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Di-pion emission in heavy quarkonia decays

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The di-pion spectrum for the $\Upsilon(nS) \to \Upsilon(n'S)$ transition with $n \le 4$ has the form $dw/dq \sim$ (phase space) $|\eta - x|^2$, with $x = \frac{q^2-4m_\pi^2}{(\Delta M)^2-4m_\pi^2} < q^2 \equiv M_{\pi\pi}^2$, and $\Delta M = M(nS) - M(n'S)$. The parameter η is calculated and the spectrum is shown to reproduce the experimental data for all 3 types of decays: $3 \to 1, 2 \to 1$ and $3 \to 2$ with $\eta \approx 0.5; 0$, and -3, respectively.

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1. The di-pion decays of heavy quarkonia are studied for the last three decades, (see [1, 2] for references and discussion). The first observed process $V(2S) \to \to V(1S)\pi\pi, V = \psi$ or Υ yields a simple di-pion spectrum, with the amplitude $M \sim (q^2 - 4m_\pi^2)$, and theoretical methods have been envisaged (see [2] for a review and references), based on PCAC and the multipole gluon field expansion (MGFE). However, the $3 \to 1$ and $3 \to 2$ di-pion transitions in bottomonium, observed by CLEO [3], show two other types of spectra: a double peaked spectrum for $(3 \to 1)$ and shallow form for $(3 \to 2)$. Numerous modifications of MGFE and additional models were suggested [1, 2], without, however, a unique physical picture for all 3 types of decays. It is a purpose of this short note to sketch the general mechanism of di-pion transitions, leaving details to the full text in [1].

2. Our starting point is the Field Correlator Method (FCM) [4], which is based on the use of Gaussian (quadratic) field correlators, applicable, in contrast to MGFE, also for systems of large size, $R \gg \lambda$, where $\lambda \lesssim 0.2$ fm is the vacuum correlation length [4]. As a result all large systems display linear confinement and the string tension σ defines the dynamics instead of gluon condensates as in MGFE. It is important that all decaying $X(n)$ states have $R \ge 0.4$ fm $> \lambda$, and therefore should be treated in FCM rather than in MGFE approach.

The pion emission in FCM comes from the light quark loop and is decribed by the quark-pion Lagrangian [5]

$$S_{QM} = -i \int d^4x \bar{\psi}(x) M_{br} \hat{U}(x) \psi(x) \qquad (1)$$

with M_{br} treated here and in [1] as a fitting parameter, found from the pionless decay, and $\hat{U}(x)$ is a chiral matrix

$$\hat{U}(x) = \exp\left(i\gamma_5 \frac{\varphi_a \lambda_a}{f_\pi}\right);$$

$$\varphi_a\lambda_a \equiv \sqrt{2}\begin{pmatrix} \frac{\eta}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}}, & \pi^+, & K^+ \\ \pi^-, & \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}}, & K^0 \\ K^-, & \bar{K}_0, & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \qquad (2)$$

Note, that $\hat{U}(x)$ describes creation of any number of $\pi(K, \eta)$ through small dimensional factor $f_\pi \cong 93$ MeV in the denominator. This ensures strong interaction and possible multipion-quarkonuim resonances.

The $Q\bar{Q}$ Green's function with light quark loop inside and with two possible ways of emission is shown in Fig.1a and b. We shall keep here notations "a" and "b" for the amplitudes with subsequent one-pion and zero-pion–two-pion emissions, respectively.

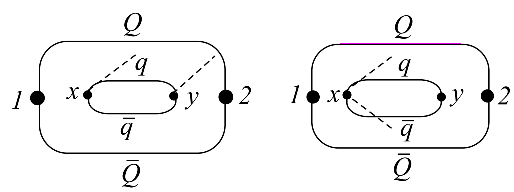

Рис.1

Using notations n and n' for $X(nS)$ and $X(n'S)$ states, and n_2, n_3 the $Q\bar{q}$ and $\bar{Q}q$ states, respectively, one can write for the amplitudes a and b,

$$a_{nn'} = \\ = \gamma \sum_{n_2 n_3} \int \frac{d^3p}{(2\pi)^3} \frac{J^{(1)}_{nn_2n_3}(\mathbf{p}, \mathbf{k}_1) J^{*(1)}_{n'n_2n_3}(\mathbf{p}, \mathbf{k}_2)}{E - E_{n_2n_3}(\mathbf{p}) - E_\pi(\mathbf{k}_1)} + (1 \leftrightarrow 2), \qquad (3)$$

$$b_{nn'} = \gamma \sum_{n_2' n_3'} \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{J^{(2)}_{nn_2'n_3'}(\mathbf{p}, \mathbf{k}_1, \mathbf{k}_2) J^*_{n'n_2'n_3'}(\mathbf{p})}{E - E_{n_2'n_3'}(\mathbf{p}) - E_\pi(\mathbf{k}_1, \mathbf{k}_2)} + \right. \\ \left. + \frac{J_{nn_2''n_3''}(\mathbf{p}) J^{*(2)}_{n'n_2''n_3''}(\mathbf{p}, \mathbf{k}_1, \mathbf{k}_2)}{E - E_{n_2''n_3''}(\mathbf{p})} \right\}. \qquad (4)$$

Here $\gamma = M_{br}^2/N_c$, $J, J^{(1)}, J^{(2)}$ are overlap integrals of heavy quarkonia wave functions $\psi_{Q\bar{Q}}$ and the product of $Q\bar{q}$ and $\bar{Q}q$ wave functions $\psi_{Q\bar{q}} \cdot \psi_{\bar{Q}q}$, multiplied by the exponentials of free relative motion $\exp(i\mathbf{pr})$ with momentum $\mathbf{p}$, and the pion plane waves with momenta $\mathbf{k}_1$ and $\mathbf{k}_2$.

