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Radiative corrections to the cross section of $e^- + p \to \nu + n$ and the crossed processes

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Born cross section and the radiative corrections to its lowest order are considered in the frame work of QED with structureless nucleons including the emission of virtual and real photons. The result is generalized to take into account radiative corrections in higher orders of perturbation theory in the leading and next-to leading logarithmic approximation. Crossing processes are considered in the leading approximation.

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1. Introduction. The process of neutron β decay has been much investigated experimentally. However, its theoretical description has not been sufficiently studied up to now because of the uncertainties due the strong interaction effects which arise from calculations of radiative corrections (RC). Research into neutrino-deuterium inelastic scattering was carried out using such experimental setups as that at the Sudbury Neutrino Observatory (SNO)[1]. The increasing accuracy of experiments and the importance of adequate theoretical description of the neutrino sector of Standard Model (SM) is the motivation of our paper. It is devoted to calculation of the RC in the framework of the QED with the point-like nucleons and omission of the electro-weak effects. The latter were considered in a series of papers (see [2] and the references therein). Below we consider the process

$$e(p_e) + P(p) \to \nu(p_\nu) + N(p_n), \tag{1}$$

and the crossed processes

$$\begin{aligned} &\nu(p_\nu) + N(p_n) \to e(p_e) + P(p), (\nu n),\\ &\bar\nu(p_\nu) + P(p) \to e^+(p_e) + N(p_n), (\nu p). \end{aligned} \tag{2}$$

Our work is performed in terms of the old formulation of weak processes, which is valid for the intermediate region of electron energies E (we suppose proton to be at rest -Laboratory Frame)

$$m = m_e \ll E \ll M_Z. \tag{3}$$

2. Born cross section. Kinematics. The matrix element of the process $ep \to \nu n$ in the Born approximation has the form:

$$M = \frac{G}{\sqrt{2}} V_{ud} \bar u_\nu(p_\nu)\gamma_\alpha(1+\gamma_5)u(p_e)\bar{u_n}\gamma_\alpha(1+\rho\gamma_5)u(p), \tag{4}$$

with G, V_{ud} being the Fermi constant, the Cabbibo-Kobayashi-Maskawa quark-mixing matrix element and $\rho = g_A/g_V = 1.25$ being the ratio of axial and vector coupling constants.

The cross-section has the form

$$d\sigma = \frac{1}{8s}\sum |M_{if}|^2 d\Gamma, \tag{5}$$

with

$$\begin{aligned} &\sum |M_{if}|^2 = 8G^2|V_{ud}|^2 S_0(p_e,p),\\ &S_0(p_e,p) = s^2(1+\rho)^2 + u^2(1-\rho)^2 -\\ &\quad -2M^2(s+u)(1-\rho^2); \end{aligned} \tag{6}$$

where

$$s = 2ME; \quad u = -2p_e p_n = -2E(E_n - P_n c),$$
$$p_e^2 = m^2, \quad p^2 = p_n^2 = M^2,$$

and the phase volume

$$d\Gamma = \frac{d^3p_\nu}{2E_\nu}\frac{d^3p_n}{2E_n}(2\pi)^{-2}\delta^4(p_e + p - p_\nu - p_n). \tag{7}$$

Performing the integration on neutrino momentum we write the phase volume as

$$d\Gamma = \frac{cdc}{2\pi}\frac{ME(M+E)^2}{[(M+E)^2 - E^2c^2]^2}, \tag{8}$$

where E- is the energy of initial electron, $c = \cos\theta$ and θ is the angle in Laboratory Frame between the 3-momenta of electron and neutron. The energy E_n and the value of 3-momentum of neutron P_n are:

$$E_n = M\frac{(E+M)^2 + E^2c^2}{(E+M)^2 - E^2c^2}; \quad P_n = M\frac{2E(E+M)c}{(E+M)^2 - E^2c^2}. \tag{9}$$

