

# Chiral symmetry and excited baryons

A. V. Nefediev, J. E. F. T. Ribeiro<sup>+</sup>, Adam P. Szczepaniak<sup>\*</sup>

*Institute of Theoretical and Experimental Physics, 117218 Moscow, Russia*

<sup>+</sup> *Centro de Física das Interações Fundamentais (CFIF), Departamento de Física, Instituto Superior Técnico, Universidade Técnica de Lisboa, Lisbon, Portugal*

<sup>\*</sup> *Department of Physics and Nuclear Theory Center Indiana University, Bloomington, IN 47405, USA*

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An approach to baryons in the framework of the microscopic Generalized Nambu–Jona-Lasinio chiral potential quark model is considered and quite general arguments are given in favor of effective restoration of chiral symmetry in excited baryons.

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Chiral symmetry breaking and confinement of color charges are the two most prominent phenomena taking place in QCD. In spite of the rather long time that has elapsed since the appearance of QCD as the theory of strong interactions, and the many theoretical efforts thereof, the fundamental underlying mechanisms of confinement and its interplay with chiral symmetry breaking cannot be derived directly from first principles. In such circumstances, quark models can be used as a source of information on various properties of hadrons. In particular, the problems related to chiral symmetry are well suited to be treated with the help of the Generalized Nambu–Jona-Lasinio (GNJL) model [1, 2]. In this model, the spontaneous breaking of chiral symmetry happens as a result of the vacuum condensation of  ${}^3P_0$  quark–antiquark pairs, an energetically favorable process as it leads to the decrease of the vacuum energy. The new BCS vacuum (as opposed to the trivial empty vacuum) is not chirally invariant and thence all hadronic states built on top of this vacuum should display the same lack of chiral invariance. With the exception of the notorious case of the pion (as its mass would be higher than its actual physical mass, by  $300 \div 400$  MeV, were it not for the mechanism of chiral symmetry spontaneous breaking) this lack of chiral invariance is quite visible with pairs of low-lying hadronic partner states, with opposite parities, being well split in mass. However, this mass split should become weaker as the hadronic excitation number increases. This phenomenon, known as the asymptotic effective restoration of chiral symmetry [3] (see also the recent review [4] and references therein) receives a natural explanation based on the fact that quantum fluctuations should progressively disappear as we climb the excitation number staircase of hadronic states

[5, 6]. In other words, in the limit of very large excitation numbers, we expect these very excited hadrons to become semiclassical objects. A pattern of chiral symmetry restoration in excited heavy–light mesons in the framework of GNJL model was studied in detail in Refs. [7–9] and its connection to the classical limit of the model was discussed in Ref. [6].

In this paper, we give general arguments concerning the aforementioned effective restoration of chiral symmetry in the spectrum of excited baryons. As in the case of mesons, chiral restoration in the baryonic sector is an inevitable consequence of the large–momentum behavior of the chiral angle – the function which naturally quantifies chiral symmetry breaking.

For this work, let us consider the simplest Hamiltonian containing the ladder Dyson–Schwinger machinery for chiral symmetry [1, 2] – in any case, the results presented here do not depend on the kernel choice (the superscripts  $i, j$  stands for the quark flavor),

$$H = \sum_{i=u,d} \int d^3x \psi_i^\dagger(\mathbf{x}) (-i\boldsymbol{\alpha} \nabla + \beta m_i) \psi_i(\mathbf{x}) + \frac{1}{2} \sum_{i,j=u,d} \int d^3x d^3y J_{i\mu}^a(\mathbf{x}) K_{\mu\nu}^{ab}(\mathbf{x} - \mathbf{y}) J_{j\nu}^b(\mathbf{y}), \quad (1)$$

with  $J_{i\mu}^a(x) = \bar{\psi}_{i\alpha}(x) \gamma_\mu (\frac{\lambda^a}{2})_\beta^\alpha \psi_i^\beta(x)$ ,  $K_{\mu\nu}^{ab}(x - y) = \delta^{ab} K_{\mu\nu}(|\mathbf{x} - \mathbf{y}|)$ . The quark field  $\psi_i^\alpha(\mathbf{x})$  is given by

