Pis'ma v ZhETF, vol. 87, iss. 6, pp. 331 – 334 © 2008 March 25

Damped dust lattice shock wave in strongly coupled complex (dusty) plasma

S. Ghosh[1)]

Government College of Engineering and Textile Technology
Berhampore, Murshidabad 742101 West Bengal, India

Submitted 22 January 2008
Resubmitted 12 February 2008

Taking into account 'hydrodynamical damping' due to irreversible processes that occur within the system and neutral drag due to dust-neutral collision, a Burgers' equation with a linear damping term is derived for 1D nonlinear longitudinal dust lattice wave (LDLW) in homogeneous strongly coupled complex (dusty) plasma. The 'hydrodynamical damping' generated dissipative effect causes generation of shock wave in dusty plasma crystal, whereas the neutral drag induced dissipative effect causes the decay of the shock intensity with time. The width of the observed compressive shock increases (decreases) with the increase of the shielding parameter κ (characteristic length L). Its implication in glow-discharge plasma are briefly discussed.

PACS: 52.27.Lw, 52.35.−g

A complex (dusty) plasma constitute ionized gases containing charged particles of condensed matter. The system is classified as "strongly coupled" or "weakly coupled" for $\Gamma > 1$ or $\Gamma \ll 1$, where $\Gamma\left[= \left(Q^2/4\pi\epsilon_0\Delta T_d\right)e^{-\kappa}\right]$ and $\kappa(= \Delta/\lambda_D)$ are the coupling and shielding parameters with dust charge Q, dust temperature T_d, interdust spacing Δ and Debye length λ_D. Complex (dusty) plasmas can spontaneously form ordered crystalline structures when $\Gamma > \Gamma_{cr}$ (critical value), so-called plasma crystal [1] that supports variety of dust lattice wave modes such as longitudinal, transverse and sloshing modes [2 – 9]. These wave modes are the elastic deformations of the lattice.

The longitudinal dust lattice wave (LDLW) is a compressional wave in atomic chain that propagates parallel to the dust particle motion in the chain. As a matter of fact, because of intrinsic nonlinearities of interatomic interaction mechanisms or on-site substrate potentials [10], LDLWs are known to be dominated by nonlinear phenomena like dislocations in crystals, energy localization, coherent signal transmission in electric lines, optical pulse propagation, charge and information transport in bio-molecules and DNA strands, etc. [11]. The nonlinear phenomena such as formation of solitons, wave-wave interactions were observed in laboratory experiments [12 – 14] and the characteristics of these nonlinear phenomena agreed with theoretical predictions [15 – 17].

Another interesting nonlinear phenomena is the generation of shock. It was shown previously that weakly coupled complex plasmas (gaseous phase) can sustain shock waves [18 – 20]. Later, it was also shown that longitudinal dust acoustic wave also sustain shock waves in strongly coupled quasi-crystal dusty plasma ($1 \ll \Gamma < < \Gamma_{cr}$), where the shock was generated due to charging-delay induced anomalous dissipation [21]. Moreover, recent experimental observations [22, 23] revealed that LDLW can sustain shock waves in dusty plasma crystal ($\Gamma > \Gamma_{cr}$). It is observed that strong shocks melt or even vaporize the solid and thereby produce phase transitions [23]. However, the physics of formation of shock in dusty plasma crystal is not well investigated as in case of weakly coupled dusty plasma.

In this letter a theoretical investigation for the generation of shock wave in dusty plasma crystal is reported. It is shown that the dissipation arises due to 'hydrodynamical damping' that takes place within the system generates the shock wave in dusty plasma crystals.

The complex (dusty) plasma consists of electrons, ions and the dust grains with mass m and negative charge Q (both are assumed to be constant for simplicity). The repulsive inter dust potential is shielded by the electron-ion plasma, characterized by Debye length λ_D [22]. A simplified 1D particle string model [15] and a weak correlation of fluctuations on the neighbour particles are assumed. The interaction potential between each particle is the Yukawa potential and the Debye-Huckel energy of this electrostatic coupling between j-th and $j \pm 1$-th particle of the string is given by

$$W_{j,j\pm1} = \frac{Q^2}{4\pi\epsilon_0 \mid x_j - x_{j\pm1} \mid} \exp\left(-\frac{\mid x_j - x_{j\pm1} \mid}{\lambda_D}\right) \tag{1}$$

