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# The critical behavior of the mixed ferrimagnetic ternary alloy on the Bethe lattice

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The critical behavior of the mixed Ising model of the type $AB_pC_{1-p}$ ternary alloy consisting of spins $\sigma$=1/2, $S$=1, and $m$=3/2 is investigated on the Bethe lattice by using the exact recursion relations. The exact expressions for the magnetizations and magnetic susceptibilities are found, and thermal behaviors of the magnetizations and susceptibilities are studied. We also investigate the behavior of magnetizations and susceptibilities as a function of the crystal-field interaction or single-ion anisotropy. We construct the phase diagram and find that the system always undergoes a second-order phase transition for the coordination number $q \leqslant 3$ and a second- and first-order phase transitions for $q > 3$; hence the system gives a tricritical point. The system also exhibits the reentrant behaviors.

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In recent years, a considerable research interest has been devoted to study magnetic properties of the $AB_pC_{1-p}$ ternary alloy consisting of three kinds of magnetic ion A, B, and C with different Ising spins. For example, the magnetic properties of a mixed ferro-ferrimagnetic ternary alloy of the type $AB_pC_{1-p}$ consisting of three different metal ions with Ising spins 1/2, 1, 3/2 [1, 2] and [3]; with Ising spins 3/2, 1, 5/2 [4 – 6] and [7]; with Ising spins 3/2, 2, 5/2 [8] and [9]; and with Ising spins 1/2, 1, 5/2 [10] have been investigated. These studies were done within the effective-field theory (EFT) [1, 3] and [6], the Monte Carlo (MC) simulation [7] and [10], the mean-field theory (MFT) based on Bogoliubov inequality for the Gibbs free energy [2, 4] and [8] and the MFT [5] and [9]. The ground-state phase diagrams of the $AB_pC_{1-p}$ ternary alloy consisting of Ising 3/2, 1, and 5/2 in the presence of a single-ion anisotropy were also constructed [11]. Since these studies have been done by approximate methods, due to the reason that the exact solutions for the realistic lattices are generally unavailable, one should study the exact solution at least for special cases, such as on the Bethe lattice. It is worth mentioning that Bethe or Bethe like lattice calculations are more reliable than conventional mean-field calculations; hence the behaviors of the systems on Bethe or Bethe like lattices are qualitatively correct even when conventional a mean-field theories fail [12].

Therefore, our aim in this work is to investigate the critical behavior of the mixed Ising model of the type $AB_pC_{1-p}$ ternary alloy consisting of spins $\sigma$=1/2, $S$=1, and m=3/2 on the Bethe lattice by using the exact recursion equations. Especially, we obtain the exact expressions for the magnetizations and magnetic susceptibilities and study their thermal behaviors, and as well as their behavior of magnetizations and susceptibilities as a function of the crystal-field interaction. We also present the exact phase diagrams. We should also mention that the mixed Ising models of binary alloys on the Bethe lattice consisting of spins $\sigma$=1/2, $S$=1 [13]; $\sigma$=1/2, $S$=3/2 [14]; $\sigma$=3/2, $S$=5/2 [15]; $\sigma$=2, $S$=5/2 [16], etc have been studied extensively, but the mixed Ising models of ternary alloy, best of our knowledge, has not been investigated.

The mixed Ising model of the type $AB_pC_{1-p}$ ternary alloy consisting of spins $\sigma$=1/2, $S$=1, and $m$=3/2 on the Bethe lattice is defined by the Hamiltonian

$$H = -J\Big(\sum_{\langle ij\rangle} \sigma_i s_j + \sum_{\langle jk\rangle} S_j m_k + \sum_{\langle ik\rangle} \sigma_i m_k\Big) + + \Delta \sum_{\langle i\rangle} (S_i^2 + m_i^2), \quad (1)$$

where each $\sigma_i$, $S_j$ and $m_k$ located at the sites $i$, $j$ and $k$ are a spin-1/2 with the discrete values $\pm 1/2$, a spin-1 with the three discrete values +1 and 0, and a spin-3/2 with the four discrete values $\pm 3/2$ and $\pm 1/2$, respectively. $J$ is the bilinear exchange interaction parameter and $\Delta$ is the crystal-field interaction or single-ion