$$\psi_i^\alpha(\mathbf{x}) = \sum_{s=\uparrow,\downarrow} \int \frac{d^3p}{(2\pi)^3} e^{i\mathbf{p}\mathbf{x}} [b_{ips}^\alpha u_s(\mathbf{p}) + d_{ips}^{\alpha\dagger} v_s(-\mathbf{p})], \quad (2)$$

which is an abstraction, independent of any particular choice of the representation for the quark creation/annihilation operators. If we choose the trivial,

empty, chirally symmetric vacuum  $|0\rangle_0$  to be the ground state, then the operators  $b^\dagger$  and  $d^\dagger$  create bare quarks, and the spinors can be conveniently written as,

$$\begin{aligned} u_{\text{bare}}(\mathbf{p}) &= \frac{1}{\sqrt{2}} [1 + (\boldsymbol{\alpha}\hat{\mathbf{p}})] u_0(\mathbf{p}), \\ v_{\text{bare}}(-\mathbf{p}) &= \frac{1}{\sqrt{2}} [1 - (\boldsymbol{\alpha}\hat{\mathbf{p}})] v_0(-\mathbf{p}), \end{aligned} \quad (3)$$

where  $u_0(\mathbf{p})$  and  $v_0(-\mathbf{p})$  are the rest-frame spinors. However, it was found long ago [1, 2] that the true vacuum state of the theory (1) is given by chirally nonsymmetric state  $|0\rangle = e^{Q_0^\dagger - Q_0} |0\rangle_0$ , with

$$Q_0^\dagger = \frac{1}{2} \sum_{i=u,d} \int \frac{d^3p}{(2\pi)^3} \varphi_p^{(i)} b_{i\alpha p s}^\dagger [\mathfrak{M}_{3P_0}]_{s's'} d_{i p s'}^{\alpha\dagger}, \quad (4)$$

and the  ${}^3P_0$  matrix being  $\mathfrak{M}_{3P_0} = (\boldsymbol{\sigma}\hat{\mathbf{p}})i\sigma_2$  [2]. In Eq. (4) and in what follows summation over repeated color and spin indices is understood. The quantity  $\varphi_p^{(i)}$  is known as the chiral angle and it defines the distribution of the  ${}^3P_0$  quark–antiquark pairs of the given flavor in the vacuum according to their relative momentum. Dependence of the chiral angle on flavor appears entirely due to different masses  $m_i$ . In what follows we assume an exact  $SU(2)_f$  symmetry, so that  $m_u = m_d = m$  and, as a result,  $\varphi_p^{(u)} = \varphi_p^{(d)} \equiv \varphi_p$ . The choice of the profile of the function  $\varphi_p$  ensures that the vacuum energy is minimal,

$$\frac{\delta}{\delta\varphi_p} \langle 0|H|0\rangle = 0, \quad (5)$$

which leads to the equation for the chiral angle, known as the mass-gap equation [1, 2]<sup>1)</sup>. For illustrative purposes we quote here the form the mass-gap equation for the simplest Lorentz structure of the inter-quark interaction in the Hamiltonian (1) compatible with the requirements of chiral symmetry and confinement,  $K_{\mu\nu}(|\mathbf{x} - \mathbf{y}|) = g_{\mu 0} g_{\nu 0} V_0(|\mathbf{x} - \mathbf{y}|)$  ( $V(|\mathbf{x} - \mathbf{y}|) = C_F V_0(|\mathbf{x} - \mathbf{y}|)$ ,  $C_F$  being the fundamental Casimir operator):

