frac{e^{-ik_0(t-T_0)}}{(k_0 - i\epsilon)^2 - \underline{k}^2}. \quad (10)$$

For an even number of flat extra dimensions the integrands can be analytically evaluated [40] and give

$$G^{3+d}_{ret} = \frac{1}{4\pi} \left[\frac{-1}{2\pi r} \frac{\partial}{\partial r} \right]^{d/2} \left[\frac{\delta((t-t_0)-r)}{r} \right], \; d \text{ even}. \quad (11)$$

Note that this is true only for infinitely expanded extra dimensions or in the case of compact extra dimensions (see Eq. (1)) as long as $r < R$. The case $r > R$ will be discussed in the next section. In the present study, we restrain ourselves to the discussion of scenarios with even numbers of extra dimension. It is convenient to shift all derivatives in Eq. (11) to the right hand side. Therefore we define the commutator brackets

$$\begin{aligned} [\partial_r, 1/r]_{-1} &:= 1, \\ [\partial_r, 1/r]_0 &:= 1/r, \\ [\partial_r, 1/r]_1 &:= -1/r^2, \\ &\dots \\ [\partial_r, 1/r]_n &:= (-1)^n n! \frac{1}{r^{n+1}}. \end{aligned} \quad (12)$$

Now we decompose $(\partial_r, 1/r)^n \delta$ into a number ($\mathrm{A}(k,n)$) times the k^{th} derivative of the δ-function with respect to its argument

$$(\partial_r, 1/r)^n \delta := \sum_{k=0}^{n} A(k,n) \delta^{(k)}. \quad (13)$$

Using the definitions Eqs. (12) and (13) a recursion equation for Eq. (11) can be obtained. Thus, knowing the Greens function for $d-2$ extra dimensions the Greens function for d extra dimensions can be calculated from

$$G^{3+d}_{ret} = \frac{1}{4\pi r} \left(\frac{1}{2\pi} \right)^{d/2} \sum_{i=0}^{d/2} \delta^{(i)} [(t-t_0) - r] \times \\ \times \left\{ \sum_{l=0}^{d/2-i} \left| \left[\partial_r, \frac{1}{r} \right]_l \right| \mathrm{A}(l+i-1, d/2-1) \frac{(l+i)!}{l! i!} \right\} \\ := \frac{1}{4\pi r} \left(\frac{1}{2\pi} \right)^{d/2} \sum_{i=0}^{d/2} K_d(r,i) \delta^{(i)} [(t-t_0) - r]. \quad (14)$$

Next we assume that the observer ($|\underline{x}|$) is at large distance in comparison to the extension of the source ($|\underline{y}|$). This means for $|\underline{x}| \gg |\underline{y}|$ that

$$\tau = t - t_0 - |\underline{x} - \underline{y}| \approx t - t_0 - |\underline{x}| + \underline{y} \frac{\underline{x}}{|\underline{x}|}. \quad (15)$$

From this the far field approximation of the gravitational wave can be written as

$$h^{(0)}_{MN}(x) = \int d\omega \exp(-i\omega(t - |\underline{x}|)) e_{MN}(\underline{x}, \omega). \quad (16)$$

The polarization tensor e_{MN} is given by

$$e_{MN} = \frac{N}{2(2\pi)^{(d+3)/2}} \sum_{j=0}^{d/2} \frac{K_d(|\underline{x}|, j)}{|\underline{x}|} (i\omega)^j \hat{\mathcal{S}}_{MN}(\omega) + \text{c.c.}, \quad (17)$$

where

$$\hat{\mathcal{S}}_{MN}(\omega) = \int d^3y d^dy_\perp \mathcal{S} \exp\left(-i\omega \underline{y}\frac{\underline{x}}{|\underline{x}|}\right) \tag{18}$$

and

$$\mathcal{S}_{MN}(\tau, \underline{y}) = \frac{1}{\sqrt{2\pi}} \int d\omega \exp\left(-i\omega(t - |\underline{x}|)\right) e_{MN}(\underline{x}, \omega), \tag{19}$$

The charge conjugated part (abbr. as c.c.) is not shown explicitly, but is taken into account in the further calculations. When calculating the source term $\mathcal{S}_{MN}(\omega, \underline{y})$ it is useful to remember that the "time" coordinate corresponding to ω is τ.

