Pis'ma v ZhETF, vol. 87, iss. 10, pp. 605 – 610 © 2008 May 25

Manifestation of a singlet heavy up-type quark in the branching ratios of rare decays $K \to \pi\nu\bar{\nu}$, $B \to \pi\nu\bar{\nu}$ and $B \to K\nu\bar{\nu}$

P. N. Kopnin[+*], *M. I. Vysotsky*[+1)]

[+]*Institute for theoretical and experimental physics, 117218 Moscow, Russia*

[*]*Moscow Institute of Physics and Technology (State University), 117303 Moscow, Russia*

Submitted 3 April 2008

We investigate the implications of the model with a $SU(2)$-singlet up-type quark, heavy enough not to be produced at the LHC, namely, the contribution of the new quark to the branching ratios of the $K \to \pi\nu\bar{\nu}$, $B \to \pi\nu\bar{\nu}$ and $B \to K\nu\bar{\nu}$ decays. We show that the deviation from the Standard Model can be up to 10% in the case of a 5 TeV quark. Precise measurements of these branching ratios at the future experiments will allow to observe the contributions of the new quark or to impose stronger constraints on its mass.

PACS: 12.15.–y, 12.60.–i, 13.20.–v

1. Introduction. When we discuss the corrections to the observables in flavor physics due to various types of New Physics, the processes that we investigate are determined by loop diagrams of two types: box-diagrams and the so-called penguin diagrams. In [1, 2] the contribution of a singlet heavy up-type quark to the observables determined by the box-diagrams (the mass differences of the neutral B-mesons Δm_{B_d}, Δm_{B_s} and CP-violation parameter in the oscillations of K-mesons ε_K) is discussed[2)]. The rare decays $K \to \pi\nu\bar{\nu}$, $B \to \pi\nu\bar{\nu}$ and $B \to K\nu\bar{\nu}$ are the processes determined in the Standard Model (SM) by penguin diagrams.

The possibility of the presence of new particles inside the loops makes the observables determined by loop diagrams quite useful when obtaining experimental bounds on the parameters of the models of New Physics. But the most important point here is the following. The formulae for the observables Δm_{B_d}, Δm_{B_s} and ε_K involve the pseudoscalar decay constants $f(B_d,\ B_s)$ and bag parameters $B(K,\ B_d,\ B_s)$ defined (in the case of the B_d-meson) as follows:

$$\begin{gathered}\langle 0|[\bar{b}\gamma_\mu(1+\gamma^5)d]|B_d\rangle = i f_{B_d} p_\mu^{B_d},\\ \langle \bar{B}_d|[\bar{b}_L\gamma_\mu d_L][\bar{b}_L\gamma_\mu d_L]|B_d\rangle = \tfrac{8}{3}B_{B_d}\times\\ \times\langle \bar{B}_d|[\bar{b}_L\gamma_\mu d_L]|0\rangle\langle 0|[\bar{b}_L\gamma_\mu d_L]|B_d\rangle.\end{gathered} \qquad (1)$$

The accuracy with which they are known is quite poor. For example, the evaluation of f_B^2 and $B(B_d,\ B_s)$ based on QCD lattice calculations has a 10% accuracy. Theoretical uncertainties of this kind make it impossible to detect the contributions of the New Physics to these observables if they are not well above 10%.