<sup>1)</sup>Notice that a more correct definition of the chiral angle and the mass-gap equation follows from the requirement that anomalous Bogoliubov terms are missing in the normally ordered Hamiltonian (1) [2]. Indeed, the theory is actually well-defined in terms of dressed quarks only for a discrete set of chiral angles – solutions of the mass-gap equation. Any variation of the chiral angle leads not only to a change in the vacuum energy but also in an unbalanced creation of quark–antiquark pairs (described by anomalous Bogoliubov terms in the Hamiltonian). The latter effect cannot be taken into account with the simple variational procedure (5). Meanwhile, formally, one and the same mass-gap equation arises from both aforementioned procedures.

$$\begin{aligned} & m \cos \varphi_p - p \sin \varphi_p = \\ &= \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} V(\mathbf{p} - \mathbf{k}) [\sin \varphi_k \cos \varphi_p - \\ & \quad - (\hat{\mathbf{p}}\hat{\mathbf{k}}) \cos \varphi_k \sin \varphi_p]. \end{aligned} \quad (6)$$

The properties of this equation and its solutions for various forms of the quark kernel, in two and four dimensions, can be found in Ref. [1, 2, 10, 11]. It is convenient to define the chiral angle such that  $-\pi/2 < \varphi_p \leq \pi/2$  and  $\varphi(0) = \pi/2$ ,  $\varphi(p \rightarrow \infty) \rightarrow 0$ . The trivial solution of the mass-gap Eq. (6) defines the trivial vacuum  $|0\rangle_0$ , whereas the nontrivial solution (see Fig.1 for the profile of this solution) defines the BCS chirally nonsymmetric

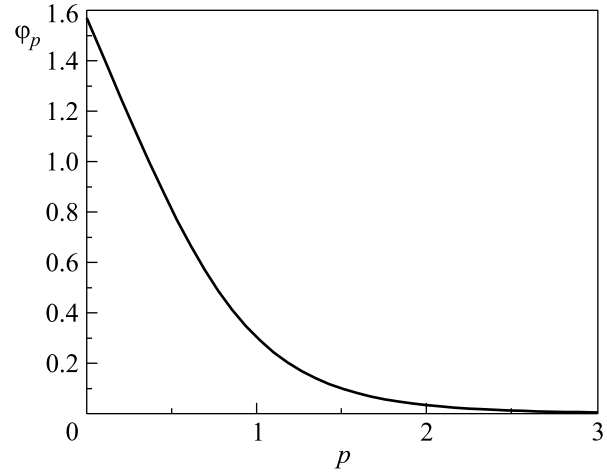


Fig.1. A typical profile of the chiral angle – solution to the mass-gap equation (6) which defines the broken BCS vacuum. The momentum  $p$  is measured in the units of strength of the potential  $V(|\mathbf{x} - \mathbf{y}|)$

vacuum  $|0\rangle$ , which is the physical vacuum of the theory. From now on, we choose to deal with the physical vacuum only, so that the operators  $b^\dagger$  and  $d^\dagger$  in the definition (2) create dressed quarks, that is, bare quarks enshrouded by an entire quark–antiquark cloud appearing as a consequence of quark selfinteractions. The spinors  $u(\mathbf{p})$  and  $v(-\mathbf{p})$  depend on the chiral angle now and can be written as

$$\begin{aligned} u(\mathbf{p}) &= \left[ \sqrt{\frac{1 + \sin \varphi_p}{2}} + (\boldsymbol{\alpha}\hat{\mathbf{p}}) \sqrt{\frac{1 - \sin \varphi_p}{2}} \right] u_0(\mathbf{p}), \\ v(-\mathbf{p}) &= \left[ \sqrt{\frac{1 + \sin \varphi_p}{2}} - (\boldsymbol{\alpha}\hat{\mathbf{p}}) \sqrt{\frac{1 - \sin \varphi_p}{2}} \right] v_0(-\mathbf{p}). \end{aligned} \quad (7)$$