4. The Greens function in compactified space. In the ADD model the greens functions for gravitational radiation have to fulfill the boundary conditions

$$G^{3+d}_{\text{ret}}(t, \mathbf{x}, y_i) = G^{3+d}_{\text{ret}}(t, \mathbf{x}, y_i + 2\pi R). \tag{20}$$

Therefore $G^{3+d}_{\text{ret}}(t, \mathbf{x}, y_i)$ can be expressed in form of its discrete Fourier modes

$$G^{3+d}_{\text{ret}}(t, \mathbf{x}, y) = \sum_{(n)} \frac{G^{3+d\ (n)}_{\text{ret}}(t, \mathbf{x})}{\sqrt{V_d}} \exp\left(i\frac{n^j y_j}{R}\right), \tag{21}$$

where $(n) = (n_1, .., n_d)$ and V_d is the volume of the compactified space

$$V_d = (2\pi R)^d. \tag{22}$$

From the ansatz (21) the retarded Greens function for Eq. (8) is found by the reverse transformation of the solution in Fourier space

$$G^{3+d}_{\text{ret}} = \frac{-1}{(2\pi)^4\sqrt{V_d}} \times$$
$$\times \sum_{(n)} \int d^3\mathbf{k} \int dk_0 \frac{e^{i\mathbf{k}\mathbf{x} - ik_0(t) + i\sum_i \frac{n^i y_i}{R}}}{(k_0 - i\epsilon)^2 - \mathbf{k}^2 - \sum_i \frac{n^i y_i}{R}}. \tag{23}$$

After performing a contour integral for $\int dk_0$ and the angular integrals from $d^3\mathbf{k} = |\mathbf{k}|^2 \cos(\beta) d|\mathbf{k}|\, d\alpha\, d\beta$ one is left with

$$G^{3+d}_{\text{ret}} = \frac{1}{(2\pi)^2\sqrt{V_d}} \sum_{(n)} \int d|\mathbf{k}| e^{i\sum_i \frac{n^i y_i}{R}}$$
$$\frac{\theta(t)|\mathbf{k}| \sin\left(t\sqrt{|\mathbf{k}|^2 + \sum_i \frac{n^i y_i}{R}}\right) \sin(|\mathbf{k}||\mathbf{x}|)}{|\mathbf{x}|\sqrt{|\mathbf{k}|^2 + \sum_i \frac{n^i y_i}{R}}}. \tag{24}$$

Due to the non-trivial factors $\sqrt{|\mathbf{k}|^2 + \sum_i (n^i y_i)/R}$, the integral $\int d|\mathbf{k}|$ can not be performed in general. Still from Eq. (24) the long wave length limit $\omega \to 0$ can be obtained by taking the $(n) = (0, ...0)$ part of $\sum_{(n)}$. This corresponds to the case of mass-less gravitons in field theory where the integral $\int d|\mathbf{k}|$ can be performed and gives

$$G^{3+d}_{\text{ret}}(t, \mathbf{x}, y)|_{n=0} = \frac{1}{4\pi\sqrt{V_d}} \frac{\delta(t - |\mathbf{x}|)}{|\mathbf{x}|}. \tag{25}$$

So we have shown that in the long wave length limit $\omega \to 0$ the polarization tensor (17) has to approach

$$e_{MN}(x, \omega)_{\omega\to 0} \to N \frac{1}{4\pi\sqrt{2\pi}\sqrt{V_d}} \frac{1}{|\underline{x}|} \hat{\mathcal{S}}_{MN}(\omega) + \text{c.c.} \tag{26}$$

and therefore

$$\left.\sum_{j=0}^{d/2} K_d(|\underline{x}|, j)\right|_{\omega\to 0} \to K_d(|\underline{x}|, 0) = \frac{(2\pi)^{d/2}}{\sqrt{V_d}}. \tag{27}$$

5. Gravitational radiation from quasi-elastic scattering. The energy and momentum of a gravitational wave is given by

$$\langle t_{MN} \rangle = \langle R^{(2)}_{MN} \rangle =$$
$$= \frac{k_M k_N}{16\pi G} (\langle e^{SL*}(\underline{x}, \tau) e_{SL}(\underline{x}, \tau) \rangle - \frac{1}{2} |\langle e^L_L \rangle|^2). \tag{28}$$