The theoretical expressions for the decay widths $\Gamma(K^+ \to \pi^+\nu\bar{\nu})$, $\Gamma(K_L \to \pi^0\nu\bar{\nu})$, $\Gamma(B_u \to \pi^+\nu\bar{\nu})$ and $\Gamma(B_d \to \pi^0\nu\bar{\nu})$ include matrix elements $\langle\pi^+|\bar{s}_L\gamma_\mu d_L|K^+\rangle$, $\langle\pi^0|\bar{s}_L\gamma_\mu d_L|K^0\rangle$, $\langle\pi^+|\bar{b}_L\gamma_\mu d_L|B_u\rangle$ and $\langle\pi^0|\bar{b}_L\gamma_\mu d_L|B_d\rangle$. Fortunately, these matrix elements are equal to the matrix elements $\langle\pi^0|\bar{s}_L\gamma_\mu u_L|K^+\rangle$, $\langle\pi^-|\bar{s}_L\gamma_\mu u_L|K^0\rangle$, $\langle\pi^0|\bar{b}_L\gamma_\mu u_L|B_u\rangle$ and $\langle\pi^-|\bar{b}_L\gamma_\mu u_L|B_d\rangle$ respectively with the accuracy of the isospin $SU(2)$ symmetry violation, which is approximately $(m_u - m_d)/\Lambda_{QCD} \approx 1\%$. The latter can be extracted from the data on the $K^+ \to \pi^0\nu e^+$, $K^0 \to \pi^-\nu e^+$, $B_u \to \pi^0\nu e^+$ and $B_d \to \pi^-\nu e^+$ decay widths. (For the $B \to K\nu\bar{\nu}$ width the corresponding accuracy is worse – of order of the $SU(3)$ symmetry violation $\approx$ 20%.) For this reason the branching ratios $Br(K^+ \to \pi^+\nu\bar{\nu})$, $Br(K_L \to \pi^0\nu\bar{\nu})$, $Br(B_u \to \pi^+\nu\bar{\nu})$ and $Br(B_d \to \pi^0\nu\bar{\nu})$ are well calculable, can serve as indicators of New Physics and help to establish its parameters.

2. Neutral and charged currents in the extended model. We are discussing a model with New Physics proposed in [1]. For its detailed description see [1] and [2]. Now we will present the formulae necessary for the calculations using the notations of [2].

Our model Lagrangian is the following [3)] ([1, 2]):

1) e-mail: kopnin@itep.ru, vysotsky@itep.ru

2) In the paper [3] the properties of the $SU(2)$-singlet down type quarks within the framework of the Minimal Flavor Violation are discussed. In [4] the tree-level contributions to different observables of a down type singlet quark are investigated. A model with a singlet up-type quark was studied in [5] in which the formulae are applicable only when the additional quark is comparatively light (just above the reach of the Tevatron) and mixes strongly with the t-quark.

3) $\mathcal{L}_{SM}$ is the SM Lagrangian with a zero Yukawa coupling of the Higgs to the t-quark.

$$\mathcal{L} = \mathcal{L}_{SM} + \bar{Q}'\left(i\gamma^\mu D_\mu - M\right)Q' + \\ + \left[\mu_R \bar{Q}'_L t'_R + \frac{\mu_L}{\eta/\sqrt{2}} H_c^+ \bar{Q}'_R \cdot \begin{pmatrix} t' \\ b^V \end{pmatrix}_L + \text{c.c.}\right]. \quad (2)$$

Here $D_\mu = \partial_\mu - i\frac{2}{3}g'B_\mu - ig_s G^a_\mu \frac{\lambda^a}{2}$. $t'_L = U_{t't''}t''_L + U_{t'c'}c'_L + U_{t'u'}u'_L$, where t''_L, c'_L, u'_L are the fields of the SM in the so-called flavor basis, U is the matrix that rotates them into the mass eigenstates c_L, u_L and t'_L (t' would have a zero mass if not for the mixing with the Q), and

$$b^V_L = V_{tb}b_L + V_{ts}s_L + V_{td}d_L,$$

where b_L, s_L, d_L are the fields in the mass basis, V is the CKM (Cabibbo–Kobayashi–Maskawa) matrix. H_c is the $SU(2)$-conjugate of the Higgs isodoublet H.

To establish the connection between the Q' and t' fields and the mass eigenstates one has to diagonalize the relevant bilinear terms of the Lagrangian. The result is the following:

$$t' \equiv t'_R + t'_L = N_R t_R - \frac{m_t}{\mu_L} N_L Q_R + N_L t_L - \frac{m_t}{\mu_R} N_R Q_L, \quad (3)$$

$$Q' \equiv Q'_R + Q'_L = N_R Q_R + \frac{m_t}{\mu_L} N_L t_R + N_L Q_L + \frac{m_t}{\mu_R} N_R t_L, \quad (4)$$

where

$$N_L = \left[1 + \frac{\mu_L^2}{M^2}\left(1 - \frac{m_t^2}{\mu_L^2}\right)^2\right]^{-1/2},$$
$$N_R = \left[1 + \frac{\mu_R^2}{M^2}\left(1 - \frac{m_t^2}{\mu_R^2}\right)^2\right]^{-1/2}. \quad (5)$$

The masses of Q and t are the following:

$$m_Q = M + O\left(\frac{\mu^2}{M}\right), \; m_t = \frac{\mu_L \mu_R}{m_Q}, \quad (6)$$

and the whole mass of the t-quark originates from its mixing with the heavy singlet quark Q'.