One can see therefore, that the GNJL model gives an explicit microscopic description of the effect of spontaneous breaking of chiral symmetry. Indeed, in the chiral limit  $m = 0$  the Hamiltonian (1) is invariant under the transformation  $\psi_i \rightarrow [\exp(i\alpha\gamma_5\tau^a/2)]^{ij}\psi_j$  (from

now onwards  $a$  stands for the index of the flavor  $\tau$  matrices), whereas the BCS vacuum  $|0\rangle$ , contains the chiral condensate

$$\langle \bar{\psi}\psi \rangle_u = \langle \bar{\psi}\psi \rangle_d = -\frac{N_C}{\pi^2} \int_0^\infty dp p^2 \sin \varphi_p \neq 0, \quad (8)$$

and therefore, is not. In this vacuum, the Hamiltonian (1), takes a diagonal form

$$H = E_{\text{vac}} + \sum_{i=u,d} \int \frac{d^3p}{(2\pi)^3} E_p [b_{i\alpha p s}^\dagger b_{i\alpha p s}^\alpha + d_{i\alpha p s}^{i\alpha\dagger} d_{i\alpha p s}] + \dots, \quad (9)$$

where  $E_p$  stands for the dressed-quark dispersive law and the ellipsis denotes the terms which are responsible for the formation of bound states of dressed quarks – hadrons. In Ref. [11] these terms were considered in detail and a second, nonlocal Bogoliubov-like transformation was applied in order to diagonalize the Hamiltonian in the mesonic sector of the theory. Alternatively, one can employ an effective diagrammatic technique with dressed quarks, having quark–antiquark Salpeter amplitudes as building blocks, in order to derive the Bethe–Salpeter equation – the bound–state equation for quark–antiquark mesons. This equation was derived and studied in a vast number of papers – see, for example, Ref. [1, 2, 11, 9]. A key ingredient of this equation is a two-component w.f. of the meson which describes simultaneously time-forward and time-backward motion of a single, dressed, quark–antiquark pair. In other words, quark pair creation–annihilation involves couplings of positive-energy to negative-energy Salpeter amplitudes. The baryon case is simpler. Due to confinement and to the fact (contrary to the mesonic case) that one cannot simultaneously annihilate three quarks with a two body interaction, we do not have such transitions for baryons and, therefore, to zero order in  $N_C$ , we are only left with a dressed, positive-energy, three–quark system — see Fig.2.

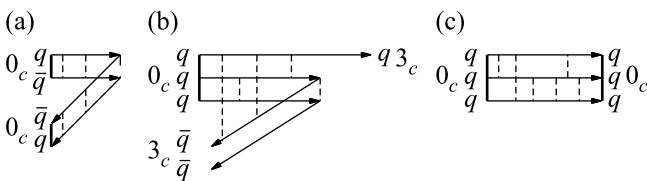


Fig.2. Diagram (A): a typical allowed (singlet  $0_c \rightarrow 0_c$ ) quark-antiquark pair annihilation transition from mesonic positive-energy to negative-energy Salpeter amplitudes; Diagram (B): a similar pair annihilation cannot proceed as it will involve non-singlets  $3_c$  in color; Diagram (C): a typical diagram pertaining to the Dyson ladder for baryons

Production of dressed quark-antiquark pairs is still possible but this is an  $N_C$  suppressed effect, responsible, for instance, for the baryon hadronic decays and virtual hadronic loops. Consideration of such effects goes beyond the scope of the present paper. Therefore, because of color confinement the w.f. of any baryon can be written as

$$\Psi_B = \Psi_{\text{color}} \otimes \Psi_{\text{flavor}} \otimes \Psi_{\text{spin}} \otimes \Psi_{\text{space}}, \quad (10)$$