[1)]e-mail: sran_g@yahoo.com

where x_j and $x_{j\pm1}$ are the coordinates of the particles in the string. The corresponding inter particle force acting on the j-th particle is $F_{j,j\pm1} = -\partial W_{j,j\pm1}/\partial x_j$ and the corresponding Yukawa force on the j-th particle in the string is given by

$$F_Y = F_{j,j-1} + F_{j,j+1} = F(x_{j-}) - F(x_{j+}),\\ x_{j-} = | x_j - x_{j-1} |, \quad x_{j+} = | x_{j+1} - x_j |, \tag{2}$$

where

$$F(x_{j-}) = \frac{Q^2}{4\pi\epsilon_0 x_{j-}} \exp\left(-\frac{x_{j-}}{\lambda_D}\right)\left[\frac{1}{x_{j-}} + \frac{1}{\lambda_D}\right],\\ F(x_{j+}) = \frac{Q^2}{4\pi\epsilon_0 x_{j+}} \exp\left(-\frac{x_{j+}}{\lambda_D}\right)\left[\frac{1}{x_{j+}} + \frac{1}{\lambda_D}\right]. \tag{3}$$

Introducing the displacement from the steady state as $r_j(= x_j + j\Delta)$ and expanding $F(x_{j-})$, $F(x_{j+})$ in Taylor series about Δ in powers of r_j, the Yukawa force can be rewritten as

$$F_Y = m\frac{C_{DL}^2}{\kappa^2\lambda_D^2}\Big[(r_{j+1} - 2r_j + r_j) + \\ + \frac{\Lambda}{2\kappa\lambda_D}\left((r_j - r_{j-1})^2 - (r_{j+1} - r_j)^2\right)\Big] + \cdots, \tag{4}$$

where

$$\Lambda = \frac{(\kappa^3 + 3\kappa^2 + 6\kappa + 6)}{(\kappa^2 + 2\kappa + 2)} \tag{5}$$

and C_{DL} is the dust lattice speed. For nearest neighbour approximation i.e. for $\kappa(= \Delta/\lambda_D) \gg 1$, the dust lattice speed can be approximated as [12, 15]

$$C_{DL}^2 \approx \frac{Q^2 e^{-\kappa}}{4\pi\epsilon_0 m\kappa\lambda_D}\left(\kappa^2 + 2\kappa + 2\right). \tag{6}$$

In solids, shocks are characterized by the damage they inflict. An elastic shock is characterized by non damaging stresses below the elastic limit. Therefore, 'hydrodynamical damping' which is extensively used in elasticity theory [24] is proposed for the dissipative mechanism in dusty plasma crystal. This damping force due to finite velocity of internal motions of the system, namely time derivative of the relative displacement between dusts in the chain can be expressed through the following discrete relation

$$F_{\text{damp}} = m\nu_d\frac{d}{dt}\left[(r_{j+1} - r_j) - (r_j - r_{j-1})\right] = \\ = m\nu_d\frac{d}{dt}\left[(r_{j+1} - 2r_j + r_{j-1})\right], \tag{7}$$

where ν_d is the damping frequency. Also it is assumed that the ratio of the damping frequency to dust lattice oscillation frequency $\omega_L\left(= \sqrt{Q^2 e^{-\kappa}(\kappa^2 + 2\kappa + 2)/4\pi\epsilon_0 m\kappa^3\lambda_D^3}\right)$ is finite i.e.

$$\nu_d \sim O(\omega_L) \Rightarrow \frac{\nu_d}{\omega_L} = \bar{\nu}_d \sim O(1) \tag{8}$$

For simplicity, the external forces such as initial laser excitation and / or the parabolic confinement forces which are often arise in experiments [12, 13, 23] are neglected. The damping due to neutral drag is considered under the assumption that dust-neutral collision rate ν_{drag} [25] is low compared to ω_L i.e. $\nu_{\text{drag}} \ll \omega_L$. Thus, the equation of motion of the jth dust grain in the chain is given by

$$m\left(\frac{d^2 r_j}{dt^2} + \nu_{\text{drag}}\frac{dr_j}{dt}\right) = F_Y + F_{\text{damp}}. \tag{9}$$