Another form of phase volume can be used for investigating the neutron energy distribution

$$d\Gamma = \frac{dE_n}{8\pi E}. \qquad (10)$$

3. Virtual photon emission and contribution counter term. Consider now the one-loop virtual radiative correction: the doubled interference of Born amplitude with the one where the emission of virtual photon is taken into account. As in Born amplitude, we suggest that the loop momentum, as well as the external ones, is much smaller compared to the W-boson mass. Only one triangle Feynman diagram is relevant. The corresponding contribution to the summed up spin states of the square of matrix element is

$$\Delta \sum |M|^2_{virt} = (G_F V_{ud}/\sqrt{2})^2 \frac{8\alpha}{\pi} \int \frac{d^4k}{i\pi^2} \frac{T}{(0)(1)(2)},$$
$$(0) = k^2 - \lambda^2); \quad (1) = (p_- - k)^2 - m^2 + i0;$$
$$(2) = (p+k)^2 - M^2 + i0, \qquad (11)$$

where k is the loop 4-momentum, λ is the fictitious photon mass, and

$$T = \frac{1}{4}\mathrm{Tr}\, \hat{p}_\nu \gamma_\alpha (1+\gamma_5)(\hat{p}_e - \hat{k})\gamma_\mu \hat{p}_e \gamma_\eta (1+\gamma_5) \times$$
$$\times \frac{1}{4}\mathrm{Tr}(\hat{p}_n + M)\gamma_\alpha(1+\rho\gamma_5)(\hat{p}+\hat{k}+M) \times$$
$$\times \gamma_\mu (\hat{p}+M)\gamma_\eta(1+\rho\gamma_5). \qquad (12)$$

The next step is to perform the loop momentum integration. To avoid ultraviolet divergences we must to introduce the cut off parameter $|k^2| < \Lambda^2$. It is natural to choose the cut-off parameter to be equal to the W-boson mass:

$$\Lambda = M_W.$$

Using the Feymnan trick to join the denominators we obtain the expression

$$\int \frac{d^4k}{i\pi^2} \int_0^1 dx \int_0^1 2y dy \frac{T}{[(k - yp_x)^2 - D]^3},$$

with $D = y^2 p_x^2 + (1-x)\lambda^2 + i0, p_x^2 = x^2 m^2 + (1-x)^2 M^2 - sx(1-x)$.

The computation of the traces leads to:

$$T = A_0 + A_e(2kp_e) + A_n(2kp_n) + A_p(2kp) +$$
$$+ A_{en}(2kp_e)(2kp_n) + A_{ep}(2kp_e)(2kp) +$$
$$+ A_{pn}(2kp)(2kp_n) + A_{ee}(2kp_e)^2 + A_{nn}(2kp_n)^2 + A_{kk}k^2, \qquad (13)$$

with

$$A_0 = 2sS_0; \quad A_e = 2(s+M^2)[s(1+\rho)^2 - u(1-\rho)^2];$$
$$A_n = 2s[u(1-\rho)^2 - M^2(1-\rho^2)];$$
$$A_p = -2s^2(1+\rho)^2 + 2(2s+u)M^2(1-\rho^2);$$
$$A_{np} = u(1-\rho)^2;$$
$$A_{ee} = (s+u)(1-\rho)^2 - 4M^2\rho(1-\rho);$$
$$A_{kk} = -2u(u+s)(1-\rho)^2 - 4s^2(1+\rho)^2 +$$
$$+ 4M^2(s+u)(1+\rho-2\rho^2);$$
$$A_{ep} = u(1-\rho)^2 - 4M^2(1-\rho^2);$$
$$A_{en} = -(2s+u)(1-\rho)^2 + 4M^2\rho(1-\rho). \qquad (14)$$

Further integration (we are interested only in the real part) gives

$$[i; i_x; i_{\bar{x}b}; i_{x\bar{x}}; i_{x\bar{x}b}; i_{\bar{x}b\bar{x}b}] =$$
$$= \int_0^1 \frac{dx}{p_x^2}[1; x, 1-x; x^2; x(1-x); (1-x)^2] =$$
$$= \begin{cases} -\frac{1}{s}L; -\frac{1}{s}L + \frac{1}{a}L_M; -\frac{1}{a}L_M; \\ -\frac{1}{s}L + \frac{1}{a} + \frac{s+2M^2}{a^2}L_M; -\frac{1}{a} - \frac{M^2}{a^2}L_M; \\ \frac{1}{a} - \frac{s}{a^2}L_M, \end{cases} \qquad (15)$$

$$i_p = \int_0^1 dx \ln\frac{p_x^2}{M^2} = -2 + \frac{s}{a}L_M, \qquad (16)$$

with $L = \ln(4E^2/m^2)$ being the so-called "large logarithm", $L_M = \ln(2E/M)$ and $a = s + M^2$. Finally,

$$I = \int \frac{d^4k}{i\pi^2}\frac{1}{(0)(1)(2)} = -\int_0^1\int_0^1 \frac{dx y dy}{D} = -\int_0^1 \frac{dx \ln\frac{p_x^2}{\lambda^2}}{2p_x^2}. \qquad (17)$$