Substituting the expressions for the primed fields into the Lagrangian one gets the explicit form of the charged current (CC) interactions of the t- and Q-quarks with the gauge (W) and goldstone (G) bosons[4]:

$$\mathcal{L}_{CC} = \left(\frac{g}{\sqrt{2}}\bar{b}^V_L \gamma_\mu W^\mu t_L + \frac{m_t\sqrt{2}}{\eta}\bar{b}^V_L G t_R\right) N_L - \\ - \left(\frac{g}{\sqrt{2}}\bar{b}^V_L \gamma_\mu W^\mu Q_L + \frac{m_Q\sqrt{2}}{\eta}\bar{b}^V_L G Q_R\right) N_R \frac{\mu_L}{m_Q} + \text{c.c.}, \quad (7)$$

[4] G is the charged unphysical Higgs field that appears in the 't Hooft R_ξ-gauge.

– see [2].

The penguin decays $K \to \pi\nu\bar{\nu}$, $B \to \pi\nu\bar{\nu}$ and $B \to K\nu\bar{\nu}$ mentioned above are described by five diagrams (Fig.1).

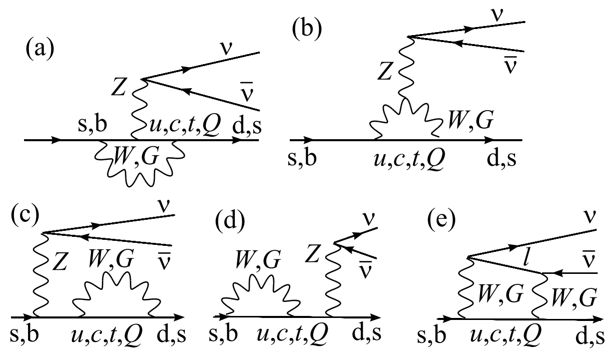

Fig.1. Diagrams contributing to the $s \to d\nu\bar{\nu}$, $b \to d\nu\bar{\nu}$ and $b \to s\nu\bar{\nu}$ transitions in the extension of the SM. Diagrams with self-energy insertions (c), (d) have to be multiplied by 1/2

In our model besides the u-, c- and t-quarks we will have to take into account the Q-quark exchange as well. The diagrams include the interaction of the quarks with the W-boson and the goldstone G. The first four of them also include the Z-boson exchange between the fermionic currents. To calculate them we have to establish the way in which the new particle Q interacts with Z.

The interaction of the Q-quark with Z will arise from the following terms of the Lagrangian[5] :

$$\mathcal{L}_{Q,t\leftrightarrow Z} = i\bar{g}Z_\mu \left[\bar{t}'\gamma^\mu(\hat{T}_3 - \hat{\mathcal{Q}}\sin^2\theta_W)t' - \\ + \frac{2}{3}\sin^2\theta_W \bar{Q}'\gamma^\mu Q'\right]. \quad (8)$$

Taking into account (3), (4) and (5) we obtain the following expression for the ttZ, QQZ and tQZ neutral current (NC) interactions in our model:

$$\mathcal{L}_{NC} = i\bar{g}Z_\mu \left[\frac{1}{2}N_L^2 \cdot \bar{t}_L\gamma^\mu t_L - \frac{2}{3}\sin^2\theta_W \cdot \bar{t}\gamma^\mu t - \\ - \frac{2}{3}\sin^2\theta_W \cdot \bar{Q}\gamma^\mu Q + \frac{1}{2}\frac{\mu_L^2}{m_Q^2}N_R^2 \cdot \bar{Q}_L\gamma^\mu Q_L - \\ - \frac{1}{2}\frac{\mu_L}{m_Q}N_R N_L \cdot \left(\bar{t}_L\gamma^\mu Q_L + \bar{Q}_L\gamma^\mu t_L\right)\right]. \quad (9)$$

In our model the flavor changing tQZ neutral current (FCNC) appears.

