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Bottomonium $\Upsilon(5S)$ decays into BB and $BB\pi$

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Two- and three-body decays of $\Upsilon(5S)$ into BB, BB^*, B^*B^*, B_sB_s, $B_sB_s^*$, $B_s^*B_s^*$ and $BB^*\pi, B^*B^*\pi$ are evaluated using the theory, developed earlier for dipion bottomonium transitions. The theory contains only two parameters, vertex masses M_{br} and M_ω, known from dipion spectra and width. Predicted values of $\Gamma_{tot}(5S)$ and six partial widths $\Gamma_k(5S), k = BB, BB^*, ...$ are in agreement with experiment. The decay widths $\Gamma_{5S}(\pi BB^*)$ and $\Gamma_{5S}(\pi B^*B^*)$ are also calculated and found to be of the order of 10 keV.

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1. Introduction. The experimental data [1] on dipionic and dikaonic transitions of $\Upsilon(5S)$, together with the earlier data on BB and total width [2] present an interesting challenge to theorists, asking for explanation of several unexpected features: 1) dipionic widths of $\Upsilon(5S)$ are almost 10^3 larger than of $\Upsilon(nS), n \leq 4$; 2) the BB, BB^* etc. partial widths of $\Upsilon(5S)$ are large, and $\Gamma_{\rm tot}(5S) \approx 110\,\mathrm{MeV}$ [2]; 3) a hierarchy exists in $\Gamma_k(5S), k = 1, 2, ..6$, e.g. $\Gamma_{BB} < \Gamma_{BB^*} < \Gamma_{B^*B^*}$.

In addition there is an open problem of finding the decay width into the $BB^*\pi, B^*B^*\pi$ channels, not yet known experimentally.

From theoretical point of view a calculation and comparison of two types of decays (6 channels for BB and 2 channels for $BB\pi$) is important, because it allows to estimate two independent types of light $q\bar{q}$ creation vertices: $L = M_i \int \bar{\psi}(x)\psi(x)d^4x, \;\; i = \omega, br$, where M_ω governs pair creation without pions and was found previously [3 – 6] (both experimentally and theoretically) to be of the order of $1\,\mathrm{GeV}$ and M_{br}, which is responsible for pair creation with additional pion emission, M_{br} being of the order of $f_\pi = 0.093\,\mathrm{GeV}$ [3 – 6]. It is clear, that M_ω enters in the 6 channels of the BB type and M_{br} enters in two $BB\pi$ channels, therefore the resulting widths would strongly differ from each other. It is purpose of the present paper to calculate all 8 widths and compare results with existing experimental data, answering in this way to the points 2) and 3) stated above.

In doing so we shall use the values of M_ω and M_{br} from the analysis of $\Upsilon(nS)$ decays with $n \leq 4$ and hence no new parameters will be involved.

The paper is organized as follows. In the next section the general equations from [3 – 6] are written for the widths $\Gamma_k(5S)$ and evaluated, using realistic one-channel wave-functions of $\Upsilon(nS)$, calculated earlier. In section 3 the πBB widths are written and calculated, and in the concluding section results are discussed and perspectives are outlined.

2. The BB decay channels of $\Upsilon(5S)$. We numerate 6 channels of BB type as follows; $k = 1, 2, ..6$ for $B\bar{B}$, $B\bar{B}^*$, $B^*\bar{B}^*$, $B_s\bar{B}_s$, $B_s\bar{B}_s^*$, $B_s^*\bar{B}_s^*$ respectively, and define the partial widths Γ_k, which according to general formulas (see e.g. Eq. (63) in [3], and Eq. (28) in [5])

$$\Gamma_k(\Upsilon(5S)) \equiv \Gamma_k = \left(\frac{M_\omega}{2\omega}\right)^2 \frac{p_k^3 M_k}{6\pi N_c}(Z_k)^2 |J_{5S}(p_k)|^2. \tag{1}$$

