

Bremsstrahlung photon polarization for $ee^\pm \rightarrow (e\gamma)e^\pm$, and $ep \rightarrow (e\gamma)p$ high energy collisions

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The polarization of bremsstrahlung photon in the processes $ee^\pm \rightarrow (e\gamma)e^\pm$, and $ep \rightarrow (e\gamma)p$ is calculated for peripheral kinematics, in the high energy limit where the cross section does not decrease with the incident energy. When the initial electron is unpolarized (longitudinally polarized) the final photon can be linearly (circularly) polarized. The Stokes parameters of the photon polarization are calculated as a function of the kinematical variables of the process: the energy of recoil particle, the energy fraction of scattered electron, and the polar and azimuthal angles of photon. Numerical results are given in form of tables, for typical values of the relevant kinematic variables.

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I. Introduction.

It is known that the bremsstrahlung photon in electron-positron and electron-proton scattering can be polarized [1, 2]. If the initial electron is unpolarized the bremsstrahlung photon may acquire linear polarization; if the initial electron is longitudinally polarized then the photon may acquire left or right circular polarization.

Bremsstrahlung photons obtained in the scattering of electron beams on positrons or protons (not excited) can be considered as a photons source where polarization characteristics are determined by the kinematics of the measurement (the energy and the angle relative to the beam). The cross-section of the bremsstrahlung emission is enough large (about some percent from elastic cross section in similar kinematic conditions for the scattered lepton). Moreover, selecting events in the same kinematical conditions, one such photon source has determined properties of polarization, which can be used in manifold goals.

Polarized beams allow to access more detailed information about the target properties in comparison with unpolarized reactions, which give differential cross sections averaged over the amplitudes. As an example, the circular polarization of bremsstrahlung photon in the scattering of charged leptons contains information about the standard model concerning heavy vector bosons [3]: one could detect neutral current effects by looking to the helicity of the outgoing particles emitted in unpolarized fermion scattering.

Polarization phenomena for pair production in photon electron collisions (crossing process of the one considered here) was investigated in Ref. [4].

Nowadays this process is used for normalization procedures and for monitoring the luminosity of the beam. The reason is the well known theoretical description (including RC) and the large cross section.

A lot of attention was paid to bremsstrahlung process for electron-electron(positron) collisions [2, 1, 4], where the complete expressions for differential cross section were obtained. Some partial distributions for the case of polarized initial or final particles was obtained in [2, 1]. Complete (but rather complicated) formulae were derived by E. Haug [5] for the case of unpolarized particles. Compact expressions in terms of chiral amplitudes were obtained by CALCUL group [6] for the high energy limit. The crossed process of pair photoproduction on electron was studied as well for the case of initial polarized particles (see [4]).

In this paper we focus on the possibility to use the bremsstrahlung process in peripheral kinematics as a source of polarized photons. The polarization characteristics of the photon (Stokes parameters) depend on the kinematical conditions of the emission.

In Ref. [2] the expression for the polarization observables were obtained in the approximation of very small scattering angles, $\theta_\gamma, \theta_e \sim m/E$ (m is the mass of electron and E is the incident energy of the electron in the laboratory (Lab) system). A similar problem was discussed in previous papers [4, 7] (and Refs. therein) for the case of photoproduction of pairs of charged particles. The results were obtained in the so called Weitszacker-Williams approximation. Such kinematics provides the main contribution to the cross section and corresponds to small invariant mass of final particles,

moving close to the initial particles (e^- or γ) direction.

Unfortunately these results can not be directly applied to real experiments, with thresholds for the emission angles and the energy of the scattered particles.

Photoproduction of electron pairs in triplet state has been suggested as a possible way to measure the degree of polarization of photon beams, in a wide energy range and it was shown that the analyzing powers may reach 14% [4].

In the present work, we calculate the polarization of bremsstrahlung photons from electron scattering on electron or proton in the high energy limit. As the cross section and the degree of polarization are sufficiently large to be measured, we suggest that this process could be used for electron polarimetry.

Let us consider the bremsstrahlung process for the scattering of an electron on an electron(positron) or on a proton:

$$e^-(p_1, \lambda) + P(p) \rightarrow e^-(p'_1) + \gamma(k, e) + P(p'), \quad (1)$$

where λ is the longitudinal polarization and p_1 the momentum of the incident electron, with $p_1^2 = m^2$, k is the four-momentum of the photon ($k^2 = 0$), and e its polarization vector. P denotes the target particle, $p^2 = p'^2 = M^2$, where $M = M_p$ (m) for the case of a proton (e^\pm) target.