For a quasi-elastic collision the source tensor that appears in the polarization tensor e_{MN} can be split up into a part for the incoming particles

$$\hat{\mathcal{S}}^{(in)}_{MN}(\omega) =: \hat{T}^{(in)}_{MN} - \eta_{MN}\hat{T}^{(in)L}_L =$$
$$= (\eta_{M\mu}\eta_{N\nu} - \frac{\eta_{MN}\eta_{\mu\nu}}{2+d}) \sum_{j=1}^{C} \frac{P^\mu_{(j)} P^\nu_{(j)}}{P_{(j)}k}, \tag{29}$$

and for the outgoing particles

$$\hat{\mathcal{S}}^{(out)}_{MN}(\omega) =: \hat{T}^{(out)}_{MN} - \eta_{MN}\hat{T}^{(out)L}_L$$
$$= (\frac{\eta_{MN}\eta_{\mu\nu}}{2+d} - \eta_{M\mu}\eta_{N\nu}) \sum_{j=1}^{C} \frac{P^\mu_{(j)} P^\nu_{(j)}}{P_{(j)}k}, \tag{30}$$

where k is the $(4+d)$ momentum of the emitted gravitational wave and $P^\mu_{(i)}$ are the four momenta of the scattering particles. Note that for the scattering standard model particles the energy momentum tensor of the ADD model

$$T_{MN}(x) = \eta^\mu_M \eta^\nu_N T_{\mu\nu}(x) \delta^d(x_\perp). \tag{31}$$

was used. The connection between radiated energy and the energy momentum tensor carrying this energy is

$$\frac{dE}{d\Omega d\omega} = |x|^{2+d} n_i \langle t^{0i} \rangle. \tag{32}$$

For the case of gravitational waves this is

$$\frac{dE}{d\Omega d\omega} = |x|^{2+d}\frac{\omega^2}{16\pi}(\langle e^{SL*}(\underline{x},\omega)e_{SL}(\underline{x},\omega)\rangle - \frac{1}{2}|\langle e^L_L\rangle|^2), \quad (33)$$

where $k_0^2 = k_i^2 = \omega^2$ was used. One can now insert the polarization tensor (17)

$$\langle e_{MN}\rangle = \frac{N\hat{\mathcal{S}}_{MN}(\omega)}{2(2\pi)^{(d+3)/2}}\langle\sum_{j=0}^{d/2} K_d(|\underline{x}|, j)\frac{1}{|\underline{x}|}(i\omega)^j\rangle, \quad (34)$$

into Eq. (33), where $\hat{\mathcal{S}}_{MN}(\omega) = (\hat{\mathcal{S}}_{MN}(\omega)^{(in)} + \hat{\mathcal{S}}_{MN}(\omega)^{(out)})$. By defining

$$\eta_I = \begin{cases} +1 & \text{for a particle in the initial state,} \\ -1 & \text{for a particle in the final state,} \end{cases} \quad (35)$$

the energy radiated gravitational radiation, which is induced by a quasi elastic $N \to N$ particle scattering is found to be

$$\frac{dE}{d\Omega d\omega} = \frac{G_d\,|\underline{x}^{2+d}|}{2\pi^2(2\pi)^d}\times$$
$$\times\sum_{j,k=0}^{d/2}\langle K_d(|\underline{x}|, j)K_d(|\underline{x}|, k)\frac{1}{|\underline{x}|^2}(i\omega)^{j+k}\rangle\times$$
$$\sum_{I,J}\frac{\eta_I\eta_J}{(P_{(I)}k)(P_{(J)}k)}\left[(P^\mu_{(I)}P_{(J)\mu})^2 - \frac{1}{2+d}P^2_{(I)}P^2_{(J)}\right], \quad (36)$$

with G_d being the d-dimensional gravitational constant. In the limit without extra dimensions ($d = 0$), Eq. (36) agrees with the well known result given by Weinberg [42]. For $d > 0$ one obtains additional contributions: There is always a ω^{d+2} dependence and there are terms with the same mass dimension, but containing a $\omega^{d+2-i}/|\underline{x}|^i$ dependence. For an uncompactified $3+d$ dimensional space the additional terms vanish for a distant observer and only the ω^{2+d} term survives, in line with [41].

