The influence of spin-dependent phases of tunneling electrons on the conductance of a point ferromagnet/isolator/superconductor contact

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The Andreev reflection probability for a ferromagnet/isolator/superconductor (FIS) contact at the arbitrary spin-dependent amplitudes of the electron waves transmitted through and reflected from the potential barrier is found. It is shown that Andreev reflection probabilities of electron and hole excitations in the FIS contact are different. The energy levels of Andreev bound states are found. The ballistic conductance of the point FIS contact is calculated.

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One of the manifestations of the exchange field in a ferromagnetic metal (F) is the presence of electron spin subbands with different values of Fermi momenta: $p_\uparrow$ for the subband with spin up and $p_\downarrow$ for the subband with spin down. As a consequence, for layered F/S structures (S stands for superconductor) the spatial dependence of the anomalous Green's function (GF) in a ferromagnet has an oscillatory character. One of the impressive manifestations of such oscillations and related phase shifts is a recent observation of spontaneous zero-field supercurrents at temperature lower than the junction $0-\pi$ transition temperature in superconducting networks of SFS junctions with weakly ferromagnetic barriers [1]. The influence of the oscillatory character of the anomalous GF in a ferromagnet on the properties of various hybrid F/S structures is studied well enough (see reviews [2 – 4]).

Another consequence is the suppression of Andreev reflection [5]. When a polarized electron from the subband with, for example, spin up gets into a superconductor, the reflected hole moves into the subband with spin down. Consequently, the efficiency of Andreev reflection is determined by the number of conducting channels in a subband with a smaller value of the Fermi momentum. As a result, the subgap conductance of an F/S contact decreases with the increase of the polarization of a ferromagnet [6].

Effects of spin filtering [7 – 9] and spin mixing [10] are manifested in the dependence of moduli and phase shifts of the amplitudes of electron states on the Fermi surface reflected from r_α ($r_\alpha = \sqrt{R_\alpha}\exp(i\theta^r_\alpha)$; $R_\alpha = 1 - D_\alpha$) and transmitted through a potential barrier d_α ($d_\alpha = \sqrt{D_\alpha}\exp(i\theta^d_\alpha)$) on α ($\alpha = \uparrow,\downarrow$ is the spin index). These effects are the consequence of the presence of the exchange field in a ferromagnet as well.

The possibility to study the influence of the spin mixing effect on the $I-V$ characteristics of superconducting weak links containing a magnetically active interface appeared after the boundary conditions (BCs) for the quasiclassical GF were obtained.

In paper [11], BCs for the quasiclassical GF for two metals in contact via a magnetically active interface in terms of an interface scattering matrix were derived. These equations were solved for a junction in the tunneling limit [11] and for a contact of a superconductor with a ferromagnetic insulator [12]. In paper [10], BCs for the retarded and advanced quasiclassical GFs were obtained in terms of Riccati amplitudes [13, 14]. In paper [15], BCs in terms of Riccati amplitudes were obtained for the nonequilibrium quasiclassical GF.

The equations, obtained in papers [10] and [15], were solved for magnetically active interfaces with finite transmission (for SFS [8, 10], for NFS [15] (N stands for normal metal), for S-FIF-S [16]). These solutions show that Andreev bound states appear within the superconducting gap [8, 10, 15], and the $0-\pi$ transition in the SFS junction is possible [8, 10].

In papers [9] and [17], quasiclassical equations of superconductivity for metals with a spin-split conduction band were derived and BCs for the temperature quasiclassical GF for the F/S interface were obtained. The model interface was the same as in [11, 18].

The aim of this work is to study the influence of spin-dependent phases of the amplitudes of the electron states reflected from and transmitted through a potential barrier on Andreev reflection in a point FIS contact.

Calculations are carried out by the method of quasiclassical GFs with BCs for GFs obtained in papers

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[9, 17]. Below the dependence of the Andreev reflection probability on spin-dependent phase shifts θ^d_α and θ^r_α will be found and the results of the numerical calculation of the dependence $G_{FIS}(V)$ for a rectangular potential barrier and ferromagnets with high polarization will be discussed.

1. Differential conductance of a point FIS contact. In various hybrid F/S structures Andreev reflection is modified. The reflected hole has some parameters (for example, the velocity modulus and phase shift) different from those of the incident electron because it moves in a subband with the opposite spin. Such spin-discriminating processes due to the exchange field in a ferromagnet lead to the formation of Andreev bound states inside the gap [8, 10].

