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On mass corrections to the decay $P \to l^+l^-$

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We use the Mellin-Barnes representation in order to improve the theoretical estimate of mass corrections to the width of light pseudoscalar meson decay into a lepton pair, $P \to l^+l^-$. The full resummation of the terms $\ln(m_l^2/\Lambda^2)\left(m_l^2/\Lambda^2\right)^n$ and $\left(m_l^2/\Lambda^2\right)^n$ to the decay amplitude is performed, where m_l is the lepton mass and $\Lambda \approx m_\rho$ is the characteristic scale of the $P \to \gamma^*\gamma^*$ form factor. The total effect of mass corrections for the e^+e^- channel is negligible and for the $\mu^+\mu^-$ channel its order is of a few per cent.

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I. Introduction. Rare decays of mesons serve as a low-energy test of the Standard Model. Accuracy of experiments has increased significantly in recent years. Theoretically, one of the main limitations comes from the large distance contributions of the strong sector of the Standard Model where the perturbative QCD theory does not work. However, in some important cases the result can be essentially improved by relating these poorly known contributions to other experimentally known processes. The famous example is the calculation of the hadronic vacuum polarization contribution to the anomalous magnetic moment of muon $(g-2)_\mu$ where the data of the processes $e^+e^- \to$ *hadrons* and $\tau \to$ *hadrons* are essential to reduce the uncertainty (see for review [1 – 4]). It turns out that this is also the case for the rare decays of light pseudoscalar mesons into a lepton pair [5]. Interest in these processes revived after new precise measurement of the decay $\pi_0 \to e^+e^-$ by the KTeV collaboration [6]. The Standard Model prediction [5] disagrees with the KTeV measurement by 3.3σ, with the theoretical accuracy exceeding the experimental one.

In the lowest order of QED perturbation theory, the photonless decay of the neutral meson, $P(q) \to l^-(p_-) + l^+(p_+)$, $q^2 = M^2$, $p_\pm^2 = m^2$, (M meson mass, m lepton mass) is described by the one-loop Feynman amplitude (Fig.1) corresponding to the conversion of the meson through two virtual photons into a lepton pair. The normalized branching ratio is given by [7 – 9]

$$R_0(P \to l^+l^-) = \frac{B_0\left(P \to l^+l^-\right)}{B\left(P \to \gamma\gamma\right)} = = 2\beta\left(M^2\right)\left(\frac{\alpha m}{\pi M}\right)^2 |\mathcal{A}\left(M^2\right)|^2, \quad (1)$$

where $\beta\left(q^2\right) = \sqrt{1 - 4m_l^2/q^2}$ and the reduced amplitude is

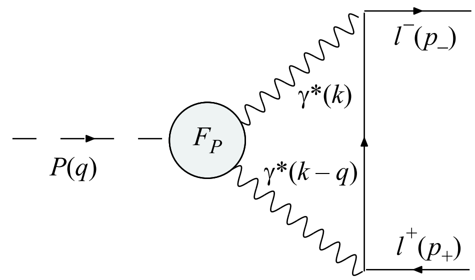

Fig.1. Triangle diagram for the $P \to l^+l^-$ process with the pseudoscalar meson form factor $P \to \gamma^*\gamma^*$ in the vertex

$$\mathcal{A}\left(q^2\right) = \frac{2}{q^2} \times \times \int \frac{d^4k}{i\pi^2} \frac{(qk)^2 - q^2k^2}{(k^2+i\epsilon)\left[(q-k)^2+i\epsilon\right]\left[(p_- - k)^2 - m^2 + i\epsilon\right]} \times \times F_{P\gamma^*\gamma^*}(-k^2, -(q-k)^2), \quad (2)$$

with the transition form factor $F_{P\gamma^*\gamma^*}(-k^2, -q^2)$ being normalized as $F_{P\gamma^*\gamma^*}(0,0) = 1$.