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anisotropy. In this work, we assume the simple case that the bilinear exchange interactions are the same for all three interactions between Ising spins. The first three sums run over all nearest-neighbor (NN) pairs, the last sum runs over all the spin-1 and spin-3/2 sites. In this ternary alloy case, we arrange the Bethe lattice such that the central spin is spin-1/2, $\sigma_0$; the second generation is spin-1, $S_0$; the third generation is a spin-3/2, $m_0$; the fourth generation is again spin-1/2, $\sigma_1$; the fifth generation is again spin-1, $S_1$; the sixth generation is again spin-3/2, $m_1$; so on to infinity, as seen in Fig.1. We

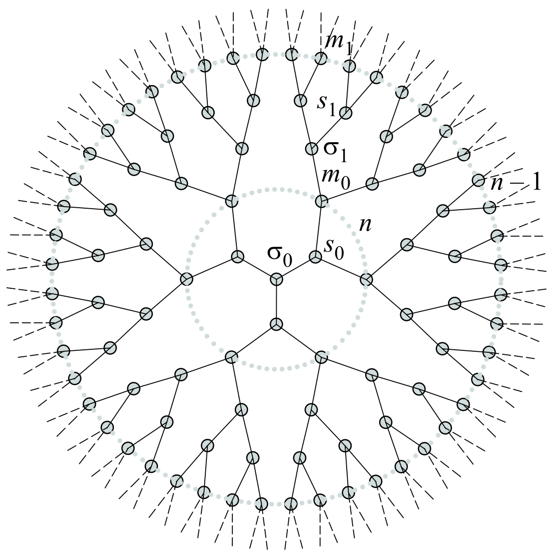


Fig.1. Bethe lattice, or regular tree, of coordination number 3 for the ternary alloy consisting of spins $\sigma$=1/2, $S$=1, and $m$=3/2. The Bethe lattice is arranged such that the central spin is spin-1/2, $\sigma_0$; the second generation is spin-1, $S_0$; the third generation is a spin-3/2, $m_0$; the fourth generation is again spin-1/2, $\sigma_1$; the fifth generation is again spin-1, $S_1$; the sixth generation is again spin-3/2, $m_1$; so on to infinity

should also mention that the central spin has $q$ NN, $q$ is the coordination number, which forms the second generation spins. Each spin in the second generation is joined to $(q-1)$ NN's. Therefore, in total the second generation has $q(q-1)$ NN spins that forms the third generation spins and so on to infinity. The Bethe lattice consideration for any model is based on the exact recursion relations.

The partition function is the main ingredient to obtain a formulation in term of the recursion relations. Using the definition, the partition function of the system is given by

$$Z = \sum_{\{\sigma,S,m\}} \exp[-\beta H] =$$
$$= \sum_{\{\sigma,S,m\}} \exp[\beta J(\sum_{\langle ij\rangle} \sigma_i s_j + \sum_{\langle jk\rangle} S_j m_k + \sum_{\langle ik\rangle} \sigma_i m_k)\times$$
$$\Delta \sum_{\langle i\rangle} (S_i^2 + m_i^2)], \qquad (2)$$

where the summation is over all spin sets, $\{\sigma, S, m\}$. If we cut the Bethe lattice in some central point deep inside with a spin $\sigma_0$, that is a spin-1/2, it splits up to $q$ identical branches in which each of these branches is a rooted tree at the central spin $\sigma_0$. Therefore the partition function for the central site on the Bethe lattice can be written as

$$Z = \sum_{\{\sigma_0\}} [g_n(\sigma_0|S_0, m_0)]^q, \qquad (3)$$

where $\sigma_0$ is the central spin value on the lattice, and $g_n(\sigma_0|S_0, m_0)$ is the partition function of an individual branch and the suffix $n$ represents the fact that the sub-tree has $n$-shells, i.e., $n$ steps from the root to the boundary sites. Each branch can be cut on the site $S_0$ which is the nearest to the central point. Therefore, $g_n(\sigma_0|S_0, m_0)$ is written in terms of the summation over spin set $\{S_0\}$ as

$$g_n(\sigma_0|S_0, m_0) = \sum_{\{S_0\}} \exp[\beta(J\sigma_0 S_0 - \Delta S_0^2)] \times$$
$$\times [g_n(S_0|m_0, \sigma_1)]^{q-1}, \qquad (4)$$