Here ω is the light quark average energy in B or B_s mesons, $\omega_B = 0.587\,\mathrm{GeV}$, $\omega_{B_s} = 0.639\,\mathrm{GeV}$ (see Appendix 1 of [3]), Z_k are spin-isospin multiplicity factors,

$$Z_1^2 = 2Z_4^2 = 1, \quad Z_2^2 = 2Z_5^2 = 4; \quad Z_3^2 = 2Z_6^2 = 7. \tag{2}$$

M_k is the doubled reduced mass in channel k, and p_k- the corresponding momentum. Finally, $J_{5S}(p)$ is the overlap matrix element of the wave functions Ψ_5 of $\Upsilon(5S)$ and ψ_B or B mesons,

$$\frac{p_i}{\omega} J_{5S}(p) = \int \frac{d^3q}{(2\pi)^3} \bar{y}_{123} \Psi_5^*(\mathbf{p}+\mathbf{q}) \Psi_B^2(\mathbf{q}), \tag{3}$$

where $\bar{y}_{123}$ is the P-wave vertex factor given in [3 – 6], $\bar{y}_{123} = q_i - \bar{c}p_i$, $\bar{c} \ll 1$. In [6] we have used for $\Psi_5(q)$ the realistic wave function for $\Upsilon(5S)$ obtained in [7] in the one-channel approximation, i.e. without influence of decay channels, and expanded in a series of oscillator functions for convenience. As a result $\Psi_5(q)$ contains 4 real zeros as a function of q^2, which yields $J_{5S}(p)$ strongly varying in region of physical values of $p_k = 0.48 \div 1.26\,\mathrm{GeV}$. This makes Γ_k to depend on the shape of $\Psi_5(q)$ and details of its approximation. At the same time one realizes, that the same decay interaction due to $M_\omega \bar{\psi}\psi$ strongly mixes the wave functions of n^3S_1 and n^3D_1 with different n, so that the $5S$ state gets (complex due to open channels) admixture of n' states with $n' \neq 5$. The preliminary analysis shows that this contribution fills the minima of wave function and overlap integral, yielding a smooth function $J_{5S}(p)$. Therefore in the Table 1 we display the "averaged" values $J^{(0)}_{5S}(p) = \exp(-p^2/\Delta_5)\bar{I}_{5S}$, with $I_{5S} = 1\,\mathrm{GeV}^{3/2}$, the latter figure approximating the average value of $I_{5s}(p)$ in the interval $0.5\,\mathrm{GeV} \leq p \leq 1.2\,\mathrm{GeV}$. For comparison also the actual values $J^{(1)}_{5S}(p_k)$ are given in the Table, obtained with the $5S$ wave function, calculated in [7] and approximated by $k_{\max} = 5$ oscillator functions. The widths Γ_k, calculated with $J^{(1)}_{5S}(p_k)$ using Eq.(1) are given in the last line of the Table.

Partial widths Γ_k and overlap integrals $J^{(1)}_{5S}(p_k)$ for six decay channels

k	$B\bar{B}$	$B\bar{B}^*$	$B^*\bar{B}^*$	$B_s\bar{B}_s$	$B_s\bar{B}^*_s$	$B^*_s\bar{B}^*_s$
p_k, GeV	1.26	1.16	1.05	0.84	0.68	0.48
M_k, GeV	5.28	5.30	5.32	5.37	5.39	5.41
Z_k	1	4	7	1/2	2	7/2
$\lvert\bar{J}^{(0)}_{5S}(p_k)\rvert$	0.27	0.33	0.40	0.56	0.68	0.82
$J^{(1)}_{5S}(p_k)$	−0.34	−0.44	−0.42	0.08	0.58	1.0
Γ_k, MeV	11	58	66	0.09	10	19

As shown in [5], for $\Upsilon(4S)$ state the similar computation yields $\Gamma_{\mathrm{tot}} \simeq (M_\omega/2\omega)^2 40\,\mathrm{MeV}$ and comparison with $\Gamma^{\exp}_{\mathrm{tot}} \approx 20\,\mathrm{MeV}$ leads to the estimate $(M_\omega/2\omega)^2 \approx$ $\approx 1/2$. As a result one obtains Γ_k for $k = 1, \ldots 6$ as shown in bottom line of Table 1 with $\Gamma_{\mathrm{tot}} \approx 150\,\mathrm{MeV}$.