Performing the integration on Feynman parameters we obtain

$$\mathrm{Re}\, I = \frac{1}{2s}\left[\frac{1}{2}L^2 + 2L\ln\frac{m}{\lambda} - L_M^2 - \pi^2 - 2Li_2\left(1+\frac{M^2}{s}\right)\right], \qquad (18)$$

with the Euler dilogarithm defined as

$$Li_2(z) = -\int_0^z \frac{dx}{x}\ln(1-x). \qquad (19)$$

The result is:

$$2\sum M_B^* M_{\rm virt} = = -\frac{\alpha}{\pi}S_0\Big[-L + L\ln\frac{m}{\lambda} + \frac{1}{4}L^2 - \frac{1}{2}L_m^2 - -\frac{\pi^2}{2} - Li_2\left(1+\frac{M^2}{s}\right) - AL_\Lambda + K_V\Big], \quad (20)$$

where $L_\Lambda = \ln(M_W^2/M^2)$ and

$$A = \frac{1}{2S_0}[4s^2(1+\rho)^2 + u^2(1-\rho)^2 - 5M^2(s+u)(1-\rho^2)]. \quad (21)$$

A rather complicated expression for K_V is given in Appendix.

Since we work within the unrenormalized theory, we must take into account the fermion (of mass m) wave function renormalization:

$$Z = 1 - \frac{\alpha}{2\pi}\left[\frac{1}{2}\ln\frac{\Lambda^2}{m^2} + \ln\frac{\lambda^2}{m^2} + \frac{9}{4}\right]. \quad (22)$$

Keeping in mind to apply this procedure for both electron and proton we obtain the contribution to the summed on spin states of the matrix element squared

$$\Delta|M|_c^2 = -\frac{\alpha}{2\pi}S_0\left[L_\Lambda + \frac{9}{2} - \frac{1}{2}L + L_M\right]. \quad (23)$$

4. Real soft and hard photon emission. A standard way to take into account the emission of additional soft (in the Laboratory frame) real photon consists in calculation of the 3-dimensional integral

$$\frac{d\sigma_{\rm soft}}{d\sigma_B} = -\frac{4\pi\alpha}{16\pi^3}\int\frac{d^3k_1}{\omega_1}\left(\frac{p_e}{p_ek_1} - \frac{p}{pk_1}\right)^2\Big|_{\omega_1<\Delta E}, \quad \Delta E \ll E. \quad (24)$$

The standard calculation leads to [3]:

$$d\sigma_{\rm soft} = d\sigma_B\frac{\alpha}{\pi}\Big[(L-1)\ln\frac{\Delta E}{E} + +(L-2)\ln\frac{m}{\lambda} + \frac{1}{4}L^2 + 1 - \xi_2\Big], \quad \xi = \frac{\pi^2}{3}. \quad (25)$$

The total sum including the Born cross section and corrections arising from the wave function renormalization, and the ones arising from taking into account emission of virtual and soft real photons is free from infrared singularities

$$\frac{d\sigma_B + d\sigma^{s+v+c}}{d\sigma_B} = = 1 + \frac{\alpha}{2\pi}\left[P_\Delta(L-1) + \frac{\alpha}{\pi}K_{svc} - \frac{\alpha}{\pi}AL_\Lambda\right] - \frac{\alpha}{\pi}\ln\frac{\Delta E}{E}, \quad (26)$$

with

$$P_\Delta = 2\ln\frac{\Delta E}{E} + \frac{3}{2}. \quad (27)$$

We note that the term $(L-1)P_\Delta$ containing the "large logarithm" can be associated with the kernel of the Altarelly-Lipatov-Parisi evolution equation of twist-2 operators [4].

Results obtained here for virtual and soft real photons emission are in agreement with ones obtained in [5, 6]

The emission of the real hard photon matrix element has the form

$$M^\gamma = \frac{G}{\sqrt{2}}V_{ud}\bar u_\nu(p_\nu)\gamma_\alpha(1+\gamma_5)\left[\frac{\hat p_e - \hat k + m}{-2p_ek}\hat e\right]u(p_e)\times \times\bar u(p_n)\gamma_\alpha(1+\rho\gamma_5)u(p) - \bar u_\nu(p_\nu)\gamma_\alpha(1+\gamma_5)u(p_e)\times \times\bar u(p_n)\gamma_\alpha(1+\rho\gamma_5)\frac{\hat p - \hat k + m}{-2pk}\hat e u(p). \quad (28)$$