with the color w.f. being antisymmetric,  $\Psi_{\text{color}} = \frac{1}{3!} \varepsilon_{\alpha\beta\gamma} q^\alpha q^\beta q^\gamma$ . Thus we need to be only concerned with the symmetric  $\Psi_{\text{flavor}} \otimes \Psi_{\text{spin}} \otimes \Psi_{\text{space}}$  component of the baryon w.f. This implies that, in general, the spatial component of the baryon w.f.  $\Psi_{\text{space}}^{\mathcal{Y}}(\boldsymbol{\rho}, \boldsymbol{\lambda})$  ( $\boldsymbol{\rho}$  and  $\boldsymbol{\lambda}$  are the standard Jacobi coordinates) must contain all the three-body permutation symmetry components  $\mathcal{Y}$ : antisymmetric ( $\mathcal{A}$ ), symmetric ( $\mathcal{S}$ ), mixed type  $F$  ( $\mathcal{D}_F$ ) and, finally, mixed type  $D$  ( $\mathcal{D}_D$ ). Recoupling of these spatial w.f.'s with their  $\mathcal{Y}$  flavor–spin counter parts will produce the completely symmetric w.f.  $\Psi_{\text{flavor}} \otimes \Psi_{\text{spin}} \otimes \Psi_{\text{space}}$ .

The Noether charge for the global chiral symmetry written in terms of the quark field is:

$$Q_5^a = \int d^3x \bar{\psi}_i \gamma_0 \gamma_5 \left( \frac{\tau^a}{2} \right)^{ij} \psi_j. \quad (11)$$

The charge (11) is to be calculated for the dressed quarks with the help of Eqs. (2) and (7). The result reads:

$$Q_5^a = \left( \frac{\tau^a}{2} \right)^{ij} \int \frac{d^3p}{(2\pi)^3} \times \\ \times [\cos \varphi_p (\boldsymbol{\sigma} \hat{\mathbf{p}})_{ss'} (b_{i\alpha p s}^\dagger b_{j\alpha p s'}^\alpha + d_{j\alpha p s}^{\alpha\dagger} d_{i\alpha p s'}) + \\ + \sin \varphi_p (i\sigma_2)_{ss'} (b_{i\alpha p s}^\dagger d_{j\alpha p s'}^{\alpha\dagger} + d_{i\alpha p s} b_{j\alpha p s'}^\alpha)]. \quad (12)$$

The two terms in square brackets in Eq. (12) have different physical meanings. The second term is known as the anomalous Bogoliubov term responsible for pion creation. Indeed, in the chiral limit, the axial charge (12) creates a nontrivial state,  $Q_5^a |0\rangle = |\pi^a\rangle$ . Being a Noether charge, the axial charge commutes with the Hamiltonian,  $[Q_5^a, H] = 0$ , which ensures that the state  $|\pi^a\rangle$  is degenerate in energy with the vacuum. This is the Goldstone boson – the chiral pion [12]. Its w.f. can be extracted either from the form of the mass–gap equation or from the Bethe–Salpeter equation for the lowest  $^1S_0$  bound quark–antiquark state (see, for example, Ref. [11] for a detailed discussion of the issue). This w.f. is given by  $\sin \varphi_p$ . Finally, the matrix  $i\sigma_2$  in Eq. (12) provides the  $^1S_0$  coupling of the quark with the antiquark. For quark masses different from zero, the pion

acquires a small finite mass. Furthermore, a simple examination of the form of the first term in square brackets in Eq. (12) shows that this operator, when applied to a given hadronic state, maps it into another hadronic state, with opposite parity, which is ensured by the matrix  $(\sigma\hat{\mathbf{p}})$ .

Consider the diagonal baryon axial charge operator defined as,

$$\mathcal{Q}_5 \equiv \mathcal{Q}_5^3 = \sum_{n=1}^3 \mathcal{Q}_{5n}^3, \quad (13)$$

where index  $n$  numerates quarks in the baryons, so that the baryon total axial charge is given by the sum of three individual charges, one for each quark.

It is obvious from Eq. (12) that this diagonal axial charge acts on baryonic states in a two-fold way, which can be schematically written in the form ( $|\pi\rangle$  denotes the neutral pion):

$$\mathcal{Q}_5|B\rangle = |B'\rangle + |B\pi\rangle, \quad (14)$$

where, for the sake of simplicity, we suppressed all unnecessary indices. The states  $|B\rangle$  and  $|B'\rangle$  have opposite parity, and the relative weight of the two terms on the r.h.s. of Eq. (14) is defined by the value of the chiral angle  $\varphi_p$ , namely by the value of the  $\sin\varphi_p$  versus the value of the  $\cos\varphi_p$ . In case of the maximal symmetry breaking ( $\varphi_p = \pi/2$ ), only the second term on the r.h.s. of Eq. (14) survives. On the contrary, if chiral symmetry is not broken in the vacuum ( $\varphi_p \equiv 0$ ) then the Goldstone does not exist and only the first term survives. In the latter limit, the following properties of  $\mathcal{Q}_5$  are obvious:

$$\mathcal{Q}_5^\dagger = \mathcal{Q}_5, \quad \langle B_2|\mathcal{Q}_5^2|B_1\rangle \propto \langle B_2|B_1\rangle = \delta_{B_1B_2}. \quad (15)$$

Therefore we must have

$$\mathcal{Q}_5|B^\pm\rangle = G_{\pm\mp}^A|B^\mp\rangle, \quad (16)$$

with  $B^\pm$  representing baryons with the parity  $\pm$  and  $G_{\pm\mp}^A$  being a  $c$ -number axial charge. The latter relation, together with the fact that  $[\mathcal{Q}_5, H] = 0$ , ensures that, in the chiral limit, the two states  $|B^+\rangle$  and  $|B^-\rangle$  must be degenerate in mass. They form a chiral doublet. This is exactly the regime expected to be approximately realized in highly excited hadrons. Indeed, in Ref. [7], the microscopic mechanism for chiral symmetry restoration in excited hadrons was investigated in detail for the case of heavy-light mesons. The analysis performed in Ref. [7] applies directly to a heavy-heavy-light baryon with two heavy quarks sitting on top of each other. In this case, the static particles degrees of freedom decouple from the system, and the baryon can be described

with the light-quark w.f.  $\psi$ . In agreement with the general consideration given above, the parity partner of this state has the w.f.  $\psi' = (\sigma\hat{\mathbf{p}})\psi$  and the two states come out approximately degenerate in mass in the limit of high orbital or radial excitations, when the momentum of the quark is large and the small value of the  $\sin\varphi_p$  (see Fig.1) suppresses the splitting between these two states (see Ref. [7] for the details). Notice, however, that in the physical world we do not have massless quarks and therefore we expect corrections to this degeneracy arising from pion loops (this corresponds to proceeding beyond the BCS approximation and taking into account the interaction between dressed quarks). These corrections are known to be, for the lowest lying baryons, of the order of a few hundred MeV. Thus the question of chiral restoration lies precisely in whether or not these corrections go to zero for highly excited baryonic states, that is, whether we can approximate  $\langle B'|\mathcal{Q}_5|B\rangle$  by

$$\begin{aligned} \langle B'|\tilde{\mathcal{Q}}_5|B\rangle &= \sum_{n=1}^3 \left(\frac{\tau_n^3}{2}\right)^{ij} \int \frac{d^3p_n}{(2\pi)^3} (\sigma_n\hat{\mathbf{p}}_n)_{ss'} \times \\ &\times \langle B'|b_{i\alpha p_n s}^{(n)\dagger} b_{j p_n s'}^{(n)\alpha} + d_{j p_n s}^{(n)\dagger} d_{i\alpha p_n s'}^{(n)}|B\rangle, \end{aligned} \quad (17)$$

where  $B$  and  $B'$  denote baryons and the index  $n$  numerates quarks.

In other words, the issue of chiral restoration in baryons is reduced to the question whether the average quark momentum inside a sufficiently excited baryon is high enough to have  $\sin\varphi_p \simeq 0$ . In the baryon case and for a generic spatial w.f. with a given permutation symmetry  $\mathcal{Y}$ , we have the following expansion:

$$\Psi_B^{\mathcal{Y}N} = \sum_C C^{\mathcal{Y}N} \sum_{\nu_1\nu_2} D_{\nu_1\nu_2}^{\mathcal{Y}N} \Phi_{\nu_1}(\mathbf{p}_\lambda) \Phi_{\nu_2}(\mathbf{p}_\rho), \quad (18)$$