The enegy of Andreev bound states depends on the spin index [8, 10]. As a result, the spectral density of condauctance G_{FIS} of the FIS contact at zero voltage is no longer a symmetrical function of energy ε. The condition of the time reversal invariance has the form $G_{FIS}(\varepsilon,\alpha) = G_{FIS}(-\varepsilon,-\alpha)$. The generalization of the conductance $G_{FIS}(V)$ [9, 19] for this case results in the following expression for $G_{FIS}(V)$:

$$G_{FIS}(V) = \frac{e^2A}{32\pi^2 T}\sum_\alpha \mathrm{Tr}\left[\int \frac{d\mathbf{p}_\|}{(2\pi)^2}\int\limits_{-\infty}^{\infty} d\varepsilon \times \right.$$
$$\times\frac{1}{\coth^2(\frac{\varepsilon - eV\hat\tau_z}{2T})}[1 - \hat g^A_s \tau_z \hat g^R_s \hat\tau_z - \hat g^A_a \hat\tau_z \hat g^R_a \hat\tau_z +$$
$$\left. + \hat\Upsilon^A_s \hat\tau_z \hat\Upsilon^R_s \hat\tau_z - \hat\Upsilon^A_a \hat\tau_z \hat\Upsilon^R_a \hat\tau_z]\right]. \quad (1)$$

In Eq. (1), A is the contact area; $\hat\tau_z$ is the Pauli matrix; $p_\|$ is the momentum in the contact plane; ($\hat g_s$, $\hat\Upsilon_s$) and ($\hat g_a$, $\hat\Upsilon_a$) are quasiclassical retarded (R) and advanced (A) GFs symmetric and antisymmetric with respect to the projection of the momentum $\hat{\mathbf{p}}$ on the Fermi surface on the axis x, respectively [9]. Calculations in Eq. (1) are to be carried out on the boundary of any contacting metal.

2. Finding GFs and conductance. Let us assume that the barrier with the width d is located in the region $a < x < b$ ($d = b - a$), the superconductor occupies the region $x > b$, and the ferromagnet occupies the region $x < a$. To find GFs, for each metal one has to solve quasiclassical equations of superconductivity for metals with a spin-split conductivity band simultaneously with their BCs derived in paper [9]:

$$\mathrm{sign}(\hat p_x)\frac{\partial}{\partial x}\hat g + \frac{1}{2}\mathbf{v}_\|\frac{\partial}{\partial \rho}(\hat v_x^{-1}\hat g + \hat g \hat v_x^{-1}) + [\hat K, \hat g]_- = 0,$$

$$\hat K = -i\hat v_x^{-\frac12}(i\varepsilon_n\hat\tau_z + \hat\Delta - \hat\Sigma)\hat v_x^{-\frac12} - i(\hat p_x - \hat\tau_x\hat p_x\hat\tau_x)/2,$$
$$[a,b]_- = ab - ba. \quad (2)$$

In this section, $\varepsilon_n = (2n+1)\pi T$ is the Matsubara frequency; $\hat\Sigma$ is the self-energy part; $\hat g$ are matrix temperature GFs:

$$\hat g = \begin{pmatrix} g_{\alpha\alpha} & f_{\alpha-\alpha} \\ f^+_{-\alpha\alpha} & -g_{-\alpha-\alpha}\end{pmatrix}, \hat g = \begin{cases} \hat g_> \hat p_x > 0, \\ \hat g_< \hat p_x < 0\end{cases}.$$

Moreover,

$$\hat\Delta = \begin{pmatrix} 0 & \Delta \\ -\Delta^* & 0\end{pmatrix}, \qquad \hat p_x = \begin{pmatrix} p_{x,\alpha} & 0 \\ 0 & p_{x,-\alpha}\end{pmatrix},$$

where Δ is the order parameter, and p_x is the projection of the momentum on the Fermi surface on the axis x. Matrices $\hat v$ have the same structure as $\hat p_x$.

BCs for the specular reflection of electrons from the boundary: $p_\| = p_\downarrow \sin\vartheta_\downarrow = p_\uparrow \sin\vartheta_\uparrow = p_S \sin\vartheta_S$, have the form [9]:

$$(\hat{\check g}^S_a)_d = (\hat{\check g}^F_a)_d, \quad (\hat{\check\Upsilon}^S_a)_d = (\hat{\check\Upsilon}^F_a)_d,$$
$$(\sqrt{\hat R_\alpha} - \sqrt{\hat R_{-\alpha}})(\hat{\check\Upsilon}^+_a)_n = \alpha_3(\hat{\check g}^-_a)_n,$$
$$(\sqrt{\hat R_\alpha} - \sqrt{\hat R_{-\alpha}})(\hat{\check\Upsilon}^-_a)_n = \alpha_4(\hat{\check g}^+_a)_n,$$
$$-\hat{\check\Upsilon}^-_s = \sqrt{\hat R_\alpha}(\hat{\check g}^+_s)_d + \alpha_1(\hat{\check g}^+_s)_n,$$
$$-\hat{\check\Upsilon}^+_s = (\hat R_\alpha)^{-\frac12}(\hat{\check g}^-_s)_d + \alpha_2(\hat{\check g}^-_s)_n, \quad (3)$$