Advancing along the any branch, we get a site that next-nearest to the central spin, hence $g_n(S_0|m_0, \sigma_1)$ is expressed as follows

$$g_n(S_0|m_0, \sigma_1) = \sum_{\{m_0\}} \exp[\beta(JS_0 m_0 - \Delta m_0^2)] \times$$
$$\times [g_n(m_0|\sigma_1, S_1)]^{q-1}, \qquad (5)$$

Finally, we will continue until we reach a recursive point and terminate the calculation of $g_n$ function. Therefore, $g_n(m_0|\sigma_1, S_1)$ can be written as

$$g_n(m_0|\sigma_1, S_1) = \sum_{\{\sigma_1\}} \exp[\beta(J m_0 \sigma_1)][g_{n-1}(\sigma_1|S_1, m_1)]^{q-1}, \qquad (6)$$

where $g_{n-1}(\sigma_1|S_1, m_1)$ is the partition function of the next shell, i.e., the $(n-1)$ shell, seen in Fig. 1. Therefore, in this way, the expression for $g_n(\sigma_0|S_0, m_0)$ in the n-shell is obtained in terms of $g_{n-1}(\sigma_1|S_1, m_1)$ in the $(n-1)$ shell. As seen from Fig. 1, all the shells are identical.

In order to find recursion relations, we introduce the following variables as a ratio of the $g_n$ function for the spin-1/2

$$N_n = \frac{g_n(1/2)}{g_n(-1/2)}, \tag{7}$$

for the spin-1,

$$K_n = \frac{g_n(+1)}{g_n(0)}, \qquad L_n = \frac{g_n(-1)}{g_n(0)} \tag{8}$$

and for the spin-3/2

$$X_n = \frac{g_n(3/2)}{g_n(-1/2)}, \; Y_n = \frac{g_n(-3/2)}{g_n(-1/2)}, \; Z_n = \frac{g_n(1/2)}{g_n(-1/2)}. \tag{9}$$

We are now ready to obtain the explicit expression of recursion relations using these ratios and Eqs. (4), (5) and (6), and find as

$$N_n = \frac{e^{\beta(J/2-\Delta)}K_n^{q-1} + e^{\beta(-J/2-\Delta)}L_n^{q-1} + 1}{e^{\beta(-J/2-\Delta)}K_n^{q-1} + e^{\beta(J/2-\Delta)}L_n^{q-1} + 1}, \tag{10a}$$

$$K_n = a/b, \tag{10b}$$

$$L_n = c/b, \tag{10c}$$

$$X_n = \frac{e^{\beta(3J/4-\Delta/4)}N_{n-1}^{q-1} + e^{\beta(-3J/4-\Delta/4)}}{e^{\beta(-J/4-\Delta/4)}N_{n-1}^{q-1} + e^{\beta(J/4-\Delta/4)}}, \tag{10d}$$

$$Y_n = \frac{e^{\beta(-3J/4-\Delta/4)}N_{n-1}^{q-1} + e^{\beta(3J/4-\Delta/4)}}{e^{\beta(-J/4-\Delta/4)}N_{n-1}^{q-1} + e^{\beta(J/4-\Delta/4)}}, \tag{10e}$$

$$Z_n = \frac{e^{\beta(J/4-\Delta/4)}N_{n-1}^{q-1} + e^{\beta(-J/4-\Delta/4)}}{e^{\beta(-J/4-\Delta/4)}N_{n-1}^{q-1} + e^{\beta(J/4-\Delta/4)}}, \tag{10f}$$

where
$a = e^{\beta(3J/2-9\Delta/4)}X_n^{q-1} + e^{\beta(-3J/2-9\Delta/4)}Y_n^{q-1} + e^{\beta(J/2-\Delta/4)}Z_n^{q-1} + e^{\beta(-J/2-\Delta/4)}$,
$b = e^{\beta(-9\Delta/4)}X_n^{q-1} + e^{\beta(-9\Delta/4)}Y_n^{q-1} + e^{\beta(-\Delta/4)}Z_n^{q-1} + e^{\beta(-\Delta/4)}$,
$c = e^{\beta(-3J/2-9\Delta/4)}X_n^{q-1} + e^{\beta(3J/2-9\Delta/4)}Y_n^{q-1} + e^{\beta(-J/2-\Delta/4)}Z_n^{q-1} + e^{\beta(J/2-\Delta/4)}$.