The imaginary part of the on-shell amplitude $\mathcal{A}\left(q^2 = M^2\right)$ comes from the contribution of real photons in the intermediate state and can be found in a model independent way [8]

$$\mathrm{Im}\mathcal{A}(M^2) = \frac{\pi}{2\beta\left(M^2\right)} \ln\left(y\left(M^2\right)\right), \; y\left(q^2\right) = \frac{1-\beta\left(q^2\right)}{1+\beta\left(q^2\right)}. \quad (3)$$

A once-subtracted dispersion relation for the amplitude in Eq. (2) is written as[1)] [10]

$$\mathcal{A}\left(q^2\right) = \mathcal{A}\left(q^2 = 0\right) + \frac{q^2}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\,\mathcal{A}\left(s\right)}{s\left(s-q^2\right)}. \quad (4)$$

[1)] In this derivation it is tacitly assumed that the imaginary part of the off-shell amplitude $\mathcal{A}(q^2)$ has the same form as in (3) with M^2 substituted by q^2.

The second term in Eq. (4) takes into account strong q^2 dependence of the amplitude around the point $q^2 = 0$ occurring due to the branch cut coming from the two-photon intermediate state. Integrating Eq. (4) for $q^2 \geq 4m_e^2$ one arrives at [11 – 13]

$$\mathrm{Re}\,\mathcal{A}\left(q^2\right) = \mathcal{A}\left(q^2=0\right) + + \frac{1}{\beta\left(q^2\right)}\left[\frac{1}{4}\ln^2\left(y\left(q^2\right)\right) + \frac{\pi^2}{12} + \mathrm{Li}_2\left(-y\left(q^2\right)\right)\right], \quad (5)$$

where $\mathrm{Li}_2\left(z\right) = -\int_0^z \left(dt/t\right)\ln\left(1-t\right)$ is the dilogarithm function.

Usually, the subtraction constant in (5), containing the nontrivial dynamics of the process, is calculated within different models describing the form factor $F_{P\gamma^*\gamma^*}(k^2,q^2)$ (e.g. [5, 10, 12]). However, it has recently been shown in [5] that this constant may be expressed in terms of the inverse moment of the transition form factor given in symmetric kinematics of spacelike photons, $G(t) \equiv F_{P\gamma^*\gamma^*}\left(t,t\right)$,

$$\mathcal{A}^0\left(q^2=0\right) = 3\ln\left(\frac{m}{\mu}\right) - -\frac{3}{2}\left[\int_0^{\mu^2} dt\frac{G(t)-1}{t} + \int_{\mu^2}^{\infty} dt\frac{G\left(t\right)}{t}\right] - \frac{5}{4}. \quad (6)$$

Here, μ is an arbitrary (factorization) scale. One should note that the logarithmic dependence of the first term on μ is compensated by the scale dependence of the integrals in the brackets.

The accuracy of these calculations is determined by omitted small power corrections of the order $O(\frac{m^2}{\Lambda^2}, \frac{m^2}{\Lambda^2}\ln\frac{m^2}{\Lambda^2})$ and $O(\frac{m^2}{M^2}, \frac{m^2}{M^2}\ln\frac{M^2}{m^2})$ in the r.h.s. (5), where $\Lambda \lesssim M_\rho$ is the characteristic scale of the form factor $G(t)$. The aim of this work is to improve the result (6) for the amplitude $\mathcal{A}\left(q^2=0\right)$ of the $P \to l^+l^-$ decay by taking into account all order mass corrections $\sim \frac{m^2}{\Lambda^2}, \frac{m^2}{\Lambda^2}\ln\frac{m^2}{\Lambda^2}$.

II. Mellin-Barnes integral representation. We evaluate the amplitude $\mathcal{A}\left(q^2\right)$ following the way used in [14]. Let us transform the integral in (2) to the Euclidean metric $k_0 \to ik_4$. The corresponding integral is convergent due to decreasing of $F_{P\gamma^*\gamma^*}(k^2,(q-k)^2)$ in the Euclidean region. Then use the double Mellin transformation for the meson form factor

$$F_{P\gamma^*\gamma^*}(k^2,(q-k)^2) = =\frac{1}{(2\pi i)^2}\int_{\sigma+iR^2} dz\Phi\left(z_1,z_2\right)\left(\frac{\Lambda^2}{k^2}\right)^{z_1}\left(\frac{\Lambda^2}{\left(k-q\right)^2}\right)^{z_2}, \quad (7)$$