We will consider the kinematics related to peripheral collisions, where particles of energy E scatter on a target of mass M at small angles, in the laboratory system. This kinematical regime is characterized by

$$s = (p + p_1)^2 - M^2 = 2ME \gg s_1 = (p'_1 + k)^2 \sim |q^2| \gg m^2, \\ q = p - p',$$

where s_1 is the invariant mass squared of the scattered electron and photon.

In peripheral kinematic and in the high energy limit, the cross sections of particle production do not depend on the incident energy. It is convenient to use the Sudakov's parametrization for the kinematical variables, as defined below. The laboratory frame is taken as the reference frame all along the paper.

II. Formalism. A. Kinematics. Considering peripheral processes, it is convenient to use the Sudakov's parametrization of momenta. Any four-vector, v , can be represented as $v = (v_0, v_{||}, \mathbf{v}_\perp)$, where v_0 is the time component, $v_{||}$ is the longitudinal component with respect to the momentum of the initial electron, and \mathbf{v}_\perp is the two-dimensional vector of the transversal component. Let us introduce two light-like four-vectors, $\tilde{p} = p - p_1 M^2/s$, $\tilde{p}_1 = p_1 - pm^2/s \simeq p_1$, with

$s = 2pp_1$. The explicit components of these vectors are $p_1 = E(1, 1, 0, 0)$ and $\tilde{p} = (M/2)(1, -1, 0, 0)$.

Any four-vector can be expressed in a basis defined by \tilde{p} , and p_1 , with the help of the coefficients α_i and β_i and of the transversal plane components:

$$k = \bar{x}p_1 + \alpha_\gamma \tilde{p} + k_\perp, \\ p'_1 = xp_1 + \alpha_e \tilde{p} + p_\perp, \\ q = \beta_q p_1 + \alpha_q \tilde{p} + q_\perp, \\ e(k) = \beta p_1 + \alpha \tilde{p} + e_\perp, \quad (2)$$

where $a_\perp p_1 = a_\perp \tilde{p} = 0$, and x is the fraction of initial energy carried by the scattered electron, $\bar{x} = 1 - x$ is the energy fraction carried by the photon and $p_\perp^2 = -\mathbf{p}^2$, $k_\perp^2 = -\mathbf{k}^2$, $q_\perp^2 = -\mathbf{q}^2$, $e_\perp^2 = -\mathbf{e}^2$, $\mathbf{q} = \mathbf{p} + \mathbf{k}$ correspond to the components of the vectors p'_1 , k , q , e which are orthogonal to the vectors \tilde{p} and \tilde{p}_1 . Here \mathbf{p} , \mathbf{k} , \mathbf{q} , \mathbf{e} are two-dimensional vectors.

Applying on-mass shell conditions and gauge invariance: $e(k)k = 0$, one finds the following relations:

$$d_1 = 2p_1 k = \frac{1}{\bar{x}} (m^2 \bar{x}^2 + \mathbf{k}^2) = \frac{\Delta_1}{\bar{x}}, \quad 2p_1 e = \frac{2}{\bar{x}} \mathbf{k}e, \quad (3a)$$

$$d_2 = 2p'_1 k = \frac{1}{x\bar{x}} (m^2 \bar{x}^2 + \mathbf{b}^2) = \frac{\Delta_2}{x\bar{x}}, \quad 2p'_1 e = \frac{2}{\bar{x}} \mathbf{b}e, \quad (3b)$$

$$2pp'_1 = \frac{1}{x} (m^2(1 + x^2) + \mathbf{p}^2), \quad \mathbf{b} = x\mathbf{q} - \mathbf{p}, \quad (3c)$$

where we introduced the terms $\Delta_1 = m^2 \bar{x}^2 + \mathbf{k}^2$ and $\Delta_2 = m^2 \bar{x}^2 + \mathbf{b}^2$. The phase volume of the final state

$$d\Gamma = (2\pi)^{-5} \frac{d^3 k}{2\omega} \frac{d^3 p'_1}{2E'_1} \frac{d^3 p'}{2E'} \delta^4(p + p_1 - p'_1 - p' - k), \quad (4)$$