We note that the small width Γ_4 into B_sB_s state is due to a nearby zero of $I_{5S}(p)$ and would be of the order of $\frac{1}{2}\Gamma_5 \approx 5\,\mathrm{MeV}$ in the next approximation.

3. The πBB decays of $\Upsilon(5S)$. The $\pi B\bar{B}^*$ and $\pi B^*\bar{B}^*$ widths can be obtained directly from the Eq. (71) of [3] (see also Eq.(3) from [4]),

$$\Gamma_{5S}(\pi B\bar{B}^*) = \frac{M^2_{br}(Z^*)^2}{N_c f^2_\pi} |J^{(1)}_{5BB^*}(\mathbf{p},\mathbf{k})|^2 \times$$
$$\times \frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\frac{2\pi\delta(E^{(0)}_5 - E(\mathbf{p}) - \omega_\pi(\mathbf{k}))}{2\omega_\pi}. \qquad (4)$$

Here $J^{(1)}_{5BB^*}(\mathbf{p},\mathbf{k})$ is the overlap matrix element with pion emission defined in [3–6], namely

$$J^{(1)}_{5BB^*}(\mathbf{p},\mathbf{k}) = \int \frac{d^3q}{(2\pi)^3}\Psi_5\left(c\mathbf{p} - \frac{\mathbf{k}}{2} + \mathbf{q}\right)\psi_B(\mathbf{q})\psi_{B^*}(\mathbf{q} - \mathbf{k}). \qquad (5)$$

Note, that the decay occurs in the S- state, hence no additional momentum p_i in (5), in contrast to (3), and $\bar{y}_{123} \approx 1$.

Expanding Ψ_5, ψ_B in oscillator wave function series, one can conveniently separate $\mathbf{p}$ and $\mathbf{k}$ dependence and write

$$J^{(1)}_{5BB^*}(\mathbf{p},\mathbf{k}) = e^{-\frac{p^2}{\Delta_5} - \frac{k^2}{4\beta^2_2}} I_{5,11}(\mathbf{p}). \qquad (6)$$

Finally the width can be written as

$$\Gamma_{5S}(\pi BB^*) = (Z^*)^2\left(\frac{M_{br}}{f_\pi}\right)^2 \times$$

$$\times \int^{\omega_{\max}}_{m_\pi} \frac{e^{-\frac{2p^2}{\Delta_5} - \frac{k^2}{2\beta^2_2}}}{4\pi^3 N_c} |I_{5,11}(\mathbf{p})|^2 \tilde{M} pk d\omega_\pi \qquad (7)$$

and conservation law

$$\frac{p^2}{2\tilde{M}} + \omega_\pi = \Delta E = 0.255\ \mathrm{GeV}; \quad \tilde{M} = 2.65\ \mathrm{GeV},$$

and $\omega_{\max} = \Delta E$, $\tilde{M}$ is the reduced mass of $B\bar{B}^*$.

The $\pi B^*\bar{B}^*$ decay width is obtained by replacing $Z^* \to Z^{**}$ and $\Delta E = M(5S) - 2M_{B^*} = 0.21\,\mathrm{GeV}$. To a good accuracy $Z^* \cong Z^{**} = 1$, and one obtains

$$\Gamma_{5S}(\pi BB^*) = \left(\frac{M_{br}}{f_\pi}\right)^2 15\ \mathrm{keV},$$

$$\Gamma_{5S}(\pi B^*B^*) = \left(\frac{M_{br}}{f_\pi}\right)^2 3.3\ \mathrm{keV} \qquad (8)$$