where Λ is the characteristic scale for the form factor, $dz = dz_1dz_2$, the vector $\sigma = (\sigma_1,\sigma_2) \in \mathbb{R}^2$, and $\Phi\left(z_1,z_2\right)$ is the inverse Mellin transform of the form factor

$$\Phi\left(z_1,z_2\right) = \int_0^\infty dt_1\int_0^\infty dt_2 t_1^{z_1-1}t_2^{z_2-1}F_{P\gamma^*\gamma^*}\left(t_1,t_2\right) \quad (8)$$

which has singularities at $\mathrm{Re}(z_i) = 0,-1,\ldots$ Introducing Feynman parameters in the standard way, the denominator part of the integrand in (2) can be written as

$$\frac{1}{(k^2)^{z_1+1}\left[(k-q)^2\right]^{z_2+1}\left[(p_- - k)^2+m^2\right]} = = \frac{\Gamma\left(3+z_1+z_2\right)}{\Gamma\left(z_1+1\right)\Gamma\left(z_2+1\right)}\int\prod_{i=1}^3 d\alpha_i\delta\left(1-\sum_{i=1}^3\alpha_i\right)\times \times\frac{\alpha_1^{z_1}\alpha_2^{z_2}}{\left[k^2+D\right]^{3+z_1+z_2}}, \quad (9)$$

where $D = \alpha_3^2m^2 - \alpha_1\alpha_2q^2$. Then the k–loop integral reduces to

$$\frac{2}{q^2}\int\frac{d^4k}{\pi^2}\frac{(qk)^2 - q^2k^2}{\left[k^2+D\right]^{3+z_1+z_2}} = \frac{\Gamma\left(z_1+z_2\right)}{\Gamma\left(3+z_1+z_2\right)D^{z_1+z_2}}\times \times\left[-3+2\frac{\alpha_3^2}{D}\left(m^2-\frac{1}{4}q^2\right)\left(z_1+z_2\right)\right].$$

Combining all factors we get

$$\mathcal{A}\left(q^2\right) = \frac{1}{\left(2\pi i\right)^2}\times \times\int_{\sigma+iR^2} dz\frac{\Phi\left(z_1,z_2\right)\left(\Lambda^2\right)^{z_1+z_2}\Gamma\left(z_1+z_2\right)}{\Gamma\left(z_1+1\right)\Gamma\left(z_2+1\right)}\times \times\int\prod_{i=1}^3 d\alpha_i\delta\left(1-\sum_{i=1}^3\alpha_i\right)\frac{\alpha_1^{z_1}\alpha_2^{z_2}}{\left(\alpha_3^2m^2-\alpha_1\alpha_2q^2\right)^{z_1+z_2}}\times \times\left[-3+2\frac{\alpha_3^2\left(m^2-\frac{1}{4}q^2\right)}{\alpha_3^2m^2-\alpha_1\alpha_2q^2}\left(z_1+z_2\right)\right]. \quad (10)$$

In the general case, to step further we need to perform the third Mellin transformation for denominators containing α_i [14]. Then, considering the process $P \to l^+l^-$ with mass hierarchy $m \ll M \leq \Lambda \sim m_\rho$ we expand the integral obtained over the ratios of the lepton and meson masses to the characteristic scale of the meson form factor Λ by closing the Mellin contours in an appropriate manner. However, in the present study we are interested in the amplitude at $q^2 = 0$. In this limit, the Feynman parameter integrals in (10) can be carried out in terms of Γ-functions, and we obtain the following Mellin-Barnes representation for $\mathcal{A}\left(q^2=0\right)$