Just like in [1, 2], throughout the paper we use the following numerical values:

[5] $\hat{\mathcal{Q}}$ stands for the electric charge operator and it is not to be confused with the quark Q.

$$M = 5\ \text{TeV},\ \mu_{\rm L} = 500\ \text{GeV},\ \mu_{\rm R} = 1.7\ \text{TeV},$$
$$N_L = 0.996,\ N_R = 0.946. \qquad (10)$$

3. The effective Lagrangian of the $K \to \pi\nu\bar{\nu}$ decay. In the SM both the c- and the t-quarks contribute to the penguin diagrams. The contribution of each of the up-type quarks is proportional to $m_i^2 \cdot V_{is}V_{id}^*$. Using the data from [6], we find out that $m_t^2/m_c^2 \approx 2{\cdot}10^4$, $|V_{ts}V_{td}^*|/|V_{cs}V_{cd}^*| \approx 1.2{\cdot}10^{-3}$, but the contribution of the c-quark is numerically enhanced as compared to that of the t-quark and in general we have to take into account both of them. Nevertheless, for the reasons discussed in the following section in the case of the decays $K_L \to \pi^0\nu\bar{\nu}$, $B \to \pi\nu\bar{\nu}$ and $B \to K\nu\bar{\nu}$ the t-quark dominates.

The expression for the $\mathcal{L}_{\rm eff}(s \to d\nu\bar{\nu})$ in the SM was obtained in [7] (also [8]):

$$\mathcal{L}_{\rm eff}^{SM}(s \to d\nu\bar{\nu}) = \mathcal{L}_1(m_t) + \mathcal{L}_1'(m_c, m_l);$$
$$\mathcal{L}_1(m_t) = \frac{G_F^2 m_W^2}{4\pi^2}\ \bar{d}_L\gamma^\mu s_L \sum_{l=e,\mu,\tau} \bar{\nu}_L^{(l)}\gamma_\mu\nu_L^{(l)} \times$$
$$\times V_{td}^* V_{ts}\xi_t\ F(\xi_t)\ \eta_X; \qquad (11)$$
$$\mathcal{L}_1'(m_c, m_l) = \frac{G_F^2 m_W^2}{4\pi^2}\ \bar{d}_L\gamma^\mu s_L \times$$
$$\times \sum_{l=e,\mu,\tau} \bar{\nu}_L^{(l)}\gamma_\mu\nu_L^{(l)} V_{cd}^* V_{cs}\ X^l(m_l, m_c),$$

here $\xi_t \equiv m_t^2/m_W^2$, m_t is the mass of the top quark.

$$F(\xi) \equiv \left[\frac{\xi+2}{\xi-1} + \frac{3\xi-6}{(\xi-1)^2}\ \ln\xi\right], \qquad (12)$$

numerically equals $F(\xi_t) = 2.744$. The sum is over the neutrino flavors $l = e, \mu, \tau$ and $X^l(m_l, m_c)$ accounts for the charm contribution. The factor η_X summarizes the QCD corrections, $\eta_X = 0.995 \approx 1$ [8].

When calculating the diagrams (Fig.1) we will neglect the masses and momenta of the external quarks compared to the momenta and masses of the particles inside the loop. When the external momenta are neglected, the diagrams (c) and (d) yield the same result.

Our model with a heavy singlet quark Q modifies the result of the SM in two ways: firstly, we have to take into account the modification of the coupling of t-quark with Z-boson (9) and with W- and G-bosons (7), secondly, we have to take into account the new particle Q.

The charged current interactions of the Q and t have the form described in (7). The NC in our model (9) consist of four terms:

1) the current proportional to the charge of the particles $\hat{\mathcal{Q}}$;

2) the left t-quark current with the coupling $\frac{1}{2}N_L^2$;

3) the left Q-quark current with the coupling $\frac{1}{2}\frac{\mu_L^2}{m_Q^2}N_R^2$;

4) the left FCNC (QtZ) with the coupling $-\frac{1}{2}\frac{\mu_L}{m_Q}N_R N_L$.

The first term – proportional to $\hat{\mathcal{Q}}$ (the operator of the electric charge) – does not contribute to the penguin amplitude. The crucial observation is that we are dealing here with a conserved vector current. Since all the momenta of the external particles (s, d, Z) are set to zero, we are dealing with four diagrams (Fig.1a–d), the sum of which is zero. The first two of them give the renormalization of the vertex $Z_1^{-1} - 1$ and the last two – the renormalization of the fermion wave-function $2 \cdot \frac{1}{2}Z_2 - 1$. The Ward identity in the abelian case leads to the equality $Z_1 = Z_2$. This implies that this part of the decay amplitude is equal to its tree-level value, i.e. zero.