6. Matching the obtained cross sections to compactified spaces. The compactification of the d extra dimensions is expected to have two consequences on Eq. (36). The first is the change of the gravitational coupling $G \to G_d = 1/\bar{M}_D^{2+d}$. The second is the change in the ω dependence of the cross section. To calculate the ω dependence in a compactified space one has to fulfill periodical boundary conditions and use the Greens function given in the appendix (24). Unfortunately the integrals in (24) only allow direct integration for two extreme regimes ($\omega \to 0$) and ($\omega \to \infty$). Therefore we use Eq. (36) as an effective model for the intermediate regimes and take $|\underline{x}|$ as the parameter of this model. This parameter is fixed by demanding, that the case without extra dimensions is the long wave length limit ($\omega \to 0$) of the kinematical part of (36)

$$\lim_{\omega\to 0}\int dV_{E\,d}\frac{dE(d)}{d\Omega_{3+d}d\omega} = \frac{dE(d=0)}{d\Omega_3 d\omega}$$
$$\text{in lowest order in}\,\omega^{\mathrm{i}} \Rightarrow$$
$$|\underline{x}| = \begin{cases} 2\pi R & \text{for} \quad d=2 \\ 2\pi R\sqrt{3} & \text{for} \quad d=4 \\ 2\pi R\sqrt[3]{15} & \text{for} \quad d=6 \end{cases} \quad (37)$$

Those values for the parameter $|\underline{x}|$ support the intuitive guess that $|\underline{x}|$ has to be of the order of the compactification perimeter. From those two arguments we find the radiated energy in models with compactified extra dimensions

$$\frac{dE_0}{d\Omega d\omega} = \frac{1}{\bar{M}_P^2}\frac{4}{\pi}\omega^2\sum_{I,J}\frac{\eta_I\eta_J\left[(P^\mu_{(I)}P_{(J)\mu})^2 - \frac{1}{2}P^2_{(I)}\,P^2_{(J)}\right]}{(P_{(I)}k)(P_{(J)}k)},$$
$$\frac{dE_2}{d\Omega d\omega} = \frac{\omega^4 + \frac{2\omega^3}{|2\pi R|} + \frac{\omega^2}{|2\pi R|^2}}{\bar{M}_D^4\pi^3}\sum_{I,J}\frac{\eta_I\eta_J\left[(P^\mu_{(I)}P_{(J)\mu})^2 - \frac{1}{4}P^2_{(I)}\,P^2_{(J)}\right]}{(P_{(I)}k)(P_{(J)}k)},$$
$$\frac{dE_4}{d\Omega d\omega} = \frac{\omega^6 + \frac{6\omega^5}{|2\pi R|\sqrt{3}} + \frac{5\omega^4}{|2\pi R|^2} + \frac{6\omega^3}{|2\pi R|^3\sqrt{3}} + \frac{\omega^2}{|2\pi R|^4}}{\bar{M}_D^6 4\pi^5}\times$$
$$\times\sum_{I,J}\frac{\eta_I\eta_J\left[(P^\mu_{(I)}P_{(J)\mu})^2 - \frac{1}{6}P^2_{(I)}\,P^2_{(J)}\right]}{(P_{(I)}k)(P_{(J)}k)},$$
$$\frac{dE_6}{d\Omega d\omega} = \left(\omega^8 + \frac{12\omega^7}{|2\pi R|\sqrt[3]{15}} + \frac{66\omega^6}{|2\pi R|^2 15^{2/3}} + \frac{14\omega^5}{|2\pi R|^3} + \right.$$
$$\left. + \frac{41\omega^4}{3|2\pi R|^4\sqrt[3]{15}} + \frac{30\omega^3}{|2\pi R|^5 15^{2/3}} + \frac{\omega^2}{|2\pi R|^6}\right)\times$$
$$\times\frac{1}{\bar{M}_D^8 16\pi^7}\sum_{I,J}\frac{\eta_I\eta_J\left[(P^\mu_{(I)}P_{(J)\mu})^2 - \frac{1}{8}P^2_{(I)}\,P^2_{(J)}\right]}{(P_{(I)}k)(P_{(J)}k)}. \quad (38)$$