It should be mentioned that these recursion relations have no direct physical sense, but all thermodynamic functions can be obtained from these relations and they reflect the critical behavior of the system. Thus, we can say that in the thermodynamic limit ($n \longrightarrow \infty$), the above-mentioned variables determine the status of the system.

Now we can obtain the sublattice magnetizations for spins 1/2, 1 and 3/2. If the central-spin is chosen to be a spin-1/2 ($\sigma_0$), the first sublattice magnetization or the dipole moment of the system is defined by

$$M_{1/2} = Z^{-1}\sum_{\{\sigma_0\}} \sigma_0 [g_n(\sigma_0|S_0, m_0)]^q, \tag{11}$$

which is easily expressed in terms of the recursion relations, namely Eqs. (10a)-(10f), and calculated as

$$M_{1/2} = \frac{1}{2}\frac{N_n^q - 1}{N_n^q + 1}. \tag{12}$$

The next step is to calculate the sublattice magnetization ($M_1$) for a spin-1. We can again start from the first shell on the Bethe lattice, and carry out the whole calculation by choosing the spin-1 ($S_0$) as the central-spin instead of spin-1/2 ($\sigma_0$), since all sites with the same kinds of spins are equivalent deep inside the Bethe lattice. We perform similar calculation, and rearranging it for the spin-1 as we have done for $\sigma$=1/2, we find

$$M_1 = \frac{e^{-\beta\Delta}K_n^q - e^{-\beta\Delta}L_n^q}{e^{-\beta\Delta}K_n^q + e^{-\beta\Delta}L_n^q + 1}. \tag{13}$$

Finally, if the central-spin is chosen a spin-3/2 ($m_0$), we obtain the third sublattice magnetization or the dipole moment with the similar calculation as before

$$M_{3/2} = d/e, \tag{14}$$

where
$d = 3e^{-9\beta\Delta/4}X_n^q - 3e^{-9\beta\Delta/4}Y_n^q + e^{-\beta\Delta/4}Z_n^q - e^{-\beta\Delta/4}$,
$e = 2e^{-9\beta\Delta/4}X_n^q + 2e^{-9\beta\Delta/4}Y_n^q + 2e^{-\beta\Delta/4}Z_n^q + 2e^{-\beta\Delta/4}$.
Thus, we found sublattice magnetizations in terms of the recursion relations. All information about the behavior of the system can be obtained from these equations. For example, we can easily calculate the magnetic susceptibilities in which give some important physical properties of the system for spin-1/2, 1 and 3/2 by using the Eqs. (12)–(14). The definition of susceptibility is given by

$$\chi_\alpha = \lim_{h\to 0}\frac{\partial M_\alpha}{\partial h}, \tag{15}$$

where $\alpha$=1/2, 1 and 3/2 which are the spin variables. Thus, the magnetic susceptibility on the sublattice with a spin-1/2 is obtained as:

$$\chi_{1/2} = \frac{N_n^{q-1}(\beta N_n + qN_n')}{(1+N_n^q)^2}, \tag{16}$$

where $N_n'$ is the partial derivative of the recursion relation ($N_n$), given in Eq. (10a). The magnetic susceptibility for a spin-1 is found as

$$\chi_1 = \frac{f}{K_n L_n (1 + e^{-\beta\Delta} K_n^q + e^{-\beta\Delta} L_n^q)^2}, \tag{17}$$