The form of the CC interactions of the t-quark (7) indicates that all the SM diagrams have to be multiplied by N_L^2. The second term of the NC implies that the diagram (a) with the t-quark in Fig.1 has an extra N_L^2 factor amounting to a total factor N_L^4. In the same way, the diagrams with Q-quark will have the factor $\left(\frac{\mu_L}{m_Q}N_R\right)^2$, while for the diagram (a) with Q-quark the corresponding factor is $\left(\frac{\mu_L}{m_Q}N_R\right)^4$. The same diagram with Qt FCNC has the factor $\left(\frac{\mu_L}{m_Q}N_R N_L\right)^2$. This suggests the following way of taking into account the effects of New Physics:

1) to multiply the result of the SM by N_L^2 obtaining $\mathcal{L}_1(m_t) \cdot N_L^2$;

2) to calculate the diagrams with Q-quark which do not include the QtZ FCNC (they include the factor $\left(\frac{\mu_L}{m_Q}N_R\right)^2$) obtaining $\mathcal{L}_2(m_Q) \cdot \left(\frac{\mu_L}{m_Q}N_R\right)^2$;

3) in order to take into account the QtZ FCNC to calculate diagram in Fig.2 – $\mathcal{P}(m_i, m_j)$ (it coincides with the diagram a) in Fig.1 when m_i and m_j are equal to m_t and m_Q).

The expression for the effective Lagrangian $\mathcal{L}_{\rm eff}^{NP}(s \to d\nu\bar{\nu})$ which takes into account the contribution of the New Physics obtained in this way is the following:

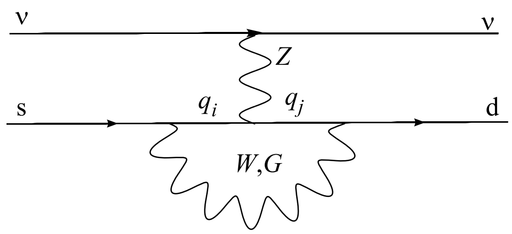

Fig.2. The diagram that defines the expression $\mathcal{P}(m_i, m_j)$. The up type quarks q_i and q_j have masses m_i and m_j respectively

$$\mathcal{L}_{\text{eff}}^{NP}(s \to d\nu\bar{\nu}) =$$
$$= \mathcal{L}_1(m_t)\ N_L^2 + \mathcal{L}_2(m_Q)\left(\frac{\mu_L}{m_Q}N_R\right)^2 +$$
$$+ \mathcal{P}(m_t, m_t)\ (N_L^4 - N_L^2) +$$
$$+ \mathcal{P}(m_Q, m_Q)\left[\left(\frac{\mu_L}{m_Q}N_R\right)^4 - \left(\frac{\mu_L}{m_Q}N_R\right)^2\right] +$$
$$+ 2\mathcal{P}(m_t, m_Q)\left(\frac{\mu_L}{m_Q}N_R N_L\right)^2 = \mathcal{L}_1(m_t)\ N_L^2 +$$
$$+ \mathcal{L}_2(m_Q)\left(\frac{\mu_L}{m_Q}N_R\right)^2 + \left(\frac{\mu_L}{m_Q}N_R N_L\right)^2 \times$$
$$\times \left(2\mathcal{P}(m_t, m_Q) - \mathcal{P}(m_t, m_t) - \mathcal{P}(m_Q, m_Q)\right). \quad (13)$$

Here we have used the identity $N_L^2 + \left(\frac{\mu_L}{m_Q}N_R\right)^2 = 1$ that can be easily established (5).

Let us investigate the contribution of the Q-quark. Since $m_Q^2/m_W^2 \approx 4\cdot 10^3$, the $m_Q^2 \gg m_W^2$ limit is applicable while calculating the diagrams with the Q-quark. In this limit we can neglect the W-boson exchanges in the diagrams; because of the heaviness of the Q the leading contribution will come from the interaction with the unphysical higgses G (the interaction with them is m_Q/m_W times stronger than the interaction with the gauge bosons W (7)). It is also important to note that taking only the G exchanges into account we will have to neglect the fifth diagram in Fig.1 (i.e. the lepton box diagram), because its contribution is proportional to the small ratio m_l^2/η^2, where m_l is the charged lepton mass.