From we Eq. (38) we see that the radiated energy increases rapidly with ω. Therefore a cut-off has to be used to estimate the amount of gravitationally radiated energy. In a $2 \to 2$ particle scattering process (radiating gravitational waves) the largest cut-off is reached as soon as the gravitational radiation takes away the invariant energy $\sqrt{s}/2$ from one of the participants. Strongest suppression of the $1/R$ terms is reached when one takes this extreme value for ω. Limits on the compactification radius down to the μm range (depending on d) have been derived from a large number of physical observations [43–46]. Thus, under the condition of

$$\omega \gg \frac{1}{2\pi R}, \quad (39)$$

Eq. (38) transforms to the result previously obtained by [41]. This shows that the additional terms become important for small $\sqrt{s}$ or very large $\bar{M}_D$. For a particle scattering with invariant energy in the TeV range, $\bar{M}_D$ would have to be > 1000 TeV, for the new terms to be relevant. However, then the whole cross-section is suppressed by a factor $1/\bar{M}_D^{2+d}$ and would be negligible. Summarizing one can say that for quasi-elastic high energetic $N \to N$ particle collisions in models with large extra dimensions the energy loss into gravitational radiation stays as given in [41]:

$$\frac{dE}{d\Omega d\omega} = \frac{4\omega^{2+d}}{\bar{M}_D^{2+d}\pi(2\pi)^d} \times \\ \times \sum_{I,J} \frac{\eta_I\eta_J \left[(P^{\mu}_{(I)}P_{(J)\mu})^2 - \frac{1}{2+d}P^2_{(I)}\, P^2_{(J)}\right]}{(P_{(I)}k)(P_{(J)}k)}. \quad (40)$$

This result is valid for quasi-elastic $N \to N$ particle scattering with highly relativistic particle velocities so that the interaction can be approximated to be instantaneous. Although the discussed terms are negligible in the standard ADD setup, still they might become important for similar models on negatively curved manifolds [47, 48]. Equation (38) was derived from classical general relativity and gives an semi-quantitative estimate for the gravitationally radiated energy. A quantum calculation for example in the ADD model was not performed, but should be considered as the next step to do.

7. Summary. The objective of this paper was to derive an effective equation of the energy emitted into gravitational radiation (38) in the quasi-elastic scattering approximation in models with extra dimensions that are compactified on a radius R. It allows quantitative predictions not only for the extreme cases ($\omega \gg 1/R$ or $\omega \to 0$) but also in the intermediate regimes ($\omega \approx 1/R$). Then we showed that for models with large compactification radii (compared to the wave length of the gravitational radiation) this transforms into Eq. (40) in line with Ref. [41] and that Eq. (38) is constructed such that the classical Weinberg formula [42] is obtained from equation (38) in the limit of small compactification radii (compared to the wave length). We discussed that for most of the physical applications within the ADD model either the total energy radiation or the new terms are negligible, but this might change for similar models on negatively curved manifolds [47, 48].

The authors thank S. Hofmann, U. Harbach and S. Hossenfelder for fruitful discussions. B.K. thanks the Frankfurt International Graduate School of Science (FIGSS) for financial support through a PhD-fellowship. This work was supported by GSI and BMBF.