Let us now consider the continuum approximation. Assume, as usual, that r_j changes appreciably only on a scale L (the typical scale length of the wave form that can be the width of a pulse or the wavelength of a sinusoidal wave) much larger than the lattice spacing (inter dust spacing) Δ. One may treat j as a quasi-continuous variable (coordinate), then in the lowest approximation $O\left(\Delta/L\right)^3$, the equation of motion (9) goes over into the continuum equation

$$\frac{d^2 r}{dt^2} + \nu_{\text{drag}}\frac{dr}{dt} = \\ = C_{DL}^2\left[\frac{\partial^2 r}{\partial x^2} - \Lambda\frac{\partial r}{\partial x}\frac{\partial^2 r}{\partial x^2} + \bar{\nu}_d\Delta^2\frac{\partial^3 r}{\partial t\partial x^2}\right]. \tag{10}$$

To derive the Burgers' equation, the following stretched coordinates are introduced

$$\xi = \epsilon L(x - C_{DL}t), \quad \tau = \epsilon^2\omega_L t, \quad \frac{\partial r}{\partial \xi} = uC_{DL}, \tag{11}$$

where ϵ is a small parameter that indicates the magnitude of the rate of change, L is a characteristic length (length of typical wave form). Also for the consistent perturbation, the following scaling for ν_{drag} is introduced

$$\nu_{\text{drag}} \sim O(\epsilon^2)\omega_L \Rightarrow \frac{\nu_{\text{drag}}}{\omega_L} = \bar{\nu}_{\text{drag}} \sim O(\epsilon^2). \tag{12}$$

Finally, substituting equations (11) and (12) into equation (10), and keeping the terms $O\left(\epsilon^3\right)$, the following (non-dimensional) Burgers' equation with a linear damping term is derived

$$\frac{\partial u}{\partial \tau} - \alpha u\frac{\partial u}{\partial \xi} + \gamma u = \mu\frac{\partial^2 u}{\partial \xi^2}, \tag{13}$$

where

$$\alpha = \frac{\kappa C_{DL}\Lambda}{2}\left(\frac{\lambda_D}{L}\right), \ \gamma = \frac{\bar{\nu}_{\text{drag}}}{2}, \ \mu = \frac{\bar{\nu}_d\kappa^2}{2}\left(\frac{\lambda_D}{L}\right)^2. \tag{14}$$

The above relation (14) shows that the Burgers' term $\mu \propto \bar{\nu}_d$ (other plasma parameters remain constant), the normalized damping frequency, whereas the damping term $\gamma \propto \bar{\nu}_{\rm drag}$, the normalized neutral drag frequency. Thus the Burgers' term in equation (13) originates from the 'hydrodynamical damping' and the damping term in the same equation originates due to neutral drag. Also the presence of Burgers' term in equation (13) implies the possibility of existence of shock structure. Thus the dissipative effect due to 'hydrodynamical damping' is responsible for the generation of shock wave in complex (dusty) plasma.

The solution of the Burgers' equation with a linear damping term (Eq. (13)) is given as [26]:

$$u(\xi,\tau)=V(\tau)\left[1+\tanh\beta(\tau)\eta(\tau)\right];\ \eta(\tau)=\xi+\theta(\tau), \quad (15)$$

where

$$V(\tau)=V_0e^{-\gamma\tau}, \quad \frac{d\theta(\tau)}{d\tau}=V_0\alpha e^{-\gamma\tau} \quad (16)$$

are the shock amplitude, shock velocity and $V_0 = V(\tau = 0)$ is the initial shock intensity. On the other hand the shock width is proportional to

$$\frac{1}{\beta(\tau)}=\frac{2\mu}{V_0\alpha}e^{\gamma\tau}. \quad (17)$$

The solution (15) together with (16) shows that the shock amplitude $V(\tau)$ and shock velocity $d\theta/d\tau$ decreases exponentially with time τ. On the other hand shock width $\beta(\tau)^{-1}$ behaves convesrely and hence the product of the shock amplitude and shock width $(V(\tau)\beta(\tau)^{-1} = 2\mu/\alpha$ is time (τ) independent) is constant as shock propagates from upstream to downstream side.