Thus, we come to the diagrams shown in Fig.3.

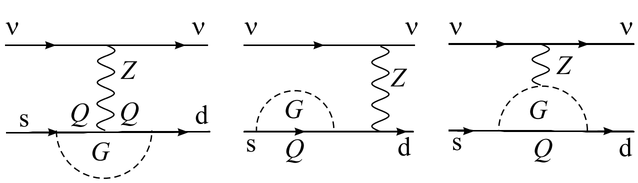

Fig.3. Diagrams contributing to the $s \to d\nu\nu$ transition with the Q-quark in the $m_Q^2 \gg m_W^2$ limit

The diagrams (a)–(c) in Fig.3 correspond to the $\mathcal{L}_2(m_Q)$ part of the interaction of the Q with the Z-boson and yield the result:

$$\mathcal{L}_2(m_Q) = \frac{G_F^2}{4\pi^2}\ \bar{d}_L\gamma^\mu s_L\ \sum_l \bar{\nu}_L^{(l)}\gamma_\mu\nu_L^{(l)} V_{td}^* V_{ts} m_Q^2. \quad (14)$$

The expression $2\mathcal{P}(m_t, m_Q) - \mathcal{P}(m_t, m_t) - \mathcal{P}(m_Q, m_Q)$ equals:

$$2\mathcal{P}(m_t, m_Q) - \mathcal{P}(m_t, m_t) - \mathcal{P}(m_Q, m_Q) =$$
$$= Z_\mu \frac{G_F^2 m_W^2}{4\pi^2}\ \bar{d}_L\gamma^\mu s_L V_{td}^* V_{ts}\ \sum_l \bar{\nu}_L^{(l)}\gamma_\mu\nu_L^{(l)} \times$$
$$\times \left[\left(2(\xi_t - 1)\ln\xi_Q - \frac{2\xi_t^2}{\xi_t - 1}\ln\xi_t\right) - \right.$$
$$\left. - \left(\xi_t - \frac{2}{\xi_t - 1}\ln\xi_t\right) - \xi_Q\right]. \quad (15)$$

The resulting expression for the effective Lagrangian is:

$$\mathcal{L}_{\text{eff}}^{NP}(s \to d\nu\bar{\nu}) =$$
$$= \frac{G_F^2}{4\pi^2}\ \bar{d}_L\gamma^\mu s_L \sum_l \bar{\nu}_L^{(l)}\gamma_\mu\nu_L^{(l)} V_{td}^* V_{ts} m_t^2 \times$$
$$\times \left[F(\xi_t)N_L^4 + \frac{\mu_L^4}{m_t^2 m_Q^2}N_R^4 + \right.$$
$$\left. + \left(\frac{\xi_t - 1}{\xi_t}\ln\frac{m_Q^2}{m_t^2} - G(\xi_t)\right)\cdot 2\frac{\mu_L^2}{m_Q^2}\ N_R^2 N_L^2\right] =$$
$$= \mathcal{L}_1(m_t) \times \left[N_L^4 + \frac{\mu_L^4}{m_t^2 m_Q^2}\cdot N_R^4\cdot F(\xi_t)^{-1} + \right.$$
$$\left. + 2\frac{\mu_L^2}{m_Q^2}\left(\frac{\xi_t - 1}{\xi_t}\ln\frac{m_Q^2}{m_t^2} - G(\xi_t)\right)\cdot N_R^2 N_L^2 \cdot F(\xi_t)^{-1}\right]. \quad (16)$$

Here the function $G(\xi)$ is defined as follows:

$$G(\xi) = \frac{\xi + 1}{\xi(\xi - 1)}\ln\xi \quad (17)$$

and numerically $G(\xi_t) = 0.513$. It is important to note that in the final expression the term proportional to $\left(\frac{\mu_L}{m_Q}N_R N_L\right)^2 \cdot m_W^2\xi_Q$ arising from the last term in (15) almost completely cancels the anomalously big contribution proportional to $\left(\frac{\mu_L}{m_Q}N_R\right)^2 \cdot m_Q^2$ from (14), giving the resulting term $\left(\frac{\mu_L}{m_Q}N_R\right)^4 \cdot m_Q^2 = \frac{\mu_L^4}{m_t^2 m_Q^2}N_R^4 \cdot m_t^2$ in (16).