where $\hat{\check g}^\pm_{a(s)} = 1/2[\hat{\check g}^S_{a(s)} \pm \hat{\check g}^F_{a(s)}]$. Functions $\hat{\check\Upsilon}^\pm_{a(s)}$ are determined analogously. The index d denotes the diagonal and n the nondiagonal part of the matrix $\hat T_{d(n)} = 1/2[\hat T \pm \tau_z\hat T\tau_z]$. Coefficients α_i are:

$$\alpha_{1(2)} = \frac{1 + \sqrt{R_\uparrow R_\downarrow} \mp \sqrt{D_\uparrow D_\downarrow}}{\sqrt{R_\uparrow} + \sqrt{R_\downarrow}},$$
$$\alpha_{3(4)} = 1 - \sqrt{R_\uparrow R_\downarrow} \pm \sqrt{D_\uparrow D_\downarrow}\,).$$

One can exclude GFs $\hat{\check\Upsilon}^F_a$ and $\hat{\check\Upsilon}^S_a$ from these relations and obtain a system of BCs only for the GF $\hat{\check g}$ [17]:

$$\begin{aligned}\hat{\check g}^+_a\widehat b_1 + \widehat b_2\hat{\check g}^+_a + \hat{\check g}^-_a\widehat b_3 + \widehat b_4\hat{\check g}^-_a &= \widehat b_3 - \widehat b_4,\\ \hat{\check g}^-_a\widehat b_1 + \widehat b_2\hat{\check g}^-_a + \hat{\check g}^+_a\widehat b_3 + \widehat b_4\hat{\check g}^+_a &= \widehat b_1 - \widehat b_2.\end{aligned} \quad (4)$$

Matrices $\widehat b_i$ in Eq. (4) are:

$$\begin{aligned}\widehat b_1 &= \hat{\check\Upsilon}^+_s\hat{\check g}^-_s + \hat{\check\Upsilon}^-_s\hat{\check g}^+_s, \quad \widehat b_2 = \hat{\check g}^+_s\hat{\check\Upsilon}^-_s + \hat{\check g}^-_s\hat{\check\Upsilon}^+_s,\\ \widehat b_3 &= \hat{\check\Upsilon}^+_s\hat{\check g}^+_s + \hat{\check\Upsilon}^-_s\hat{\check g}^-_s, \quad \widehat b_4 = \hat{\check g}^+_s\hat{\check\Upsilon}^+_s + \hat{\check g}^-_s\hat{\check\Upsilon}^-_s.\end{aligned} \quad (5)$$

GFs $\hat{\tilde{g}}$ are connected with GFs being solutions of Eq. (2) by the following relationships [9]:

$$(\hat{\tilde{g}}_s^S)_n = (\hat{g}_s^S)_n \cos(\theta_\alpha) + i\hat{\tau}_z(\hat{g}_a^S)_n \sin(\theta_\alpha)$$
$$(\hat{\tilde{g}}_a^S)_n = (\hat{g}_a^S)_n \cos(\theta_\alpha) + i\hat{\tau}_z(\hat{g}_s^S)_n \sin(\theta_\alpha)$$
$$(\hat{\tilde{g}}_s^F)_n = (\hat{g}_s^F)_n \cos(\beta_\alpha^r) + i\hat{\tau}_z(\hat{g}_a^F)_n \sin(\beta_\alpha^r)$$
$$(\hat{\tilde{g}}_a^F)_n = (\hat{g}_a^F)_n \cos(\beta_\alpha^r) + i\hat{\tau}_z(\hat{g}_s^F)_n \sin(\beta_\alpha^r)$$

$$\theta_\alpha = \frac{\theta_\alpha^r - \theta_{-\alpha}^r}{2} - (\theta_\alpha^d - \theta_{-\alpha}^d); \quad \beta_\alpha^r = \frac{\theta_\alpha^r - \theta_{-\alpha}^r}{2}. \quad (6)$$

The explicit form of functions $\hat{\tilde{\Upsilon}}$ is not needed. These functions are found from BCs. The diagonal parts of matrices $\hat{\tilde{g}}$ are equal to the corresponding matrices $\hat{g}$. Equations (2) for the ballistic contact are solved in paper [9, 18]. At the boundaries for $x = b$ and $x = a$ we have:

$$\hat{g}_s^S = \hat{g}_0^S + \hat{g}_0^S\hat{g}_a^S; \quad \hat{g}_s^F = \hat{g}_0^F - \hat{g}_0^S\hat{g}_a^F. \quad (7)$$