The resulting decay probability is

$$\frac{dw}{dq}(n\to n') \cong \frac{2}{\pi^3 N_c^2}\left(\frac{M_{br}}{f_\pi}\right)^4 \mu^2\sqrt{x(1-x)}\,d\cos\theta|a-b|^2, \tag{5}$$

where $\mu^2 = (\Delta M)^2 - 4m_\pi^2$, $x = (q^2 - 4m_\pi^2)/\mu^2$, $q^2 \equiv \equiv M_{\pi\pi}^2 = (k_1+k_2)^2$ and θ is the angle of emitted π^+ with initial $X(nS)$ direction. It is important, that a and b depend differently on q, θ: in b, the vectors $\mathbf{k}_1, \mathbf{k}_2$ enter as a sum $\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2$, while in a one has a product of decreasing functions of $|\mathbf{k}_1|$ and $|\mathbf{k}_2|$. E.g. in the Simple Harmonic Oscillator (SHO) basis one can write a form, exactly satisfying the Adler zero condition:

$$\mathcal{M} \equiv a - b = \text{const}\left(e^{-\frac{\mathbf{k}_1^2+\mathbf{k}_2^2}{4\beta_2^2}}\left(\frac{1}{\tau_{nn'}(\omega_1)+\omega_1} + \frac{1}{\tau_{nn'}(\omega_2)+\omega_2}\right) - e^{-\frac{(\mathbf{k}_1+\mathbf{k}_2)^2}{4\beta_2^2}}\left(\frac{1}{\tau_{nn'}(0)} + \frac{1}{\tau_{nn'}(\omega_1+\omega_2)+\omega_1+\omega_2}\right)\right). \tag{6}$$

Here $1/(\tau+\omega) = 1/(\langle\frac{\mathbf{p}^2}{2\tilde{M}}\rangle + \omega)$ and β_2 is the SHO parameter for the B, B^* wave functions. Fitted to known r.m.s. radius one has $\beta_2 = 0.5$ GeV. Now one can see, that $\mathbf{k}_1^2 + \mathbf{k}_2^2 = \alpha(q) + \gamma(q)\cos^2\theta$ is a weak but increasing function of q, while $\mathbf{K}^2$ is strongly decreasing function of q. Expanding a, b in powers of x at the threshold, one arrives at the expression:

$$\frac{dw}{dq} = \frac{2}{\pi^3 N_c^2}\left(\frac{M_{br}}{f_\pi}\right)^4 \frac{\mu^6}{(4\beta_2^2)^2} d\cos\theta\sqrt{x(1-x)}\,b_{th}^2|\eta - x|^2. \tag{7}$$

Here η is a real parameter below $B\bar{B}$ threshold. We have calculated the values of η for $\Upsilon(nS) \to \Upsilon(n'S)\pi\pi$, using the SHO basis with parameters fitted to all r.m.s. radii of states and finding η from (6), where $\tau_{nn'}$ were found from $\langle\mathbf{p}^2/2\tilde{M}\rangle$. The intermediate states in all three transitions $(3\to1)$, $(2\to1)$, and $(3\to2)$ were chosen as $(B\bar{B}^* + B^*\bar{B})$ for the amplitude a, and $(B\bar{B})$ for the amplitude b. We have checked that the Adler zero condition is fulfilled when the full set of intermediate states is involved, and imposed the Adler zero condition in our case of restricted set of intermediate states, i.e. we have used the form (6) with $a(k_1 = \omega_1 = 0, k_2) = b(k_1 = = \omega_1 = 0, k_2)$.

As a result we have obtained the following values of η: $\eta(3\to1) \cong 0.39$, $\eta(2\to1) = 0.05$ and $\eta(3\to2) = -3.2$. On the other hand we have independently fitted the three di-pion spectra measured by CLEO [3] using the form

$$\frac{dw(n\to n')}{dq} = \text{const}\sqrt{x(1-x)}|\eta_{\text{fit}} - x|^2, \tag{8}$$

with constant and η_{fit} as free parameters and found that $\eta_{\text{fit}}(3\to1) \approx 0.5$, $\eta_{\text{fit}}(2\to1) = 0$, $\eta_{\text{fit}}(3\to2) \approx -3$. Results of this fitting are shown in Fig.2.