4. Comparison with experimental data: prospects. The quantity δ_p that describes the deviation of the $\mathcal{L}_{\text{eff}}^{NP}(s \to d\nu\bar{\nu})$ from the $\mathcal{L}_{\text{eff}}^{SM}(s \to d\nu\bar{\nu})$ (i.e.

$\mathcal{L}_{\rm eff}^{NP}(s \to d\nu\bar{\nu}) = (1+\delta_p)\cdot\mathcal{L}_{\rm eff}^{SM}(s \to d\nu\bar{\nu}))$ equals (11), (16):

$$\delta_p = -\delta_c + N_L^4 - 1 + \frac{\mu_L^4}{m_t^2 m_Q^2}\cdot N_R^4 \cdot F(\xi_t)^{-1} + \\ + 2\frac{\mu_L^2}{m_Q^2}\left(\frac{\xi_t - 1}{\xi_t}\ln\frac{m_Q^2}{m_t^2} - G(\xi_t)\right)\cdot N_R^2 N_L^2 \cdot F(\xi_t)^{-1}. \quad (18)$$

Here δ_c accounts for the c-quark contribution (11). Substituting the numerical values of the parameters of New Physics from (10) and the numerical value $F(\xi_t) = 2.744$ (12) and neglecting for the moment δ_c we obtain:

$$\delta_p = 0.04. \quad (19)$$

The correction δ_p describes the deviation of the t-quark dominated penguin amplitudes of the decays $K_L \to \pi^0\nu\bar{\nu}$, $B \to \pi\nu\bar{\nu}$ and $B \to K\nu\bar{\nu}$ from the SM. The decay $K_L \to \pi^0\nu\bar{\nu}$ proceeds only via the CP violation mechanism and includes the CKM factor $\mathrm{Im}\,(V_{is}V_{id}^*)$ which is approximately the same for both the c- and t-quarks inside the loop. Thus the c-quark contribution is damped by a factor $m_c^2/m_t^2 \approx 10^{-4}$ as compared to t-quark. In the case of the B-meson decays the CKM factors are of the same order for the c- and t-quarks inside the loop and the charm contribution is damped again by the factor m_c^2/m_t^2. For these decays δ_c is negligible.

The case of the $K^+ \to \pi^+\nu\bar{\nu}$ is more involved. When calculating its probability we have to take into account the contribution of the lepton box diagram with the c-quark inside the loop. This effect turns out to be quite significant [8], $\tilde{\delta}_c \approx 0.3 \times \tilde{\delta}_p$. As a result, for the $K^+ \to \pi^+\nu\bar{\nu}$ decay we obtain $\tilde{\delta}_p \approx 0.03$ and the corresponding branching ratio increases by approximately $\tilde{\delta}_{br} = 2\tilde{\delta}_p$ as compared to the SM:

$$\tilde{\delta}_{br} = 0.06. \quad (20)$$

The increase of the branching ratios of the $K_L \to \to \pi^0\nu\bar{\nu}$, $B \to \pi\nu\bar{\nu}$ and $B \to K\nu\bar{\nu}$ decays will be

$$\delta_{br} = 2\delta_p = 0.08. \quad (21)$$

Thus, for 5 TeV mass of the isosinglet quark Q the branching ratios of the Z-penguin originated decays $K_L \to \pi^0\nu\bar{\nu}$, $B \to \pi\nu\bar{\nu}$ and $B \to K\nu\bar{\nu}$ are 8% larger than in the SM.