Matrices $\hat{g}_0$ are values of GFs $\hat{g}$ away from the boundary:

$$\hat{g}_0^F = \mathrm{sign}(\varepsilon_n)\hat{\tau}_z \quad (8)$$
$$\hat{g}_0^S = g_0^S\hat{\tau}_z + (\hat{g}_0^S)_n = \frac{1}{\sqrt{\varepsilon_n^2 + |\Delta|^2}}\begin{pmatrix}\varepsilon_n & -i\Delta \\ i\Delta^* & -\varepsilon_n\end{pmatrix}.$$

After the substitution of functions $\hat{\tilde{g}}_s^F$ and $\hat{\tilde{g}}_s^S$, expressed via $\hat{\tilde{g}}_a^F$ and $\hat{\tilde{g}}_a^S$ by Eq. (7), in the system of BCs Eq.(4) and their solution in the linear approximation with respect to the functions $\hat{\tilde{g}}_a^S$ and $\hat{\tilde{g}}_a^F$, we find the function $\hat{\tilde{g}}_a^F$:

$$\hat{\tilde{g}}_a^F = -\frac{\sqrt{D_\uparrow D_\downarrow}\hat{\tau}_z(\hat{g}_0^S)_n}{Z}$$
$$Z = (1 - \sqrt{R_\uparrow R_\downarrow})[g_0^S\cos(\theta_\alpha) + i\sin(\theta_\alpha)] + \quad (9)$$
$$+(1 + \sqrt{R_\uparrow R_\downarrow})\mathrm{sign}(\varepsilon_n)[\cos(\theta_\alpha) + ig_0^S\sin(\theta_\alpha)].$$

From Eqs. (3) and (4) we find the rest functions necessary to calculate conductance Eq. (1) and calculate conductance at the ferromagnet side.

After carrying out the analytical continuation in these functions (substitution $i\varepsilon_n$ for $\varepsilon \pm i\delta$ for retarded and advanced GFs, respectively), we obtain the expression for the conductance $\sigma_{F/S}(V)$:

$$\sigma_{F/S}(V) = \frac{e^2A}{\pi}\int\frac{d\mathbf{p}_\parallel}{(2\pi)^2}\left\{\int_{|\Delta|}^{\infty}\frac{d\varepsilon}{2T}\left[\frac{1}{\cosh^2(\frac{\varepsilon+eV}{2T})} + \frac{1}{\cosh^2(\frac{\varepsilon-eV}{2T})}\right]\frac{\varepsilon\xi^R(D_\uparrow + D_\downarrow) + \varepsilon(\varepsilon - \xi^R)D_\uparrow D_\downarrow}{Z_\Uparrow} + \right.$$
$$\left. + \int_0^{|\Delta|}\frac{d\varepsilon}{2T}\left[\frac{1}{\cosh^2(\frac{\varepsilon+eV}{2T})} + \frac{1}{\cosh^2(\frac{\varepsilon-eV}{2T})}\right]\frac{D_\uparrow D_\downarrow|\Delta|^2}{Z_\Downarrow}\right\},$$
$$Z_\Uparrow = [\varepsilon(1 - W) + \xi(1 + W)]^2 + 4W|\Delta|^2\sin^2(\theta_\alpha),$$
$$Z_\Downarrow = [1 + 2W\cos(2\theta_\alpha) + W^2]|\Delta|^2 - 4W\varepsilon^2\cos(2\theta_\alpha) -$$
$$-\frac{16W(|\Delta|^2 - \varepsilon^2)\varepsilon^2\sin^2(2\theta_\alpha)}{[1 + 2W\cos(2\theta_\alpha) + W^2]|\Delta|^2 - 4W\varepsilon^2\cos(2\theta_\alpha)},$$
$$W = \sqrt{R_\uparrow R_\downarrow}; \quad \xi = \sqrt{\varepsilon^2 - |\Delta|^2}. \quad (10)$$

At $\theta_\alpha = 0$ the expression for conductance obtained in paper [9] follows from Eq. (10). In the case of nonmagnetic metal, when $D_\uparrow = D_\downarrow$ this expression is the same as that obtained in paper [18], and for $D = 1/(1 + Z^2)$ this expression is the same as that obtained in paper [19].