where
$f = e^{-2\beta\Delta} q K_n^q L_n^q (e^{\beta\Delta} + 2L_n^q) K'^q_n + e^{-\beta\Delta} K_n L_n^q (\beta L_n - - qL'_n) + e^{-2\Delta} K_n^{q+1} (\beta e^{\beta\Delta} L_n + 4\beta L_n^{q+1} - 2qL_n^q L'_n)$,
and finally for a spin-3/2 is calculated as

$$\chi_{3/2} = g/h, \tag{18}$$

where
$g = qX_n^q Y_n Z_n (3Y_n^q + e^{2\beta\Delta}(2 + Z_n^q)) X'_n + X_n^{q+1}(9\beta Y_n^{q+1} Z_n - 3qY_n^q Z_n Y'_n + e^{2\beta\Delta} Y_n (4\beta Z_n + \beta Z_n^{q+1} + qZ_n^q Z'_n)) + e^{2\beta\Delta} X_n (-qY_n^q Z_n (1 + 2Z_n^q) Y'_n + e^{2\beta\Delta} Y_n Z_n^q (\beta Z_n + qZ'_n) + Y_n^{q+1}(\beta Z_n + 4\beta Z_n^q Z'_n))$,

$$h = X_n Y_n Z_n (e^{2\beta\Delta} + X_n^q + Y_n^q + e^{2\beta\Delta} Z_n^q)^2.$$

$K'_n$, $L'_n$ are the partial derivative of the recursion relations of $K_n$ and $L_n$, respectively; $X'_n$, $Y'_n$ and $Z'_n$ are the partial derivative of the recursion relations of $X_n$, $Y_n$ and $Z_n$, respectively.

Now, we should investigate the thermal behavior of the magnetizations and susceptibilities. Thermal behaviors of magnetizations are obtained solving Eqs. (12), (13) and (14) by using iterations in the recursion relations. Moreover, temperature dependence of the susceptibilities is examined by solving Eqs. (16), (17) and (18) using iterations in the recursion relations. We present two representative graphs to see the thermal behaviors of the magnetizations and susceptibilities, seen in Figs.2 and 3. Figs.2 are obtained for $\Delta$=2 with $q$=6 and show that the system undergoes only a second-order phase transition, because as seen in Fig.2a the magnetizations decrease to zero continuously as the temperature increases and a second-order phase transition occurs at $T_C$=2.0670. Fig. 2(b) illustrates the thermal behaviors of the susceptibilities $\chi_\alpha$ ($\alpha$=1/2, 1, 3/2) and it is seen that when the temperature approaches to $T_C$ the susceptibilities increases very rapidly and goes to infinity at $T_C$=2.0670. This has also been tested of our calculations, because we found exactly same $T_C$ for both calculations. Figs.3 are calculated for $\Delta$=3.3 with $q$=6 and illustrate that the system undergoes two successive phase transitions. The first one is a first-order phase transition due to the discontinuity occurs at $T_t$=0.1725, which is the first-order phase transition temperature and the second one is a second-order phase transition because magnetizations decrease to zero continuously, seen in Fig.3a. Fig.3b displays the behavior of $\chi_\alpha$ as a function of temperature and seen that $\chi_\alpha$ remains finite and exhibits the discontinuous characteristic behavior at $T_t$=0.1725, and then as the temperature increases, increases very rapidly and goes to infinity at $T_C$=0.8790, as seen in Fig.3b. If one compares Fig.3a with Fig.3b, one can see that $T_t$ and $T_C$ are found exactly same for both calculations.