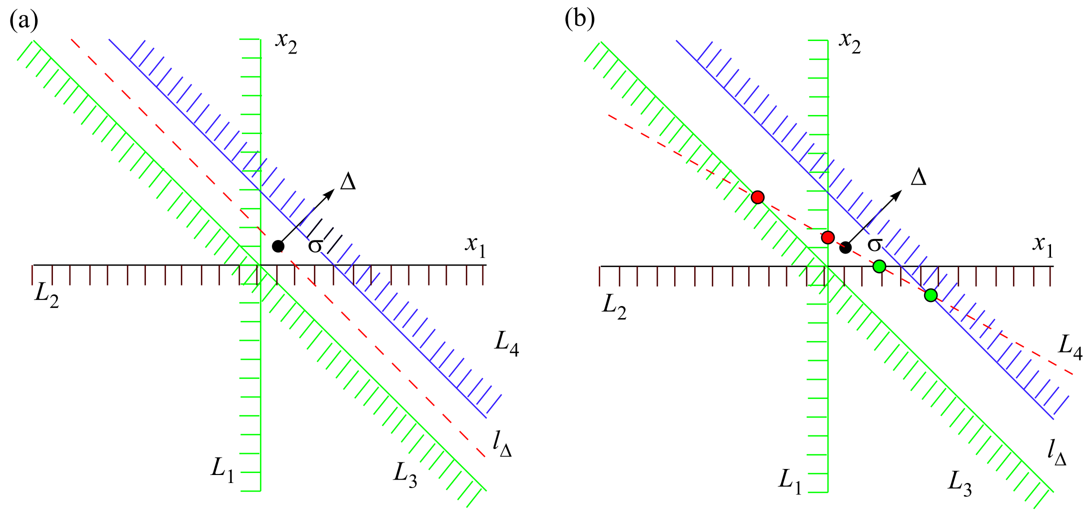

Fig.2. (a) Singularities of the integrand in (11). The semi-planes where the arguments of Γ functions produce singularities are depicted by lines $Ł_i$ with shadowed bands. The point σ characterizng the integration countor is from the triangle $\{x_1 > 0, x_2 > 0, x_2 + x_1 < \frac{1}{2}\}$. (b) Rotation of l_Δ allow ones to read off the degeneration

$$\mathcal{A}(0) = \frac{1}{(2\pi i)^2} \int_{\sigma + i\mathbb{R}^2} dz \left(\xi^2\right)^{-z_1 - z_2} \times \\ \times \frac{\Gamma(z_1)\Gamma(z_2)\Gamma(z_{12})\Gamma(1 - 2z_{12})}{\Gamma(3 - z_{12})} \times \\ \times \left[\frac{(-3 + 2z_{12})\Phi(z_1, z_2)}{\Gamma(z_1)\Gamma(z_2)}\right], \tag{11}$$

with σ in the triangle $\{x_1 > 0, x_2 > 0, x_2 + x_1 < \frac{1}{2}\}$ chosen so that the integration path $\sigma + i\mathbb{R}^2$ does not intersect the Γ-function singularities given by the polar complex lines (see illustration in Fig.2a)

$$L_1 : \{z_1 = -\nu\}, \quad L_2 : \{z_2 = -\nu\}, \\ L_3 : \{z_1 + z_2 = -\nu\}, \quad L_4 : \{1 - 2(z_1 + z_2) = -\nu\}, \\ \nu = 0, 1, 2, \ldots \tag{12}$$

In (11) we introduce the notation $\xi^2 = m^2/\Lambda^2$, $z_{12} = z_1 + z_2$ and combine the regular expression in the squared brackets.

In further analysis of the integral (11) we use the technique suggested in [15][2)]. Following this line let us associate the vectors in 2-dimensional space with the coefficients of the Γ-function arguments in the numerator and denominator of the integrand in (11) $a_1 = (1, 0), a_2 = (0, 1)$, $a_3 = (1, 1)$, $a_4 = (-2, -2)$, $c_1 = (-1, -1)$. Next, define the vector

$$\Delta = \sum a_i - \sum c_j = (1, 1) \tag{13}$$

and draw through σ the straight line $l_\Delta = \{x \in \mathbb{R}^2 : (\Delta, x) = (\Delta, \sigma)\}$ with the normal vector Δ. The scalar product is introduced as $(x, y) = x_1 y_1 + x_2 y_2$. The point σ divides l_Δ into two rays l^+ and l^- so that the pair of directions l^+ and Δ yields the same orientation of $\mathbb{R}^2$ as the pair of coordinate axes x_1 and x_2. The half-plane $\pi_\Delta = \{x \in \mathbb{R}^2 : (\Delta, x) < (\Delta, \sigma)\}$ with boundary l_Δ and the integration half-space $\Pi_\Delta = \pi_\Delta + i\mathbb{R}^2 = \{z \in C^2 : \mathrm{Re}\,(\Delta, z) < \mathrm{Re}\,(\Delta, \sigma)\}$ characterize the domain in the space of integration variables z in which the integrand is a decreasing function.