3. Andreev reflection. The quasiclassical GFs entering Eq. (8) enable the conclusion that

$$[1 - \hat{g}_s^A\tau_z\hat{g}_s^R\hat{\tau}_z - \hat{g}_a^A\hat{\tau}_z\hat{g}_a^R\hat{\tau}_z + \hat{\Upsilon}_s^A\hat{\tau}_z\hat{\Upsilon}_s^R\hat{\tau}_z -$$
$$- \hat{\Upsilon}_a^A\hat{\tau}_z\hat{\Upsilon}_a^R\hat{\tau}_z] = 4[-\hat{\tilde{g}}_a^A\hat{\tau}_z\hat{\tilde{g}}_a^R\hat{\tau}_z] \sim \hat{1}. \quad (11)$$

Now, the comparison of the form of under-gap conductances in Eq. (1) and that of the corresponding Eq. (25) in paper [19] shows that the matrix elements $(\hat{\tilde{g}}_a^R)^F$ and $(\hat{\tilde{g}}_a^A)^F$ are the amplitudes of the Andreev reflection probability $\tilde{a}(\varepsilon, \theta_\alpha)$ in FIS contacts for energies less than $|\Delta|$ ($\varepsilon^2 < |\Delta|^2$). Let us assume that $\tilde{a}(\varepsilon, \theta_\alpha)$ are matrix elements of $(\hat{\tilde{g}}_a^R)^F$.

$$\tilde{a}(\varepsilon, \theta_\alpha) = \frac{\sqrt{D_\uparrow D_\downarrow}\Delta}{Z} = a(\varepsilon, \theta_\alpha)e^{-i\beta_\alpha^r} \quad (12)$$
$$Z = (1 - \sqrt{R_\uparrow R_\downarrow})[\varepsilon\cos(\theta_\alpha) - \sqrt{|\Delta|^2 - \varepsilon^2}\sin(\theta_\alpha)] +$$
$$+ i(1 + \sqrt{R_\uparrow R_\downarrow})[\sqrt{|\Delta|^2 - \varepsilon^2}\cos(\theta_\alpha) + \varepsilon\sin(\theta_\alpha)].$$

The presence of the imaginary part in functions $a(\varepsilon, \theta_\alpha)$ means that Andreev reflection is accompanied by the phase shift. The Andreev reflection probability $A(\varepsilon, \theta_\alpha)$ ($A(\varepsilon, \theta_\alpha) = \tilde{a}(\varepsilon, \theta_\alpha)\tilde{a}^*(\varepsilon, \theta_\alpha)$) is:

$$A(\varepsilon, \theta_\alpha) = \frac{D_\uparrow D_\downarrow|\Delta|^2}{Z}, \quad (13)$$
$$Z = [1 - \sqrt{R_\uparrow R_\downarrow}]^2|\Delta|^2 +$$
$$+ 4\sqrt{R_\uparrow R_\downarrow}[\sqrt{|\Delta|^2 - \varepsilon^2}\cos(\theta_\alpha) + \varepsilon\sin(\theta_\alpha)]^2.$$

It follows from this equation that: (1) in terms of paper [10] spin-mixing angle Θ for FIS contact is equal to θ_α (for SFS and NFS contacts $\Theta=\theta^r_\uparrow-\theta^r_\downarrow=\theta^d_\uparrow-\theta^d_\downarrow$ [8, 10, 15]); (2) for $\theta_\alpha<0$ the Andreev reflection probability of the electron excitation with the spin projection α is larger than that of the hole excitation; for $\theta_\alpha>0$ the Andreev reflection probability of the hole excitation with the spin projection α is larger than that of the electron excitation; (3) the Andreev reflection probability has maxima at $\varepsilon=\varepsilon_b$ (at the values of the energy of electron (hole) excitations corresponding to the energy levels of Andreev surface bound states)

$$\epsilon_b=\begin{cases}\varepsilon=|\Delta|\cos(\theta_\alpha) & \text{for}\quad \theta_\alpha<0,\\ \varepsilon=-|\Delta|\cos(\theta_\alpha) & \text{for}\quad \theta_\alpha>0.\end{cases} \tag{14}$$

Below the results of the numerical calculations of phase shifts and conductance are presented. In the numerical calculations the relation between Fermi momenta of contacting metals was the following: $p_S=(p_\uparrow+p_\downarrow)/2$. Calculations are carried out for a rectangular barrier with the height U counted off the bottom of the conduction band of a superconductor; [$\chi(x)$ is the wave function of an electron in an isolator, $\chi(x)=C_1\exp(\gamma x)+$ $+C_2\exp(-\gamma x)$; $\gamma=\sqrt{k^2+p_\parallel^2}$; $k^2=2m_b(U-E_F^S)$; E_F^S is the Fermi energy of a superconductor, m_b is the mass of an electron in a barrier]. In this case the expressions for θ^d_α and θ^r_α have the following form:

$$\theta^d_\alpha=\widetilde{\theta}^d_\alpha+i(p^F_{x,\alpha}a-p^S_xb);\quad \theta^r_\alpha=\widetilde{\theta}^r_\alpha+2ip^F_{x,\alpha}a,$$

$$\widetilde{\theta}^d_\alpha=\arctan\left(\frac{(p^F_{x,\alpha}p^S_x-\gamma^2)\tanh(\gamma d)}{\gamma(p^F_{x,\alpha}+p^S_x)}\right), \tag{15}$$

$$\widetilde{\theta}^r_\alpha=\arctan\left(\frac{2\gamma p^F_{x,\alpha}[\gamma^2+(p^S_x)^2]\tanh(\gamma d)}{Z}\right),$$

$$Z=\gamma^2[(p^S_x)^2-(p^F_{x,\alpha})^2]+[\gamma^2-p^S_xp^F_{x,\alpha}]^2\tanh^2(\gamma d),$$

so that the angle θ_α [$\theta_\alpha=(\theta^r_\alpha-\theta^r_{-\alpha})/2-(\theta^d_\alpha-\theta^d_{-\alpha})$] $=$ $=(\widetilde{\theta}^r_\alpha-\widetilde{\theta}^r_{-\alpha})/2-(\widetilde{\theta}^d_\alpha-\widetilde{\theta}^d_{-\alpha})$ does not depend on the location of the barrier.

Figure 1 shows the dependences of the phase shifts on $\cos(\vartheta_\downarrow)$. All angles are connected by specular reflection $p_\parallel=p_\downarrow\sin\vartheta_\downarrow=p_\uparrow\sin\vartheta_\uparrow=p_S\sin\vartheta_S$.

The phase shift $\theta_\uparrow$ slowly decreases as the polarization of the ferromagnet δ decreases [from (-1.3) at $\delta=0.05$ to (-1.5) at $\delta=0.5$ $(p_\uparrow d=1; k/p_\uparrow=0.8)$] and [from (-0.7) at $\delta=0.05$ to (-1.2) at $\delta=0.5$ $(p_\uparrow d=1; k/p_\uparrow=0.2)$]. It means that the points ϵ_b approach zero as the polarization decreases and $k/p_\uparrow$ increases, however, at $k/p_\uparrow>1$, $\theta_\uparrow$ rapidly decreases down to zero. 6. With the increasing parameter $p_\uparrow d$ and other parameters fixed (but for $k/p_\uparrow<1$) the angle θ_α also tends to $\pi/2$. Note that the spin-mixing angle θ_α for ferromagnets with large polarization is practically the same for all electron trajectories.

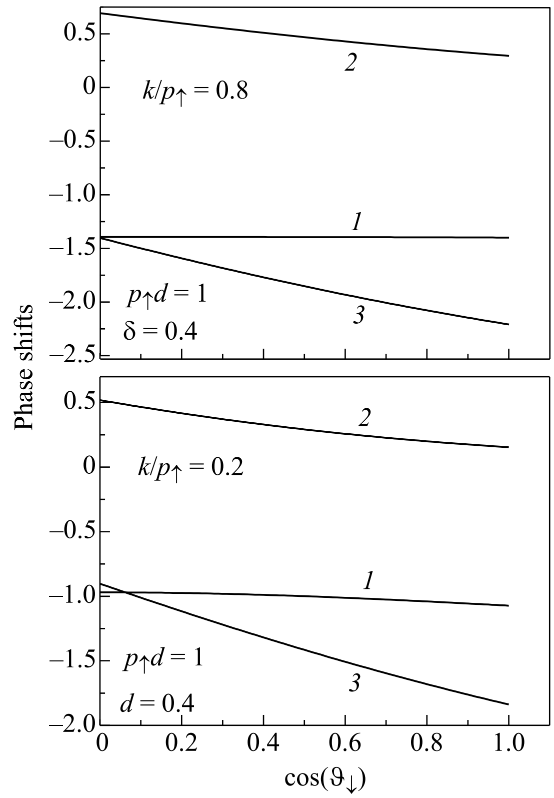

Fig.1. Dependence of the phase shifts of reflection and transmission amplitudes on $\cos(\vartheta_\downarrow)$. Lines with numbers *1*, *2*, and *3* depict these dependence on $\cos(\vartheta_\downarrow)$: $\theta_\uparrow$; $(\theta^d_\uparrow-\theta^d_\downarrow)$ and $(\theta^b_\uparrow-\theta^b_\downarrow)$, respectively

The upper panel in Fig.2 shows the results of the numerical calculations carried out according to Eq. (10) not taking into account (dashed lines) and taking into account (solid lines) the phase shift $\theta\alpha$. The peaks in the dependence of the conductance on V (Fig.2, upper panel) correspond to the motion of the energy levels of Andreev surface bound states towards each other as the parameter $k/p_\uparrow$ increases.