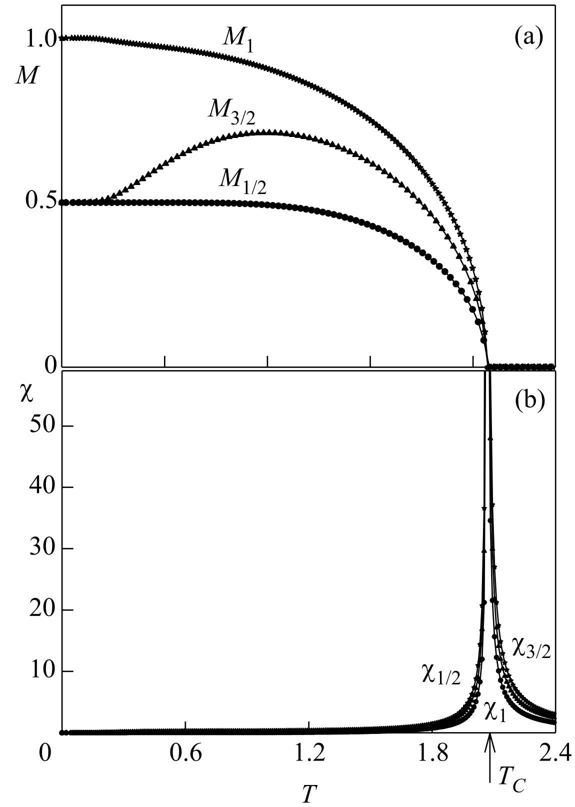


Fig.2. (a) Temperature dependence of magnetizations for $q = 6$ and $\Delta = 2$, and $T_C$ is the second-order phase transition or critical temperature. (b) Temperature dependence of magnetic susceptibility for $q = 6$ and $\Delta = 2$

We should also mention that in order to find the $T_t$ temperature precisely, one needs obtain the free energy expression. Using the Eq. (3) and Eqs. (7)–(10) together with the definition of the free energy $F$ ($F = -kT \ln Z$), the exact free energy expression in terms of the recursion relations is obtained as

$$F = -\frac{1}{\beta}\left[\frac{q(q-1)^2}{1-(q-1)^3}\ln(A) + \frac{q(q-1)}{1-(q-1)^3}\ln(B) + + \frac{q}{1-(q-1)^3}\ln(C) + \ln(D)\right], \tag{19}$$

where
$A = e^{\beta(-J/4+h/2)} N_{n-1}^{q-1} + e^{\beta(J/4-h/2)}$,
$B = e^{\beta(-9\Delta/4+3h/2)} X_n^{q-1} + e^{\beta(-9\Delta/4-3h/2)} Y_n^{q-1} + + e^{\beta(-\Delta/4+h/2)} Z_n^{q-1} + e^{\beta(-\Delta/4-h/2)}$,
$C = e^{\beta(-J/2-\Delta+h)} K_n^{q-1} + e^{\beta(J/2-\Delta-h)} L_n^{q-1} + 1$,
$D = e^{\beta(-\Delta/4+h/2)} N_n^q + 2e^{\beta(-\Delta/4-h/2)}$.

The first-order phase transition temperatures are determined by matching the values of the two branches

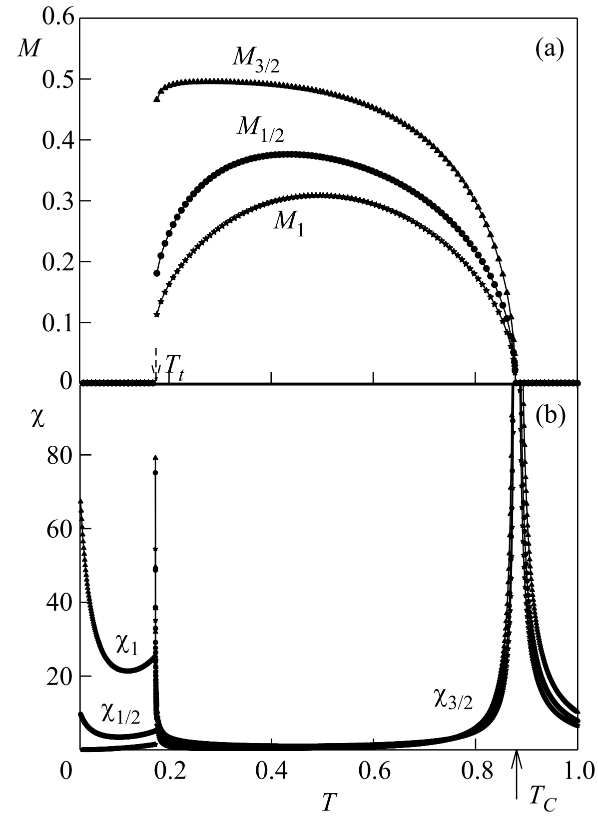


Fig.3. (a) Same as Fig.2a, but $\Delta = 3.3$. $T_t$ is the first-order phase transition temperature. (b) Same as Fig.2b, but $\Delta = 3.3$