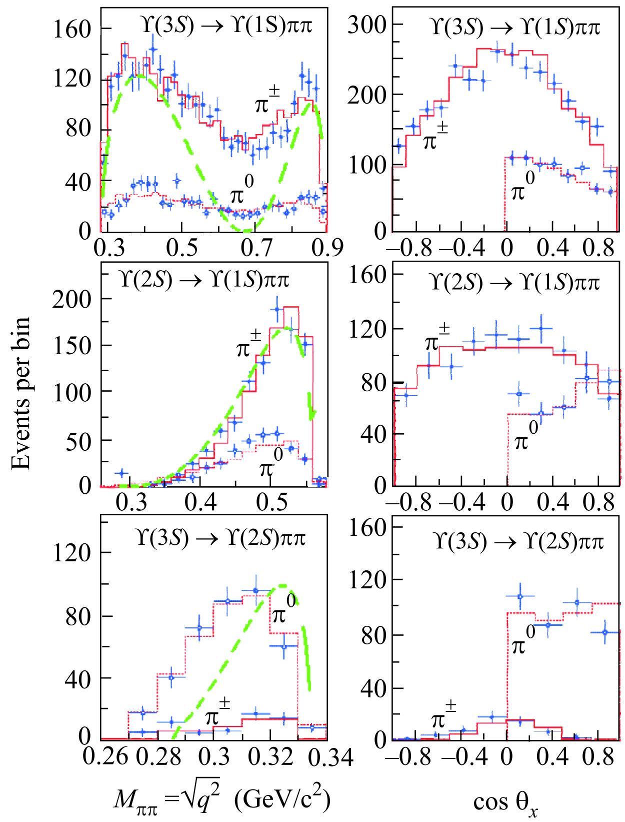

Fig.2 The experimental data for $\Upsilon(nS) \to \Upsilon(n'S)\pi\pi$ from [3] versus theoretical spectrum, Eq. (7) with $\eta = 0.55$; 0; -2.7 (top to bottom, dashed line)

3. The fitting curves in Fig.2 appear to reproduce the main features of di-pion spectra in all three cases. The deep well in the $(3\to1)$ double-peaked fitting curve is partially filled since η actually depends on $\cos\theta$ and in integration $\int d\cos\theta|\eta(\theta) - x|^2$ one has nonzero result for all x.

It is interesting that the total yield of $\pi\pi$, calculated for $\psi(2S) \to J/\psi\pi\pi$ with $M_{br} = 1$ GeV (fitted to $\psi(3770) \to D\bar{D}$), yields the value of $\Gamma_{\pi\pi}$ which is 3 times smaller than experiment. This is reasonable, since $\Gamma_{\pi\pi}$ is proportional to the fourth power of the overlap integral and the SHO wave function is certainly a very rough approximation, nevertheless, the resulting $\Gamma_{\pi\pi}$ is in the correct ballpark.

The di-pion spectrum for $\psi(2S) \to J/\psi\pi\pi$ also corresponds to $\eta \approx 0$, while those of $\Upsilon(4S) \to \Upsilon(n'S)_{\pi\pi}$ with $n' = 1, 2$ have $\eta_{fit}(4S \to 2S) = 0.30$ and $\eta_{fit}(4S \to \to 2S) = 0.61$ respectively. The calculation of $\eta(4S \to \to n'S)$, however, should be done not with SHO, but with realistic wave functions, which is in progress.

The processes $\Upsilon(5S) \to \Upsilon(n'S)\pi\pi$ with $n' = 1, 2$ were observed recently [6] with di-pion spectra of possibly three-peaked form: also the total di-pion yield $\Gamma_{\pi\pi}$ is thousand times larger, than for $n = 4S, 3S$. Both facts can be derived from our expressions, Eqs. (4), (6), if one takes into account that the mass of $\Upsilon(5S) = 10.86$ GeV is above all three thresholds $B\bar{B}, B\bar{B}^*, B^*\bar{B}^*$ and the appearing imaginary part is very large. This analysis is now in progress.

Finally, one can compare the total yield of K^+K^- to that of $\pi^+\pi^-$ in $\Upsilon(5S \to 1S)$ transition. Indeed, the only difference is in $\mu_i^2 = (\Delta M)^2 - 4m_i^2$, $i = \pi, K$. From (7) one can derive that

$$\Gamma_i = \int \frac{dw}{dq} dq \sim \mu_i^7,$$

and

$$\frac{\Gamma_{K^+K^-}(5S \to 1S)}{\Gamma_{\pi\pi}(5S \to 1S)} \sim \left(\frac{\mu_K}{\mu_\pi}\right)^7 \approx 0.104,$$

which roughly agrees with experimental data (an additional 50% reduction of this ratio follows if one use $f_K = 0.112$ GeV$\neq f_\pi = 0.093$ GeV).

Summarizing, we have suggested a new nonperturbative approach which describes all transitions $X(n) \to \to X(n')\pi\pi(KK)$ for $n = 2, 3$ and $n' = 1, 2$ without fitting parameters. It is argued that the standard MGFE approach to di-pion transitions in heavy quarkonia cannot be applied to large size quarkonia.

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