after introducing an auxiliary integration

$$\delta^4(p + p_1 - p'_1 - p' - k) = \\ = d^4 q \delta^4(p_1 + q - k - p'_1) \delta^4(q - p + p'), \quad (5)$$

can be expressed in terms of Sudakov's variables as :

$$d\Gamma = \frac{d^2 q d^2 p dx}{4s x \bar{x} (2\pi)^5} = \frac{E' dE' dx \theta_e d\theta_e d\phi}{2^7 \bar{x} \pi^4}, \quad (6)$$

where $\phi = (\widehat{\mathbf{p}}, \mathbf{q})$ is the azimuthal angle (see Fig.1), delimited by two planes: the plane which contains the momenta of the initial and final electrons, (p_1, p'_1) and the plane defined by the momenta of the initial electron and of the exchanged photon (p_1, q) , $\theta_e = |\mathbf{p}|/(Ex)$ is the angle between initial and scattered electron directions.

In the Laboratory frame the transverse momentum $|\mathbf{q}|$ is related to the energy of the recoil particle. Due to

Table I

Calculated values for the term R (Eq. (11)) for $\phi = \pi/2$, $x = 1/2$ as a function of q^2 and p^2

$q^2, \text{ GeV} / p^2, \text{ GeV}$	0.50	1.00	1.50	2.00	2.50	3.00
0.50	1.583	0.489	0.225	0.127	0.080	0.055
1.00	0.493	0.198	0.102	0.061	0.040	0.028
1.50	0.217	0.104	0.059	0.037	0.025	0.018
2.00	0.114	0.062	0.037	0.025	0.017	0.013
2.50	0.067	0.040	0.026	0.018	0.013	0.010
3.00	0.043	0.027	0.018	0.013	0.010	0.007

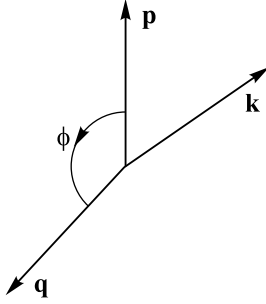


Fig.1. Definition of azimuthal angle ϕ of recoil particle

four-momentum conservation the recoil proton momentum $p' = (E', \mathbf{p}')$ can be written as:

$$\mathbf{p}'^2 = \mathbf{q}^2 + \frac{(\mathbf{q}^2)^2}{4M^2}, \quad (7)$$

and the following relations hold:

$$E'^2 = \frac{(\mathbf{q}^2 + 2M^2)^2}{4M^2}, \quad \mathbf{p}^2 = (E\theta_e x)^2, \quad \mathbf{q}^2 = 2M^2 + 2ME'. \quad (8)$$

B. Unpolarized electron. According to the formalism developed in Ref. [8], the matrix element of the process (1) (which is illustrated in Fig.2), for not negligible momentum transferred by the exchanged virtual photon, $|\mathbf{q}|$, can be written as:

$$\mathcal{M} = \frac{(4\pi\alpha)^{3/2}}{\mathbf{q}^2} \frac{2}{s} [\bar{u}(p')\Gamma_\lambda p_1^\lambda u(p)] [\bar{u}(p')O_{\mu\nu}\tilde{p}^\mu e^\nu u(p_1)], \quad (9)$$

where $\Gamma_\mu = \gamma_\mu$ in case of e^\pm target, and $\Gamma_\mu = F_1(q^2)\gamma_\mu + (1/4M_p)(\hat{q}\gamma_\mu - \gamma_\mu\hat{q})F_2(q^2)$ for a proton target, where $F_{1,2}(q^2)$ are the proton form factors. The vectors \mathbf{k} , \mathbf{q} and \tilde{p} are defined in Eqs. (2).