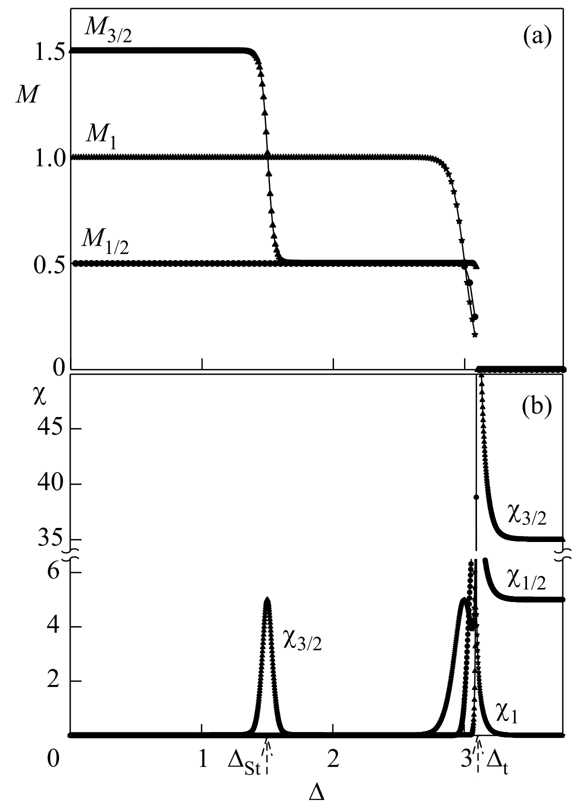


Fig.4. The behavior of magnetizations and magnetic susceptibilities as a function of the crystal-field interaction or single-ion anisotropy for $q = 6$ and $T = 0.05$. $\Delta_t$ and $\Delta_{St}$ are the first-order phase transition and the separation point, respectively. (a) For magnetizations; (b) for susceptibilities

of the free energy followed while increasing and decreasing the temperature. The temperature at which the free energy values are equal to each other is the first-order phase transition temperature.

We also investigate the behaviors of magnetizations and magnetic susceptibilities as a function of the crystal-field interaction or single-ion anisotropy and present one representative graph, seen in Fig.4. Figs.4 represent one of the more interesting behavior of the system as follows. At very low temperature $\sigma$=1/2, $S$=1 and $m$=3/2; hence we have the ferrimagnetic phase of (1/2, 1, 3/2), as the crystal-field increases the ferrimagnetic phase of (1/2, 1, 3/2) smoothly passes to the ferrimagnetic phase of (1/2, 1, 1/2) without any phase transition, seen in Fig.4a. Thus, a smooth passing, with no phase transition singularity, can occur between these two ferrimagnetic phase. We have found a similar behavior to the one seen in the phase diagrams of the two-sublattice spin-3/2 Ising model by using the renormalization group calculation [17]. The peak or maximum for the susceptibility of spin-3/2, marked with $\Delta_{St}$, is the separation point of these two ordered phases, seen in Fig.4b. Moreover, as the crystal-field increases the magnetizations decrease to zero discontinuously; hence a first-order phase transition occurs at $\Delta_t$, and the transition is from the ferrimagnetic phase of (1/2, 1, 1/2) to the paramagnetic phase. In addition, the susceptibilities remain finite and exhibit the discontinuous characteristic behavior at $\Delta_t$, seen in Fig.4b.