Now we need to define the divisors given by the condition

$$D_1 = \cup \left\{L_j : L_j \cap l^- = 0\right\}, D_2 = \cup \left\{L_j : L_j \cap l^+ = 0\right\}. \tag{14}$$

The theorem [15] states that in the nondegenerated case ($\Delta \neq 0$ and all $a_i \nparallel \Delta$) the integral like (11) is given by the sum

$$\mathcal{A}(0) = \sum_{z_r \in \Pi_\Delta} \operatorname*{res}_{z_r} [\mathrm{Integ\,rand}\mathcal{A}(0)], \tag{15}$$

where $\operatorname*{res}_{z_r} [\mathrm{Integ\,rand}\mathcal{A}(0)]$ is the residue with respect to the system of divisors $\{D_1, D_2\}$.

The integral (11) corresponds to the degenerate case since a_3 and a_4 are parallel to Δ. In this case, one has

[2)] When our study was completed we became aware of the results of the work [16] where similar technique of the two-dimensional counter integrals is used.

$L_2 \in D_1$ and $L_1 \in D_2$. However, L_3 and L_4 are parallel to l_Δ and cannot to be ascribed to any of divisors. To read off the degeneration we slightly rotate l_Δ with respect to the point σ in clock-wise or anti-clock-wise directions (Fig.2b). Now we have crossings of $L_{3,4}$ with the rotated line and are able to decide to which divisor they should be related. One has two cases

$$\begin{aligned} &1)\ D_1 = \{L_2, L_4\}, D_2 = \{L_1, L_3\}, \\ &2)\ D_1 = \{L_2, L_3\}, D_2 = \{L_1, L_4\}. \end{aligned} \tag{16}$$

We are interested in the intersection of divisors, i.e., intersections of all $L_i^{(1)} \in D_1$ and all $L_i^{(2)} \in D_2$ so that these intersections $L_i^{(1)} \cap L_j^{(2)}$ belong to the half-space Π_Δ.

Another important property is the intersection rank, the number of lines that meet at each point. If only two lines $L^{(1)}$ and $L^{(2)}$ meet at each point $z_r \in D_1 \cap D_2 \cap \Pi_\Delta$ (rank 1), then one has only simple poles. If the intersection rank is more than one, than one may either apply the theory of multiple residues or introduce small $\varepsilon-$parameters in the arguments of $\Gamma-$functions in such way that all poles become simple ones (like in the dimensional regularization method).

We prefer here the second approach, namely, L_1, L_2, L_3 in (12) meet at the same points $(-\alpha, -\beta)$ where $\alpha, \beta = 0, 1, ...$ are positive integers. In order to get rid of this kind of degeneracy, we add a small parameter ε to the argument $\Gamma(z_2) \to \Gamma(z_2 + \varepsilon)$ in (11). Now we ready to analyze the poles and their residues. Consider first the case 1) in (16). We have two sets of intersections in $D_1 \cap D_2 \cap \pi_\Delta$

$$L_2 \cap L_1, L_2 \cap L_3, \tag{17}$$

which may be parametrized as

$$L_2 \cap L_1 : \left\{ \begin{array}{c} z_2 + \varepsilon = -\alpha, \\ z_1 = -\beta, \end{array} \right. ,$$

$$L_2 \cap L_3 : \left\{ \begin{array}{c} z_2 + \varepsilon = -\alpha, \\ z_1 = \alpha - \beta + \varepsilon, \end{array} \right. . \tag{18}$$

Calculating residues we get two contributions to the integral

$$\mathcal{A}(0) = \mathcal{A}_a(0) + \mathcal{A}_b(0), \tag{19}$$

$$\begin{aligned} \mathcal{A}_a(0) = \sum_{\alpha,\beta=0}^{\infty} \frac{(-1)^{\alpha+\beta}}{\alpha!\beta!} \left(\xi^2\right)^{\alpha+\beta+\varepsilon} \times \\ \times \frac{\Gamma(-\alpha-\beta-\varepsilon)\Gamma(1+2(\alpha+\beta+\varepsilon))}{\Gamma(3+\alpha+\beta+\varepsilon)} \times \\ \times (-3-2(\alpha+\beta+\varepsilon)) \left[\frac{\Phi(-\alpha,-\beta)}{\Gamma(-\alpha)\Gamma(-\beta)}\right], \end{aligned} \tag{20}$$