We extract the terms which correspond to the collinear kinematics of photon emission (photon momentum is collinear to the electron one) and the noncollinear ones. The first contribution contains the large logarithm L, and thus includes electron mass singularities. The second contribution is finite in the mass electron zero limit

$$\sum|M^\gamma|^2 = \sum|M^\gamma|^2_{\rm coll} + \sum|M^\gamma|^2_{\rm ncoll}. \quad (29)$$

The first term (in agreement with the prescription of quasi-real electron method [7] is:

$$\frac{1}{16}\sum|M^\gamma|^2_{\rm coll} = \left(\left(\frac{G}{\sqrt{2}}V_{ud}\right)^2 S_0(p_e(1-x),p)\right)\times \times\left[\frac{1+(1-x)^2}{x(1-x)}\frac{1}{p_ek} - \frac{m^2}{(p_ek)^2}\right], \quad (30)$$

with $x = \omega/E$ energy fraction of hard photon.

The relevant contribution to the cross section is

$$d\sigma_h = \frac{\alpha}{\pi}\int_\Delta^1 dx\left[\frac{1+(1-x)^2}{2x}(L-1) + K_h\right]\times \times d\sigma_B(p_e(1-x),p) + \frac{\alpha}{\pi}\ln\Delta\, d\sigma_B(p_e,p), \quad \Delta = \frac{\Delta E}{E} \ll 1. \quad (31)$$

5. Cross section in the leading and next to leading approximation. Let us write down the results obtained above in the Born approximation and their corrections connected with emission of virtual and soft photons as:

$$d\sigma_B + d\sigma_{vs} = d\sigma_B\left[1 + \frac{\alpha}{2\pi}(L-1)\left(2\ln\Delta + \frac{3}{2}\right) + \right.$$
$$\left. + \frac{\alpha}{\pi}K_{vsc} - \frac{\alpha}{\pi}\ln\Delta\right]. \quad (32)$$

Keeping in mind the result for the contribution of hard real photon emission (see previous section) the total sum can be written in the form:

$$d\sigma = \int\limits_0^1 D(x,L)d\sigma_B(p_e(1-x),p)\left(1+\frac{\alpha}{\pi}K\right)dx, \quad (33)$$

with the non-singlet electron Structure function defined as [8]:

$$D(x,L) = \delta(x) + \frac{\alpha}{2\pi}(L-1)P^{(1)}(x) + ..., \quad (34)$$

and the kernel of the evolution equation of twist-2 operator

$$P^{(1)}(x) =$$
$$= \lim_{\Delta\to 0}\left[\delta(x)\left(2\ln\Delta + \frac{3}{2}\right) + \frac{1+(1-x)^2}{x}\theta(x-\Delta)\right]. \quad (35)$$

The smoothed form of the electron structure function is

$$D(x,L) = bx^{b-1}\left[1+\frac{3}{4}b\right] - b\left(1-\frac{1}{2}x\right) + O(b^2), \quad (36)$$

$$b = \frac{2\alpha}{\pi}(L-1); \qquad \int_0^1 D(x,L)dx = 1.$$

The so-called K-factor $K = K_{EW} + K_{vsc} + K_h$ is the smooth function, finite in the limit of zero electron mass $m \to 0$. We put this contribution in the form $K_{EW} = (K_{EW})_\Lambda + (K_{EW})_f$, where

$$(K_{EW})_\Lambda = \left[-\frac{1}{2} + A\right]\ln\left(\frac{M_W^2}{M^2}\right), \quad (37)$$

and the part which does not depend on M_W^2/M^2 is included in $(K_{EW})_f$. Numerically, it is much smaller than $(K_{EW})_\Lambda$ and is not touched upon here. For details see [2].

The K_h value depends on experimental conditions of photon detection and is will not considered here.

The function K_{svc} is presented in the Table.

6. Crossed processes. Discussion. The differential cross section of process (2) $e^+(p_e) + N(p_n) \to \to P(p) + \bar\nu(p_\nu)$ can be obtained from the results given above by replacement $s \leftrightarrow -u$ with the same definitions of kinematic invariants.

The cross section of the process $(\nu n), (\nu p)$ (see (2)) in the leading logarithmical approximation (LLA) has the form

$$\frac{d}{dy}d\sigma^i(y,s) = \int_y^1 \frac{dx}{x}d\sigma_B^i\left(\frac{p_e}{x}\right)D\left(\frac{y}{x},L\right), i=(\nu p),(\nu n), \quad (38)$$

where E_ν, yE_ν are the energies of the initial neutrino and the final lepton.