We can now present the phase diagrams of the system. The calculated phase diagrams are presented in the $(\Delta, T)$ plane for different values of the coordination number $q$. In these phase diagrams, the solid and dashed lines represent the second- and first-order phase transition lines, respectively and the tricritical point is denoted by a filled circle, and $Z$ is the zero-temperature critical point. The dotted line is an ordered line smoothly mediating, with no phase transition, between the ferrimagnetic (1/2, 1, 3/2) and ferrimagnetic (1/2, 1, 1/2) phases. We have found two different topological types of phase diagrams, that only a second-order phase transition occurs for $q \leq 3$; and a second- and first-order phase transitions for $q > 3$ in the system. (i) For $q = 3$, the phase diagram is illustrated in Fig.5a. The system always undergoes a second-order

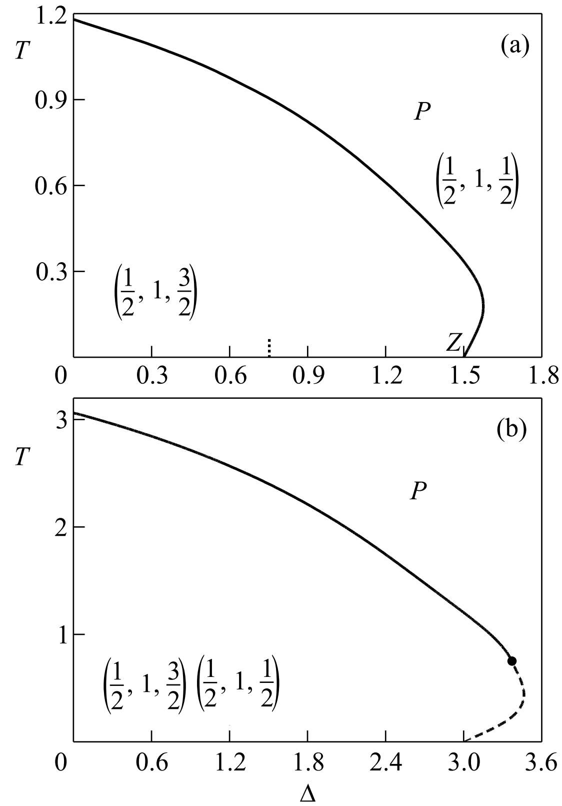


Fig.5. Phase diagrams in the $(\Delta, T)$ plane for the $AB_pC_{1-p}$ ternary alloy consisting of spins $\sigma$=1/2, $S$=1, and $m$=3/2 on the Bethe lattice. Dashed and solid lines indicate, first- and second-order phase transitions, respectively; the tricritical point is denoted by a filled circle and Z is the zero-temperature critical point. The dotted line is an ordered line smoothly mediating, with no phase transition, between the ferrimagnetic (1/2, 1, 3/2) and ferrimagnetic (1/2, 1, 1/2) phases. (a) $q$=3 and (b) $q$=6

phase transition. The system also exhibits a reentrant behavior, i.e., as the temperature is lowered, the system passes from the paramagnetic (P) (disorder phase) phase to the ferrimagnetic phase of (1/2, 1, 1/2), and back to the P phase again. (ii) For q=6, the phase diagram is illustrated in Fig.5b. The system undergoes a second-order phase transition for high values of $T$, and a first-order phase transition for low values of $T$; hence, the tricritical behavior appears in the phase diagram. A similar phase diagram has been obtained in a mixed ferro-ferrimagnetic ternary alloy of the type $AB_pC_{1-p}$ consisting of three different metal ions with Ising spins 1/2, 1, and 3/2 within the MFT based on Bogoliubov inequality for the Gibbs free energy [2] and with Ising spins 3/2, 2, and 5/2 by using the MFT [9]. Moreover, a similar phase diagram is also found in three sublattice (ternary) mixed-spin system, in consisting of spins 1/2, 1 and 3/2 [18].

In conclusion, we investigate the critical behavior of the mixed Ising model of the type $AB_pC_{1-p}$ ternary alloy consisting of spins $\sigma$=1/2, $S$=1, and $m$=3/2 on the Bethe lattice for different values of $q$ by using the exact recursion equations. We have found that the system always undergoes a second-order phase transition for $q \leq 3$; and a second- and first-order phase transitions for $q > 3$, hence the system exhibits tricritical point. The system also has the reentrant behavior. Finally, we should also mention that in this work we assume the bilinear exchange interactions are the same for the interactions between Ising spins 1/2, 1, and 3/2. Therefore, one should study this system by taking different values for bilinear exchange interactions; hence our next task will be to study this case.

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