$$\begin{aligned} \mathcal{A}_b(0) = \sum_{\alpha,\beta=0}^{\infty} \frac{(-1)^{\alpha+\beta}}{\alpha!\beta!} \left(\xi^2\right)^{\beta} \times \\ \times \frac{\Gamma(1+2\beta)}{\Gamma(3+\beta)} (-3-2\beta) \left[\frac{\Phi(\alpha-\beta+\varepsilon,-\alpha)}{\Gamma(-\alpha)}\right]. \end{aligned} \tag{21}$$

The second case in (16) is reduced to the first one because one has single parameter (ξ) integral. If one would be interested in the expansion in inverse powers of ξ one needs to consider intersections $L_4 \cap L_1, L_4 \cap L_3$ instead of (17).

By using the representation (8) one may show that the corresponding residues are

$$\begin{aligned} \frac{\Phi(-\alpha,-\beta)}{\Gamma(-\alpha)\Gamma(-\beta)} &= (-1)^{\alpha+\beta} F^{(\alpha,\beta)}_{P\gamma^*\gamma^*}(0,0), \\ \frac{\Phi(z,-\alpha)}{\Gamma(-\alpha)} &= (-1)^{\alpha} \int_0^\infty dt t^{z-1} F^{(0,\alpha)}_{P\gamma^*\gamma^*}(t,0), \end{aligned} \tag{22}$$

where $F^{(\alpha,\beta)}_{P\gamma^*\gamma^*}(0,0)$ denotes the derivatives of an order of α and β in the corresponding arguments of the form factor. After these substitutions one sum in (20) and (21) may be performed with the result

$$\begin{aligned} \mathcal{A}_a(0) = -\sum_{n=0}^{\infty} \frac{G^{(n)}(0)}{n!} \left(\xi^2\right)^{n+\varepsilon} \times \\ \times \frac{\Gamma(-n-\varepsilon)\Gamma(1+2(n+\varepsilon))}{\Gamma(3+n+\varepsilon)} (3+2(n+\varepsilon)), \end{aligned}$$

$$\begin{aligned} \mathcal{A}_b(0) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left(\xi^2\right)^n \times \\ \times \frac{\Gamma(1+2n)\Gamma(-\varepsilon)}{\Gamma(3+n)\Gamma(1-\varepsilon+n)} (3+2n) \int_0^\infty dt t^\varepsilon G^{(n+1)}(t), \end{aligned}$$

where we again use $G(t) \equiv F_{P\gamma^*\gamma^*}(t,t)$. Now we expand in ε and take the limit $\varepsilon \to 0$ with the total result

$$\begin{aligned} \mathcal{A}(0) = \sum_{n=0}^{\infty} \frac{\left(-\xi^2\right)^n}{n!} \frac{\Gamma(1+2n)}{\Gamma(1+n)\Gamma(3+n)} \times \\ \times \left\{ G^{(n)}(0) \left[2 + (3+2n)\left(\ln 4\xi^2 - \gamma - \right.\right.\right. \\ \left.\left. - \psi(n+1) + \psi\left(n+\frac{1}{2}\right) - \psi(n+3)\right)\right] + \\ \left. + (3+2n) \int_0^\infty dt G^{(n+1)}(t) \ln t \right\}. \end{aligned} \tag{23}$$

Note that the ε^{-1} poles contained in the intermediate steps of calculations are canceled in the final expression. To the lowest orders in ξ^2 expansion one gets

$$\mathcal{A}^{(0)}(0) = \frac{1}{2} \left[3 \ln \xi^2 - \frac{5}{2} + 3 \int_0^\infty dt G^{(1)}(t) \ln t\right], \tag{24}$$

$$\mathcal{A}^{(1)}(0) = -\xi^2 \frac{1}{3}\left[G^{(1)}(0)\left(5\ln\xi^2 + \frac{13}{6}\right) + \right.$$
$$\left. +5\int_0^\infty dt G^{(2)}(t)\ln t\right]. \qquad (25)$$