The lower panel in Fig.2 shows the suppression of Andreev reflection due to the reduction of the number of conducting channels in a subband with a lower value of the Fermi momentum and the effect of spin filtering. Andreev surface bound states are formed in a superconductor due to the interference of electronlike and holelike particles with different spin-dependent phase shifts. To demonstrate this, let us consider diagrams in Fig.3, corresponding to Andreev reflection of an electron with

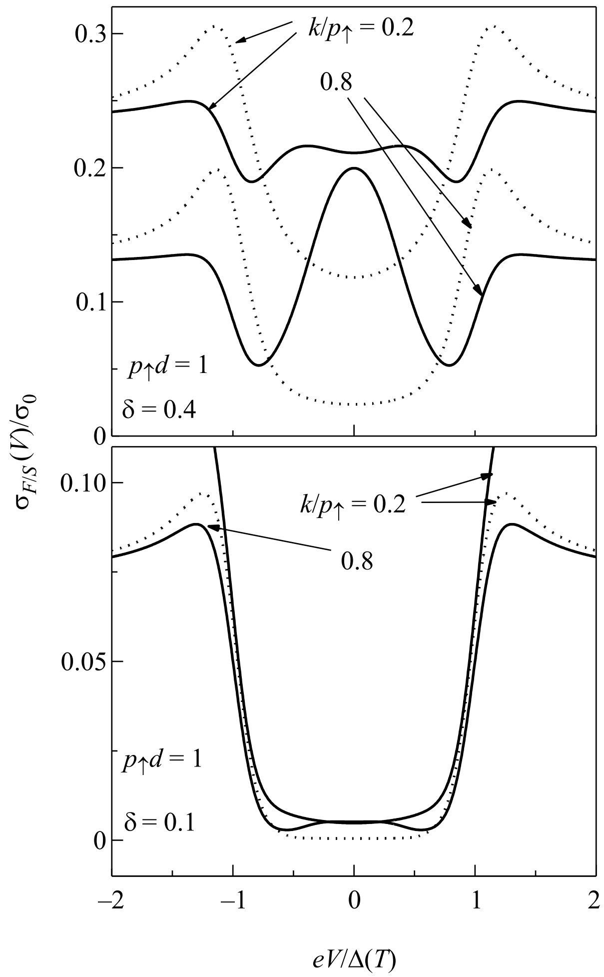

Fig.2. Dependence of the normalized conductance $\sigma_{F/S}(V)/\sigma_0$ from Eq.(10) on the applied voltage for different values of the polarization of a ferromagnet $\delta = p_\downarrow/p_\uparrow$ at the ratio $\Delta_d(T)/2T = 6$

the spin projection α and the energy less than $|\Delta|$ transmitted from a ferromagnet into a superconductor. The amplitude $a(\varepsilon, \theta_\alpha)$ is:

$$a(\varepsilon,\theta_\alpha) = d_\alpha \tilde{d}^*_{-\alpha}\beta^{eh}_{\alpha,-\alpha}[1 + \tilde{r}^*_{-\alpha}\tilde{r}_\alpha\beta^{eh}_{\alpha,-\alpha}\beta^{he}_{-\alpha,\alpha} + (\tilde{r}^*_{-\alpha}\tilde{r}_\alpha\beta^{eh}_{\alpha,-\alpha}\beta^{he}_{-\alpha,\alpha})^2 + \ldots] = \frac{d_\alpha\tilde{d}^*_{-\alpha}\beta^{eh}_{\alpha,-\alpha}}{1 - \tilde{r}^*_{-\alpha}\tilde{r}_\alpha\beta^{eh}_{\alpha,-\alpha}\beta^{he}_{-\alpha,\alpha}} = \frac{\sqrt{D_\alpha D_{-\alpha} p^F_{x\alpha}/p^F_{x-\alpha}}\, e^{i\beta^r_\alpha}\beta^{eh}_{\alpha,-\alpha}}{e^{i\theta_\alpha} - e^{-i\theta_\alpha}\sqrt{R_\alpha R_{-\alpha}}\,\beta^{eh}_{\alpha,-\alpha}\beta^{he}_{-\alpha,\alpha}}. \quad (16)$$