where  $\mathbf{p}_\lambda$  and  $\mathbf{p}_\rho$  are the momenta conjugate to the Jacobi coordinates  $\boldsymbol{\lambda}$  and  $\boldsymbol{\rho}$ ;  $\{\Phi_\nu\}$  is a convenient basis (for example, the harmonic oscillator one),  $N$  stands for the set of quantum numbers, and  $D^{\mathcal{Y}N}$  are the appropriate coefficients for the particular permutation symmetry  $\mathcal{Y}$  and for the given set of quantum numbers  $N$ . The  $D_{\nu_1\nu_2}^{\mathcal{Y}N}$ 's obey the normalization condition,

$$\sum_{\nu_1\nu_2} (D_{\nu_1\nu_2}^{\mathcal{Y}N})^2 = 1. \quad (19)$$

The sum  $\sum_C$  just reflects the fact that the quarks microscopic interaction, despite being confining and henceforth infrared divergent, differs from the force with the eigenstates  $\{\Phi_\nu\}$ . Therefore, all such states should contribute to the baryon spatial w.f. with weights given by

the coefficients  $C^{\mathcal{Y}N}$ . Then, Eq. (19) and orthonormality of baryon states would have that

$$\sum_C (C^{\mathcal{Y}N})^2 = 1. \quad (20)$$

Then it is clear that for a sufficiently excited baryon, the set  $\{C^{\mathcal{Y}N}\}$  must peak around a given  $C^{\mathcal{Y}N_0}$  — with  $N_0$  large — and, as a result, Eq. (18) will govern the average value of the quark momenta, which are also large. Sooner or later this is bound to happen, which results in the chiral angle vanishing. Then two consequences take place: (i)  $\sin \varphi_p \rightarrow 0$  and pions decouple from this excited baryon (see also Ref. [13] for a detailed discussion of the pion decoupling from excited hadrons in the framework of GNJL) and (ii)  $\cos \varphi_p \rightarrow 1$ , so that the baryon axial charge can be well approximated by the form given in Eq. (17). As was discussed before, this warrants chiral restoration. Furthermore, when  $\cos \varphi_p \neq 1$ , the state  $\mathcal{Q}_5|B\rangle$  overlaps with every state of the same parity, therefore entangling it with states belonging to other chiral multiplets. As soon as chiral symmetry gets restored and  $\cos \varphi_p = 1$ , we have zeros both for the diagonal matrix element,  $G_{\pm\pm}^A = \langle B^\pm | \mathcal{Q}_5 | B^\pm \rangle = 0$ , (because of parity), and for the matrix elements between states belonging to different multiplets (because of the w.f. orthogonality). In the meantime, for transitions inside of the same multiplet,  $G_{\pm\mp}^A = \langle B^\mp | \mathcal{Q}_5 | B^\pm \rangle \neq 0$ . As seen from Eq. (17), for highly excited baryons, the off-diagonal axial charge  $G_{\pm\mp}^A$  tends to a universal constant independent of the particular baryon multiplet it was calculated for. By an appropriate rescaling of the baryon axial charge operator, Eq. (13), this asymptotic value of  $G_{\pm\mp}^A$  can be set to unity. In actuality, since the chiral angle never vanishes identically and therefore chiral symmetry is never fully restored in the spectrum, we end up with the approximate relations,

$$G_{+-}^A = G_{-+}^A \simeq 1, \quad G_{++}^A = G_{--}^A \simeq 0, \quad (21)$$

which have been obtained microscopically.

We conclude therefore that the GNJL model gives a clear and selfconsistent pattern of effective chiral symmetry restoration in excited baryons (see the discussion in Ref. [4]) and, what is more, it provides a full *microscopic* picture of this phenomenon, which, as a matter of principle, cannot be reproduced by any naive quark model or approach [14].

As our final remark, we would like to stress that the actual form of the Hamiltonian (1) is of no importance for the above conclusions, provided that  $\mathcal{Q}_5$  is a Noether charge of the dynamics and we have a nontrivial  $\varphi_p$ .

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