After summing over the lepton quantum numbers, the square of the matrix element is:

$$\sum |\mathcal{M}|^2 = \frac{(\pi\alpha)^3}{(\mathbf{q}^2)^2} 2^{11} s^2 D \frac{1}{4} \text{Tr} \times \left[\hat{p}'_1 \left(\hat{e}\rho + \frac{\hat{e}\hat{q}\hat{p}}{sd_2} - \frac{\hat{p}\hat{q}\hat{e}}{sd_1} \right) \hat{p}'_1 \left(\rho\hat{e}^* + \frac{\hat{p}\hat{q}\hat{e}^*}{sd_2} - \frac{\hat{e}^*\hat{q}\hat{p}}{sd_1} \right) \right], \quad (10)$$

where $\rho = 1/d_2 - x/d_1$ and

$$D = \begin{cases} 1, & \text{for electron target,} \\ F_1^2(-\mathbf{q}^2) + \frac{\mathbf{q}^2}{4M_p^2} F_2^2(-\mathbf{q}^2) & \text{for proton target.} \end{cases}$$

In case of unpolarized final photon, the cross section can be written as:

$$M_p^3 \frac{2\pi d\sigma}{dx dE' d\mathbf{p}^2 d\phi} = \frac{4\alpha^3}{M_p^2} DR, \quad R = \frac{M_p^6 A}{(\mathbf{q}^2)^2 x \bar{x}}, \quad (11)$$

where

$$A = (x\bar{x})^2 \left(\frac{1}{\Delta_1} - \frac{1}{\Delta_2} \right)^2 \left[\frac{\mathbf{p}^2}{2x} + \frac{(\mathbf{q} - \mathbf{p})(x\mathbf{q} - \mathbf{p})}{\bar{x}^2} \right] + \frac{(x\bar{x})^2}{\Delta_2} \left(\frac{1}{\Delta_2} - \frac{1}{\Delta_1} \right) \left[-\mathbf{p}\mathbf{q} - \frac{2}{\bar{x}}(x\mathbf{q}^2 - \mathbf{p}\mathbf{q}) \right] - \frac{x\bar{x}^2}{\Delta_1} \left(\frac{1}{\Delta_2} - \frac{1}{\Delta_1} \right) \left[\frac{1}{2}\mathbf{p}\mathbf{q} - \frac{2x}{\bar{x}}\mathbf{q}^2 + \frac{2x}{\bar{x}}\mathbf{p}\mathbf{q} \right] + \left[\frac{(x\bar{x})^2}{\Delta_2^2} + \frac{(\bar{x})^2}{\Delta_1^2} \right] \frac{x\mathbf{q}^2}{2}, \quad (12)$$

The quantity R is calculated for $x = 1/2$ and for different values of \mathbf{q}^2 and \mathbf{p}^2 in Table I.

Introducing the photon polarization density matrix:

$$\sum_{ij} e_i^\lambda e_j^{*\lambda} = \frac{1}{2} [I + \xi_1 \sigma_1 + \xi_3 \sigma_3]_{ij}, \quad (13)$$

Eq. (10) can be written in the form

$$\sum |\mathcal{M}|^2 \sim A + B\xi_1 + C\xi_3 = A[1 + X_1\xi_1 + X_3\xi_3].$$

The polarization state of bremsstrahlung photon is characterized by the Stokes parameters, which have the form [9]

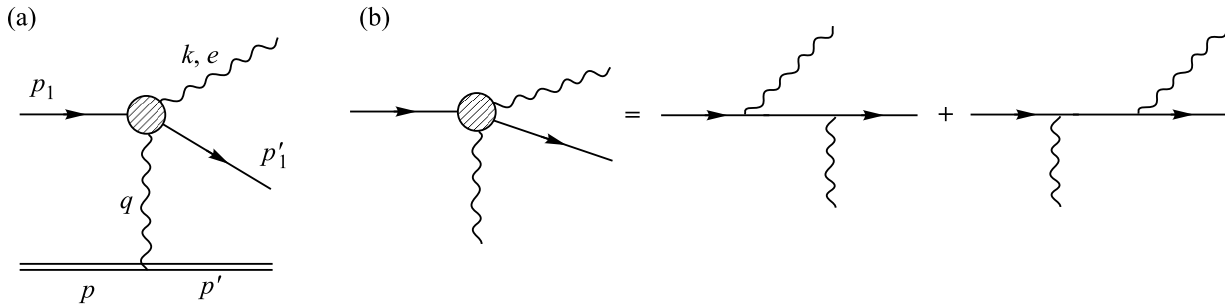


Fig.2. Feynman diagrams which are relevant in peripheral kinematics

$$X_1 = B/A, \quad X_3 = C/A. \quad (14)$$

$$X_2 = \frac{|\mathcal{M}_+|^2 - |\mathcal{M}_-|^2}{|\mathcal{M}_+|^2 + |\mathcal{M}_-|^2}, \quad (17)$$