The corresponding probability of Andreev reflection is:

$$A(\varepsilon,\theta_\alpha) = \frac{D_\alpha D_{-\alpha} p^F_{x,\alpha}/p^F_{x,-\alpha}\beta^{eh}_{\alpha,-\alpha}\beta^{*eh}_{\alpha,-\alpha}}{1 + R_\alpha R_{-\alpha}|\beta^{eh}_{\alpha,-\alpha}|^2|\beta^{he}_{-\alpha,\alpha}|^2 - Q}, \quad (17)$$

$$Q = \sqrt{R_\alpha R_{-\alpha}}\left[\cos(2\theta_\alpha)[\beta^{eh}_{\alpha,-\alpha}\beta^{he}_{-\alpha,\alpha} + \beta^{*eh}_{\alpha,-\alpha}\beta^{*he}_{-\alpha,\alpha}] + i\sin(2\theta_\alpha)[\beta^{*eh}_{\alpha,-\alpha}\beta^{*he}_{-\alpha,\alpha} - \beta^{eh}_{\alpha,-\alpha}\beta^{he}_{-\alpha,\alpha}]\right].$$

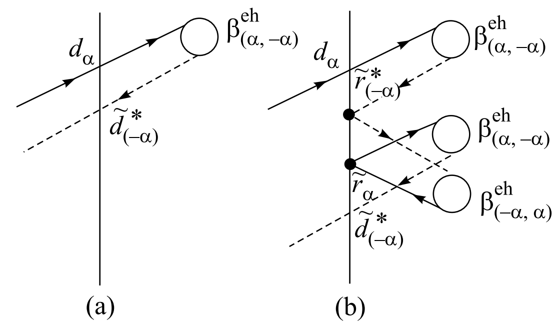

Fig.3. Structure of the diagrams corresponding to Andreev reflection in the superconductor: diagram (a) one-act process; diagram (b) two-act process. The vertex ○ is Andreev reflection of electronlike (solid lines) and holelike (broken lines) quasiparticles by the pair potential. The vertex • is the normal reflection of electronlike and holelike quasiparticles by the barrier potential. When the solid line transforms into the broken line, ○ denotes the vertex $\beta^{eh}_{\alpha,-\alpha}$. When the broken line transforms into the solid line, ○ denotes the vertex $\beta^{he}_{-\alpha,\alpha}$. Parameters $d_\alpha, \tilde{d}_\alpha, r_\alpha$ and $\tilde{r}_\alpha$ are related as follows: $\tilde{d}_\alpha = d_\alpha p^S_x/p^F_{x\alpha}$; $\tilde{r}_\alpha = -r^*_\alpha d_\alpha/d^*_\alpha$; $D_\alpha = d_\alpha\tilde{d}_\alpha$ [18]

By comparing formulas (16), (17) with formulas (12), (13) we find the vertices $\beta^{eh}_{\alpha,-\alpha}$ and $\beta^{he}_{-\alpha,\alpha}$:

$$\beta^{eh}_{\alpha,-\alpha} = \sqrt{\frac{p^F_{x,-\alpha}}{p^F_{x,\alpha}}}\;\frac{\varepsilon - i\sqrt{|\Delta|^2 - \varepsilon^2}}{|\Delta|}\frac{\Delta}{|\Delta|}, \quad (18)$$

$$\beta^{he}_{-\alpha,\alpha} = \sqrt{\frac{p^F_{x,\alpha}}{p^F_{x,-\alpha}}}\;\frac{\varepsilon - i\sqrt{|\Delta|^2 - \varepsilon^2}}{|\Delta|}\frac{\Delta^*}{|\Delta|}.$$

It follows from formula (17) that in the absence of the interferential term Q the probability of Andreev reflection is a constant (independent of the energy ε) quantity. The interference of electronlike and holelike particles reflected by the pair potential and interface results in the formation of Andreev surface bound states. At $\theta_\alpha = 0$ the maximum in the probability of Andreev reflection is at $\varepsilon = \pm|\Delta|$ [19]. At $\theta_\alpha = \pm\pi/2$ Andreev surface bound states with the width Γ equal to:

$$\Gamma = \frac{(1 - \sqrt{R\!\uparrow\! R\!\downarrow})|\Delta|}{2\sqrt[4]{R\!\uparrow\! R\!\downarrow}} \quad (19)$$

are formed at $\varepsilon = 0$ on the Fermi level. The peak in the differential conductance of an FIS contact at the zero voltage may be used to determine the polarization of strong ferromagnets by comparing experimental data with those calculated according to formula (10).

Thus, in the present paper the ballistic conductance of the point FIS contact is calculated. The dependence of Andreev surface bound states on the spin-dependent phase shifts of the electron states reflected from and transmitted through the potential barrier is found for the interface with finite transmission. By the example of a rectangular potential barrier it is shown that these states are manifested in the peaks of the dependence of the conductance of the FIS contact on the applied voltage.

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