---

1. E. Witten, Nucl. Phys. B **443**, 126 (1995).
2. P. Horava and E. Witten, Nucl. Phys. B **460**, 506 (1996).
3. L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
4. L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
5. N. Arkani-Hamed, S. Dimopoulos, and G.R. Dvali, Phys. Rev. D **59**, 086004 (1999).
6. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G.R. Dvali, Phys. Lett. B **436**, 257 (1998).
7. H. Kodama and K. Uzawa, *Comments on the four-dimensional effective theory for warped compacti fication*, 2005.
8. N. Arkani-Hamed, S. Dimopoulos, and G.R. Dvali, Phys. Lett. B **429**, 263 (1998).
9. G.F. Giudice, R. Rattazzi, and J.D. Wells, Nud. Phys. B **544**, 3 (1999).
10. P.C. Argyres, S. Dimopoulos, and J. March-Russell, Phys. Lett B **441**, 96 (1998).
11. T. Banks and W. Fischler, *A model for high energy scattering in quantum gravity*, 1999.
12. R. Emparan, G.T. Horowitz, and R.C. Myers, Phys. Rev. Lett. **85**, 499 (2000).
13. S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. **87**, 161602 (2001).
14. S.B. Giddings and S.D. Thomas, Phys. Rev. D **65**, 056010 (2002).
15. S. Hossenfelder, S. Hofmann, M. Bleicher, and H. Stocker, Phys. Rev. D **66**, 101502 (2002).
16. M. Bleicher, S. Hofmann, S. Hossenfelder, and H. Stocker, Phys. Lett. **548**, 73 (2002).
17. S.B. Giddings, *Black hole production in tev-scale gravity, and the future of high energy physics*, eConf, C **010630**, 328 (2001).
18. S.N. Solodukhin, Phys. Lett. B **533**, 153 (2002).
19. A. Jevicki and J. Thaler, Phys. Rev. D **66**, 024041 (2002).
20. I. Mocioiu, Y. Nara, and I. Sarcevic, Phys. Lett. B **557**, 87 (2003).
21. V.S. Rychkov, Phys. Rev. D **70**, 044003 (2004).
22. B. Koch, M. Bleicher, and S. Hossenfelder, JHEP **10**, 053 (2005).
23. M.B. Voloshin, Phys. Lett. B **524**, 376 (2002).
24. T.G. Rizzo, JHEP **02**, 011 (2002).
25. S. Hossenfelder, Phys. Lett. B **598**, 92 (2004).
26. C.M. Harris, P. Richardson, and B.R. Webber, JHEP **08**, 033 (2003).
27. M. Cavaglia, R. Godang, L. Cremaldi, and D. Summers, *Catfish: A monte carlo simulator for black holes at the Ihc*, 2006.
28. G. Domokos and S. Kovesi-Domokos, Phys. Rev. Lett. **82**, 1366 (1999).
29. R. Emparan, M. Masip, and R. Rattazzi, Phys. Rev. D **65**, 064023 (2002).

30. A. Ringwald and H. Tu, Phys. Lett. B **525**, 135 (2002).
31. L. A. Anchordoqui, J. L. Feng, H. Goldberg, and A. D. Shapere, Phys. Rev. D **65**, 124027 (2002).
32. B. Koch, H.-J. Drescher, and M. Bleicher, Astropart. Phys. **25/5**, 291 (2006).
33. D. Kazanas and A. Nicolaidis, Gen. Rel. Grav. **35**, 1117 (2003).
34. J. Alvarez-Muniz, J. L. Feng, F. Halzen et al., Phys. Rev. D **65**, 124015 (2002).
35. G. Bertone, G. Servant, and G. Sigl, Phys. Rev. D **68**, 044008 (2003).
36. L. A. Anchordoqui, J. L. Feng, H. Goldberg, and A. D. Shapere, Phys. Rev. D **68**, 104025 (2003).
37. L. A. Anchordoqui et al., Phys. Rev. D **72**, 065019 (2005).
38. R. C. Myers and M. J. Perry, Ann. Phys. **172**, 304 (1986).
39. S. Hassani, *Mathematical physics*, Springer-Verlag, New York, 1972.
40. D. V. Galtsov, Phys. Rev. D **66**, 025016 (2002).
41. V. Cardoso, O. J. C. Dias, and J. P. S. Lemos, Phys. Rev. D **67**, 064026 (2003).
42. S. Weinberg, *Gravitation and cosmology*, ISBN 0-471-92567-5, 1972.
43. L. Anchordoqui, T. Paul, S. Reucroft, and J. Swain, Int. J. Mod. Phys. A **18**, 2229 (2003).
44. S. Hannestad and G. G. Raffelt, Phys. Rev. Lett. **88**, 071301 (2002).
45. J. D. Barrow, Phys. Rev. D **35**, 1805 (1987).
46. S. Hossenfelder et al., Phys. Lett. B **575**, 85 (2003).
47. G. D. Starkman, D. Stojkovic, and M. Trodden, Phys. Rev. D **63**, 103511 (2001).
48. G. D. Starkman, D. Stojkovic, and M. Trodden, Phys. Rev. Lett. **87**, 231303 (2001).