with B and C given by:

$$B = (x\bar{x})^2 \left(\frac{1}{\Delta_2} - \frac{1}{\Delta_1} \right)^2 \times \\ \times [x\mathbf{q}^2 \sin(2\phi) - (1+x)|\mathbf{p}||\mathbf{q}| \sin(\phi)] + \\ + \frac{(x\bar{x})^2}{\Delta_2} \left(\frac{1}{\Delta_2} - \frac{1}{\Delta_1} \right) [x\mathbf{q}^2 \sin(2\phi) - |\mathbf{p}||\mathbf{q}| \sin(\phi)] + \\ + \frac{2x^2\bar{x}}{\Delta_1} \left(\frac{1}{\Delta_2} - \frac{1}{\Delta_1} \right) [\mathbf{q}^2 \sin(2\phi) - |\mathbf{p}||\mathbf{q}| \sin(\phi)], \quad (15)$$

$$C = x^2 \left(\frac{1}{\Delta_2} - \frac{1}{\Delta_1} \right)^2 \times \\ \times [\mathbf{p}^2 + x\mathbf{q}^2 \cos(2\phi) - 2x|\mathbf{p}||\mathbf{q}| \cos(\phi)] - \\ - \frac{2x^2\bar{x}}{\Delta_2} \left(\frac{1}{\Delta_2} - \frac{1}{\Delta_1} \right) [x\mathbf{q}^2 \cos(2\phi) - |\mathbf{p}||\mathbf{q}| \cos(\phi)] + \\ + \frac{2x^2\bar{x}}{\Delta_1} \left(\frac{1}{\Delta_2} - \frac{1}{\Delta_1} \right) [\mathbf{q}^2 \cos(2\phi) - |\mathbf{p}||\mathbf{q}| \cos(\phi)] - \\ - \frac{(x\bar{x})^2}{\Delta_1\Delta_2} \mathbf{q}^2 \cos(2\phi), \quad (16)$$

where $\Delta_{1,2}$ are defined in Eqs.(3a,3b).

The quantities X_1 and X_3 are calculated in Tables II and III for typical kinematics. As one can see, the values of X_1 are quite constant and very large, of the order of 80%. The values of X_3 are also sizable and increase with energy. The magnitude of the cross section can be calculated from Eq. (11) and Table II, the kinematical coefficient $4\alpha^3/M_p^2$ being of the order of 700 pb. So the bremsstrahlung with unpolarized initial electron can be considered an interesting reaction for polarimetry, as its cross section is large, the linear polarization of photon is sizable and depends smoothly on the relevant kinematical variables.

C. Circular photon polarization. The longitudinal polarization of the initial electron induces a circular polarization of the photon (as calculated in Ref. [10], using the method from Ref. [11]):

where \mathcal{M}_\pm is the light-cone projection of matrix element of the subprocess of creation of a photon with chirality \pm in the process where the lepton has positive chirality: $e(p_1, +) + \gamma^* \rightarrow e(p'_1, +) + \gamma(k, \pm)$.

The Stokes parameter X_2 , related to the circular polarization of the photon depends only on the energy fraction carried by the photon:

$$X_2 = \frac{1-x^2}{1+x^2} = \frac{\bar{x}(2-\bar{x})}{2-2\bar{x}+\bar{x}^2}, \quad (18)$$

One can see that the larger is the energy of the photon, the larger is the degree of polarization (Fig.3).

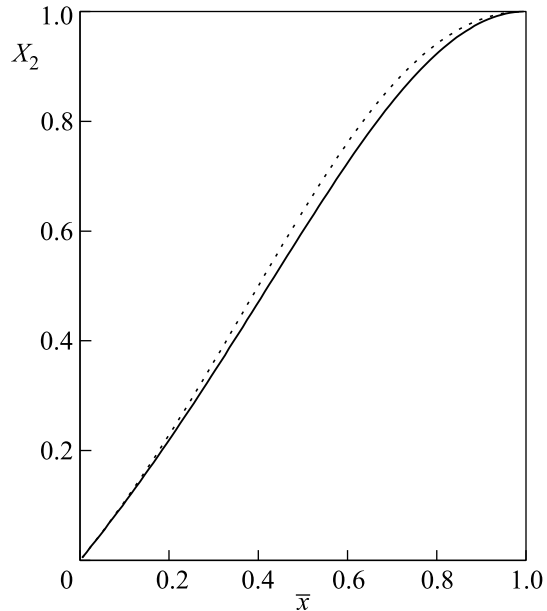


Fig.3. The Stokes parameter X_2 as a function of the energy fraction carried by the photon \bar{x} , according to Eq. (18) (solid line) and to Eq. (20) (dashed line)