The leading order expression (24) is in accordance with the result (6) obtained in [5]. In the general case it is convenient to convert the sum in (23) into the integral form

$$\mathcal{A}(0) = \frac{4}{3\pi}\int_0^1 dy\sqrt{\frac{1-y}{y}} \times$$
$$\times \left\{[(\ln 4\xi^2 - \gamma)(2+y) + 2(1-y)]G(-4y\xi^2) + \right.$$
$$+ (2+y)\int_0^\infty dt\left[\ln t G^{(1)}(t - 4y\xi^2) + \right.$$
$$\left.\left. + \frac{e^{-\frac{1}{2}t} - e^{-3t} - e^{-t}}{e^{-t}-1} G(-4ye^{-t}\xi^2) - \frac{e^{-t}}{t}G(-4y\xi^2)\right]\right\}. \quad (26)$$

Finally, let us consider the form factor we are interested in from a physical point of view

$$G(t) = \frac{1}{1+t}.$$

For this form factor from (24)–(26) one gets the coefficient of logarithmic term as

$$\mathcal{A}(0) = \frac{1}{12\xi^4} \times$$
$$\times \left[1 + 6\xi^2 - \sqrt{1-4\xi^2}\left(1+8\xi^2\right)\right]\ln\xi^2 + O\left(\xi^0\right), \quad (27)$$

or the first terms of expansion

$$\mathcal{A}(0) = \frac{3}{2}\left(1 + \frac{10}{9}\xi^2 + O\left(\xi^4\right)\right)\ln\xi^2 -$$
$$- \frac{5}{4}\left(1 + \frac{86}{45}\xi^2 + O\left(\xi^4\right)\right). \qquad (28)$$

Thus, one can see that in the realistic case for muon, $\xi^2 = m_\mu^2/\Lambda^2 \sim m_\mu^2/m_\rho^2 \approx 0.02$ the corrections to the leading order coefficients are of an order of 1% and for an electron pair they are negligible.

III. Conclusions. The aim of this paper is to clarify the situation with rare decays of pseudoscalar mesons to a lepton pair. The situation became more pressing after recent KTeV E799-II experiment at Fermilab in which the pion decay into an electron-positron pair was measured using the $K_L \to 3\pi$ process as a source of tagged neutral pions [6]. The branching ratio was determined to be equal to

$$B^{\mathrm{KTeV}}_{\mathrm{no-rad}}\left(\pi^0 \to e^+e^-\right) = (7.49 \pm 0.29 \pm 0.25)\cdot 10^{-8}. \qquad (29)$$

The standard model prediction based on the use of CELLO and CLEO data on the transition form factor $\pi \to \gamma\gamma^*$ [19, 20] gives [5]

$$B^{\mathrm{Theor}}\left(\pi^0 \to e^+e^-\right) = (6.2 \pm 0.1)\cdot 10^{-8}, \qquad (30)$$

which is 3.3σ below the KTeV result (29). Therefore, it is extremely important to trace possible sources of the discrepancy between the experiment and theory. There are a number of possibilities: (1) problems with (statistic) experiment procession, (2) inclusion of QED radiation corrections by KTeV is wrong, (3) unaccounted mass corrections are important, and (4) effects of new physics. At the moment the last possibilities was reinvestigated. In [17], the contribution of QED radiative corrections to the $\pi^0 \to e^+e^-$ decay, which must be taken into account when comparing the theoretical prediction (30) with the experimental result, (29) was reviced. Comparing with earlier calculations [18], the main progress is in the detailed consideration of the $\gamma^*\gamma^* \to e^+e^-$ subprocess and revealing of dynamics of large and small distances. Occasionally, this number agrees well with the earlier prediction based on calculations [18] and, thus, the KTeV analysis of radiative corrections is confirmed. In the present paper, we show that the mass corrections are under control and do not resolve the problem. So our main conclusion is that the inclusion of radiative and mass corrections is unable to reduce the discrepancy between the theoretical prediction for the decay rate (30) and experimental result (29). The effects of new physics were considered in [21] where the excess of experimental data over theory is explained by the contribution of low mass ($\sim$ 10 MeV) vector bosons appearing in some models of dark matter. Further independent experiments at KLOE, NA48, WASAatCOSY, BES III and other facilities will be crucial for resolution of the problem.

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