To obtain the constraints on the Q mass[6)] from the experimental data we will rewrite δ_p (18) in a more convenient form:

[6)]From now on, the parameter M will no longer be a constant. To keep μ_L equal to its value in equation (10) and, which is more important, m_t equal to its experimental value we adjust μ_R according to (6).

$$\delta_p \approx \frac{\mu_L^2}{M^2}\left[-2\left(1-\frac{m_t^2}{\mu_L^2}\right)^2 + \frac{\mu_L^2}{m_t^2 F(\xi_t)} + \\ + 2\frac{(\xi_t-1)/\xi_t\cdot\ln(M^2/m_t^2) - G(\xi_t)}{F(\xi_t)}\right] \equiv \\ \equiv \frac{\mu_L^2}{M^2}\, f(\mu_L,\ \ln M). \quad (22)$$

Since the dependence of $f(\mu_L,\ \ln M)$ on M is only logarithmical and thus very weak, we will use its value at $M = 5$ TeV:

$$f(\mu_L,\ \ln 5\ \text{TeV}) \approx 5.0. \quad (23)$$

In this way we obtain a simple expression for the branching ratios in our model:

$$Br(K,B\to\pi,K\nu\bar{\nu})_{NP} = Br(K,B\to\pi,K\nu\bar{\nu})_{SM}\times \\ \times\left(1 + 2\ f(\mu_L,\ln M)\ \frac{\mu_L^2}{M^2}\right), \quad (24)$$

which can be straightforwardly compared with the experimental data (for the $K^+ \to \pi^+\nu\bar{\nu}$ decay one should substitute 0.8 instead of unity in paren theses in (24)).

The present-day results for the branching ratios are the following [6]:

$$Br(K^+ \to \pi^+\nu\bar{\nu}) = (1.5^{+1.3}_{-0.9})\cdot 10^{-10}, \\ Br(K_L \to \pi^0\nu\bar{\nu}) < 5.9\cdot 10^{-7}, \\ Br(B_u \to \pi^+\nu\bar{\nu}) < 1.0\cdot 10^{-4}, \\ Br(B_u \to K^+\nu\bar{\nu}) < 5.2\cdot 10^{-5}. \quad (25)$$

New data were obtained in August 2007 [9]:

$$Br(B_u \to K^+\nu\bar{\nu}) < 1.4\cdot 10^{-5}, \quad (26)$$

and in December 2007 [10]:

$$Br(K_L \to \pi^0\nu\bar{\nu}) < 6.7\cdot 10^{-8}. \quad (27)$$

The current state of the experiment does not allow us to make any conclusions concerning the existence of the New Physics. The future plans are the following:

- the measurement of $Br(K^+ \to \pi^+\nu\bar{\nu})$ at the CERN SPS NA62 experiment with $\approx 10\%$ accuracy, the data taking is planned for 2009–2010 [11];
- the measurement of $Br(B_u \to K^+\nu\bar{\nu})$ at the Super B Factory experiment with the accuracy $\leq 20\%$ by 2014–2015 [12];
- the measurement of $Br(K^+ \to \pi^+\nu\bar{\nu})$ at the J-PARC experiment with the accuracy $\leq 20\%$ after 2012–2013 [13];

- the measurement of $Br(K_L \to \pi^0 \nu\bar{\nu})$ at the J-PARC experiment with the accuracy $\leq 10\%$ after 2010 [14].

5. Conclusions. The branching ratios $Br(K \to \pi\nu\bar{\nu})$, $Br(B \to \pi\nu\bar{\nu})$ and $Br(B \to K\nu\bar{\nu})$ get in our model up to 10% corrections for the mass of the isosinglet quark $M = 5$ TeV. The uncertainties coming from the poor knowledge of the CKM matrix elements should considerably improve in the near future, while the experimental data on the probabilities of the rare decays analyzed in the paper should also appear in due time [11–14]. The proper accuracy of the data will allow to discover New Physics or to establish the lower bounds on the mass of the heavy quark Q.

Soon after our paper was published in the e-print arXiv the paper [15] appeared, in which the model with a heavy singlet up-type quark was studied. In [15] our formulae were confirmed and in addition the $B_{d,s} \to \mu^+\mu^-$ decays were considered.

We thank A.E. Bondar for emphasizing the importance of the consideration of $K \to \pi\nu\bar{\nu}$ decays in our model and for the helpful information on the experimental prospects concerning $B \to K\nu\bar{\nu}$ decays. Our research is supported by the grants # NSh-4568.2008.2, # RFBR-08-02-00494 and partially by Rosatom. The research of P.N.K. is also supported by the grants # NSh-8004.2006.2, # RFBR-07-02-00878 and by the Dmitry Zimin Dynasty Foundation. The research of M.I.V. is also supported by the grant # RFBR-07-02-00021.

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