Table II

The Stokes parameter X_1 (Eq. (14)) for $\phi = \pi/2$, $x = 1/2$ as a function of q^2 and p^2

$q^2, \text{ GeV} / p^2, \text{ GeV}$	0.50	1.00	1.50	2.00	2.50	3.00
0.50	-0.841	-0.853	-0.845	-0.827	-0.806	-0.785
1.00	-0.816	-0.841	-0.851	-0.853	-0.851	-0.845
1.50	-0.799	-0.827	-0.841	-0.849	-0.853	-0.853
2.00	-0.784	-0.816	-0.831	-0.841	-0.847	-0.851
2.50	-0.769	-0.807	-0.823	-0.833	-0.841	-0.846
3.00	-0.754	-0.799	-0.816	-0.827	-0.834	-0.841

Table III

The Stokes parameter X_3 (Eq. (14)) for $\phi = \pi/2$, $x = 1/2$ as a function of q^2 and p^2

$q^2, \text{ GeV} / p^2, \text{ GeV}$	0.50	1.00	1.50	2.00	2.50	3.00
0.50	0.397	0.483	0.534	0.569	0.596	0.617
1.00	0.308	0.397	0.447	0.483	0.511	0.534
1.50	0.251	0.345	0.397	0.432	0.460	0.483
2.00	0.209	0.308	0.360	0.397	0.424	0.447
2.50	0.175	0.277	0.332	0.369	0.397	0.419
3.00	0.147	0.251	0.308	0.345	0.374	0.397

Let us stress that the present results hold for relatively large angle of the emitted photon:

$$\theta_{e,\gamma} = \frac{|\mathbf{k}|}{E(1-x)} \sim \frac{|\mathbf{p}|}{Ex} \gg \frac{m}{E}, \quad \theta_{e,\gamma} \ll 1. \quad (19)$$

Formulas previously obtained in the literature can not be derived as a limit to the present results, because chirality is not a good quantum number for small angles of e or γ emission. More precisely, Eq. (18) holds in the limit (19) and it is different from Eq. (6) of Ref. [2]:

$$X_2 = \frac{\bar{x}(4-\bar{x})}{4-4\bar{x}+3\bar{x}^2}, \quad (20)$$

which has been derived in the Weitzsacker-Williams approximation and it is strictly valid in kinematics unreachable by experiments (very small angles of emission). The different behaviors of X_2 are shown in Fig.3.

Conclusion. We calculated the degree of linear polarization of the bremsstrahlung photon for unpolarized high-energy $e-p$, $e-e^\pm$ scattering at small scattering angles and high energy. The relevant Stokes parameters are functions of kinematical variables such as the transferred momentum, the polar and azimuthal angles of the photon and its energy fraction. For the case of longitudinally polarized initial electrons, the photon has nonzero circular polarization which depends only on its energy fraction.

The emission angles of the electron and photon, $\theta_{e,\gamma}$ are assumed to be small but much larger than m/E .

The accuracy of formulae given above is determined by radiative corrections and on the omitted terms. We estimate it as

$$1 + O\left(\frac{\mathbf{p}^2}{s}, \frac{\alpha}{\pi} \ln \frac{\mathbf{p}^2}{m^2}, \frac{m^2}{\mathbf{p}^2}\right). \quad (21)$$

As an example, for the conditions of the upgraded JLab facility ($E \approx 12$ GeV) it is of order $\sim 10\%$.

In this paper we emphasized the possibility to obtain polarized photons in the process of peripheral radiative scattering of leptons. We derived the expressions that relate the polarization parameters and the cross section to the relevant kinematical variables and calculated these observables, for different kinematical conditions.

We showed that the cross section and the degree of polarization is sufficiently large and that this process should be taken into consideration for designing photon polarimeters in the GeV range.

The Stokes parameters that characterize the bremsstrahlung photon polarization do not depend neither on the initial energy nor on the target mass.

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