To analyze the equation (15) numerically, the following representative plasma parameters of glow-discharge plasma are adopted [22]: $\Delta = 256\mu$m, $\lambda_D = 67\mu$m, $\kappa = 3.8$, $\nu_{\rm drag} = 10{\rm s}^{-1}$, $a = 4.45\mu$m, $m = 5.57\cdot10^{-13}$ kg and $Q = 1.6\cdot10^4$ e. The dynamical behaviour of the approximate solution (Eq.(15)) of the modified form of Burgers' equation (Eq.(13)) is shown in Fig.1. It is observed that the shock amplitude $V(\tau)$ decays with time τ and the damped shock moves with gradually diminishing velocity. As a result, it will propagate a finite distance $D \approx \alpha V_0/\gamma$ before it dies out $[\tau \to \infty]$. The variations of shock width with shielding parameter κ for different characteristic length L are plotted in Fig.2. This figure shows that shock width increases with the increase of shielding parameter κ, whereas it decreases with the increase of characteristic length L. This is happened due to the fact that the nonlinear interactions between two neighbouring dust grains in the chain increase with the increase of κ, whereas, these interactions decrease with the increase of L.

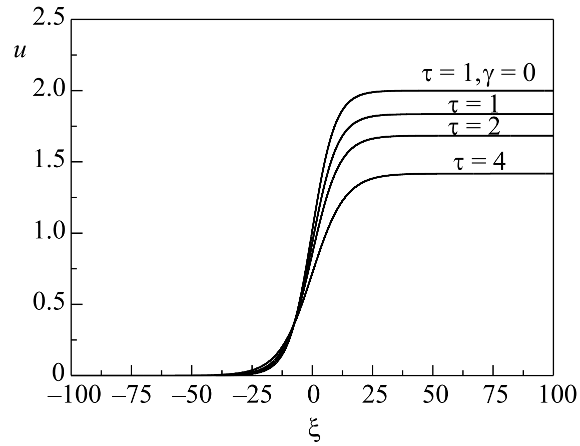

Fig.1. Monotonic shock structures of particle velocity u with ξ in complex plasma in presence of damping γ. The plasma parameters are: $L = 10\lambda_D$, $\kappa = 3.8$, $\bar{\nu}_d = 1$, $\gamma = 0.086$ and initial amplitude $V_0 = 1$

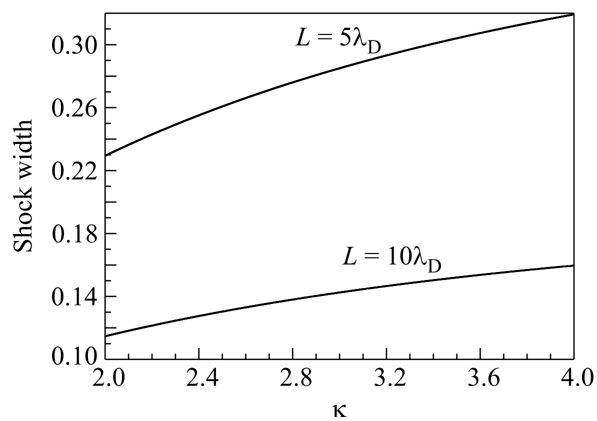

Fig.2. Variations of shock width with shielding parameter κ in complex plasma for $V_0 = 1$, $\tau = 1$, $\gamma = 0.086$, and $\bar{\nu}_d = 1$

In conclusion, it is observed that the 'hydrodynamical damping' in dusty plasma crystal may balance the wave breaking nonlinearity and generates shock wave in complex plasma. The observed shock is compressive in nature and shock width increases (decreases) with the increase of $\kappa(L)$. The neutral drag due to dust-neutral collision does not contribute to shock wave formation, but plays a predominant role in the life and death of shock structures. For the above specified glow-discharge plasma parameters, before the death of the observed shock, the shock moves a sufficiently long distance $D \approx 450\mu$m ($\gg \lambda_D = 67\mu$m) to be observed in the laboratory.

The author would like to thank Prof. M.R. Gupta, Centre for Plasma Studies, Faculty of Science, Jadavpur University, Kolkata 700 032, India for his stimulating influence and helpful suggestions during the course of this work.