The relevant cross sections in the Born approximation have a form (5) with the squared matrix element given in (6) and the phase volume defined in (7), (8).

One can be convinced in the fulfillment of the Kinoshita-Lee-Nauenberg theorem [9, 10] about cancellation of mass singularities in the fraction of final electron differential cross sections integrated by energy:

$$\int_0^1 dy\frac{d}{dy}d\sigma(y) = \int_0^1 dx d\sigma_B\left(\frac{p_e}{x}\right)\int_0^x \frac{dy}{x}D\left(\frac{y}{x},L\right) =$$
$$= \int_0^1 dx d\sigma_B\left(\frac{p_e}{x}\right). \quad (39)$$

As a result, the dependence on L disappears.

The order of value of the total cross sections considered above

$$\sigma \sim 10\text{pb}\ E(\text{GeV})$$

is too small to be measured in modern experiments.

Above, we supposed to simplify the structure of hadron weak current, which must be considered as a model approximation. In reality, the induced axial and vector nucleon formfactors must be taken into account [11]. Nevertheless, the form cross section (33) remains valid.

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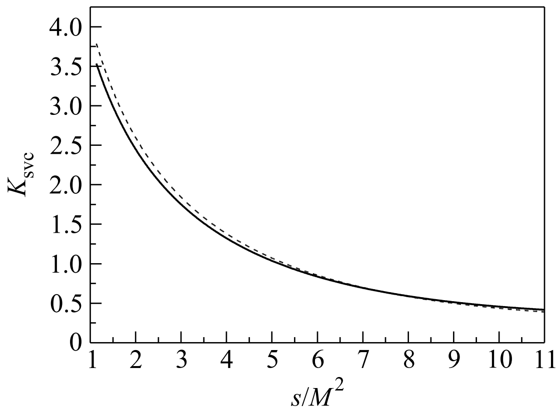
Appendix: *K*-factor. The relevant loop momentum integrals are

$$(I, I_\mu, I_{\mu\nu}, I_{kk}) = \int \frac{d^4k}{i\pi^2}\frac{1, k_\mu, k_\mu k_\nu, k^2}{(0)(1)(2)}. \quad (A.1)$$

Feynman joining denominators procedure leads to (scalar 3-denominator integral I is given above):

$$I_\mu = -p_{e\mu}i_x + p_\mu i_{\bar x};$$
$$I_{\mu\nu} = \frac{1}{4}g_{\mu\nu}\left[L_\Lambda - i_p - \frac{1}{2}\right] -$$
$$-\frac{1}{2}[p_{e\mu}p_{e\nu}I_{xx} + p_\mu p_\nu i_{\bar x\bar x} - (p_{e\mu}p_\nu + p_{e\nu}p_\mu)I_{x\bar x}];$$
$$I_{kk} = L_\Lambda - i_p - 1. \quad (A.2)$$



Dependence of K_{svc} on (s/M^2) for $u = -s$ (dashed line) and $u = 0$ (solid line) (see Appendix)

The values of the contributions to the $K_{svc} = -K_V + K_S + K_C$-factor are:

$$K_S = 1 + \xi_2; \quad K_C = -\frac{9}{4} - \frac{1}{2}L_M, \tag{A.3}$$

and

$$K_V = -\frac{1}{2}L_M^2 - 3\xi_2 - Li_2\left(1 + \frac{M^2}{s}\right) + + \frac{1}{2S_0 a}\left[B_1 + \frac{1}{a}L_M B_M\right], \tag{A.4}$$

where

$$B_1 = -M^4(s+u)(1-\rho)(7+11\rho) + (s+M^2)u^2(1-\rho)^2 + +8s^3(1+\rho)^2 + M^2s^2(1+\rho)(-3+19\rho) - 9suM^2(1-\rho^2);$$

$$B_M = -4M^6(s+u)(1-\rho^2) + s^2u^2(1-\rho)^2 + 4s^4(1+\rho)^2 + +u^2(s+M^2)M^2(1-\rho)^2 - M^2s^2u(1-\rho)(7+5\rho) + +M^2s^3(1+\rho)(-1+13\rho) - 10M^4su(1-\rho^2) + +M^4s^2(-7+2\rho+13\rho^2). \tag{A.5}$$

In Figure the dependence of $K_{svc}(s,u)$ is presented as a function of s for fixed values of u.

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