1. J. H. Chu and L. Li, Phys. Rev. Lett. **72**, 4009 (1994); H. Thomas, G. E. Morfill, V. Demmel et al., Phys. Rev. Lett. **73**, 652 (1994).
2. J. B. Pieper and J. Goree, Phys. Rev. Lett. **77**, 3137 (1996).
3. A. Homann, A. Melzer, S. Peters et al., Phys. Lett. A **242**, 173 (1998).
4. G. Morfill, A. V. Ivlev, and J. R. Jokipii, Phys. Rev. Lett. **83**, 971 (1999).
5. S. Nunomura, D. Samsonov, and J. Goree, Phys. Rev. Lett. **84**, 5141 (2000); S. Nunomura, J. Goree, S. Hu et al., Phys. Rev. Lett. **89**, 035001 (2002); S. Nunomura, T. Misawa, N. Ohno, and S. Takamura, Phys. Rev. Lett. **83**, 1970 (1999).
6. A. P. Nefedov, O. F. Petrov, V. I. Molotkov, and V. E. Fortov, JETP Lett. **72**, 313 (2000).
7. H. Ohta and S. Hamaguchi, Phys. Rev. Lett. **84**, 6026 (2000).
8. X. Wang, A. Bhattacharjee, and S. Hu, Phys. Rev. Lett. **86**, 2569 (2001).
9. B. Klumov, M. Rubin-Zuzic, and G. E. Morfill, JETP Lett. **84**, 542 (2007).
10. A. Tsurui, Prog. Theo. Physics **48**, 1196 (1972); M. Wadati, J. Phys. Soc. Jpn. **38**, 673, 681 (1975).
11. A. S. Davydov, *Solitons in molecular systems*, Riedel Publ.-Kluwer, Dordrecht, 1985; A. C. Newell and J. V. Moloney, *Nonlinear Optics*, Addison-Wesley Publ. Co. Redwood City Ca., 1992.
12. D. Samsonov, A. V. Ivlev, R. A. Quinn et al., Phys. Rev. Lett. **88**, 095004 (2002); V. Nosenko, S. Nunomura, and J. Goree, Phys. Rev. Lett. **88**, 215002 (2002).
13. S. Nunomura, S. Zhdanov, G. Morfill, and J. Goree, Phys. Rev. E **68**, 026407 (2003).
14. V. Nosenko, K. Avinash, J. Goree, and B. Liu, Phys. Rev. Lett. **92**, 085001-1 (2004).
15. F. Melandso, Phys. Plasmas **3**, 3890 (1996).
16. S. Zhdanov, D. Samsonov, and G. Morfill, Phys. Rev. E **66**, 026411 (2002).
17. K. Avinash, P. Zhu, V. Nosenko, and J. Goree, Phys. Rev. E**68**, 046402 (2003).
18. Y. Nakamura, H. Bailung, and P. K. Shukla, Phys. Rev. Lett. **83**, 1602 (1999); Q. Z. Luo, N. D'Angelo, and R. L. Merlino, Phys. Plasmas **6**, 3345 (1999).
19. S. I. Popel, A. P. Golub, T. V. Losseva, and R. Bingham, JETP Lett. **73**, 258 (2001); S. I. Popel, A. P. Golub, and T. V. Losseva, JETP Lett. **74**, 396 (2001).
20. Samiran Ghosh, S. Sarkar, M. Khan, and M. R. Gupta, Phys. Lett. A **274**, 162, (2000); M. R. Gupta, S. Sarkar, Samiran Ghosh, and M. Khan, Phys. Rev. E **63**, 046406, (2001).
21. Samiran Ghosh and M. R. Gupta, Phys. Plasmas **12**, 092306-1 (2005); Samiran Ghosh, Phys. Plasmas **13**, 0223016-1 (2006).
22. D. Samsonov, J. Goree, Z. W. Ma et al., Phys. Rev. Lett. **83**, 3469 (1999).
23. D. Samsonov, S. K. Zhdanov, R. A. Quinn et al., Phys. Rev. Lett. **92**, 255004-1 (2004).
24. L. D. Landau and E. M. Lifshitz, *Theory of elasticity*, Pergamon Press, London, 1959, p. 122.
25. P. Epstein, Phys. Rev. **23**, 710 (1924).
26. Samiran Ghosh, M. Khan, and M. R. Gupta, Phys. Plasmas **13**, 102312 (2006).