Defining $(M_{br}/f_\pi)^2$ from dipion widths, one has an estimate $(M_{br}/f_\pi)^2 \approx 1.5 \div 2$, leading to the predictions

$$\Gamma_{5S}(\pi BB^*) \approx (23 \div 30)\ \mathrm{keV},$$

$$\Gamma_{5S}(\pi B^*B^*) \approx (5 \div 6.6)\ \mathrm{keV}$$

4. Results and discussion. We start with the results for the BB decays, shown in Table. Estimating $(M_\omega/2\omega)^2$ from $\Gamma_{\mathrm{tot}}(4S)$ one finds that our calculated $\Gamma_{\mathrm{tot}}(5S)$ is approximately 150 MeV to be compared with the experimental value from [2], $\Gamma^{\exp}_{\mathrm{tot}}(5S) =$ $= (110 \pm 13)\,\mathrm{MeV}$. One can see a reasonable $\sim 25\%$ agreement, taking into account a strong sensitivity to the actual form of the $5S$ wave function.

An experimental hierarchy among the channels 1–3 is given by relations [2].

$$\frac{\Gamma^{\exp}_1}{\Gamma^{\exp}_2} < 0.92; \quad \frac{\Gamma^{\exp}_1}{\Gamma^{\exp}_3} < 0.3; \quad \frac{\Gamma^{\exp}_2}{\Gamma^{\exp}_3} = 0.324, \qquad (9)$$

while for B_sB_s channels one has [2]

$$\frac{\Gamma_4^{\rm exp}+\Gamma_5^{\rm exp}+\Gamma_6^{\rm exp}}{\Gamma_{\rm tot}}=0.16\pm 0.02\pm 0.058 \tag{10}$$

and also

$$\frac{\Gamma_4^{\rm exp}}{\Gamma_6^{\rm exp}}<0.16;\quad \frac{\Gamma_5^{\rm exp}}{\Gamma_6^{\rm exp}}<0.16. \tag{11}$$

Comparing with our calculated Γ_k in Table one concludes, that all relations (9)–(11) are satisfied except for the last ones in (9) and (11).

The situation is better for the averaged widths $\bar{\Gamma}_k$ obtained from $\bar{J}_{5S}^{(0)}$. It is clear that more efforts are needed both on the theoretical side (improvement of wave functions) and on the experimental side (improvement of accuracy of $\Gamma_k^{\rm exp}$).

As a whole, we have explained the points 2) and 3) of Introduction, obtaining large $\Gamma_{\rm tot}\approx\Gamma_{tot}^{\rm exp}$ and an approximate experimental hierarchy among the Γ_k, $k=1,...6$ In addition, we have calculated for the first time the $B\bar{B}^*\pi, B^*\bar{B}^*\pi$ widths of $\Upsilon(5S)$, which are as large as $O(10\text{ keV})$. This magnitude makes it possible to observe the $BB\pi$ decays by the Belle and BaBar Collaborations.

On theoretical side the large factor of 10^3 between $\Gamma_{5S}(BB)$ and $\Gamma_{5S}(BB\pi)$ is due to different vertex masses M_ω and M_{br} respectively, and experimental confirmation of the magnitude of $\Gamma_{5S}(BB\pi)$ is important for understanding of quark pair creation mechanism. In a less direct way, M_ω and M_{br} yield the values of $\Gamma_{\pi\pi}(nS)$ and explain the large ratio $\Gamma_{\pi\pi}(5S)/\Gamma_{\pi\pi}(4S)\approx O(10^3)$, as it is discussed in [5, 6].

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1. K. Abe et al., (Belle Collaboration), arXiv:0710.2577.
2. W.-M. Yao et al., Particle Data Group, J.Phys. G **32**, 1 (2006).
3. Yu. A. Simonov, arXiv: 0711,3626, Phys. Atom. Nucl. (in press).
4. Yu. A. Simonov, JETP Lett. **87**, 147 (2008).
5. Yu. A. Simonov and A. I. Veselov, arXiv:0804.4635.
6. Yu. A. Simonov and A. I. Veselov, arXiv:0805.4499.
7. A.M. Badalian, A.I. Veselov, and B.L.G. Bakker, J. Phys. G **31**, 417 (2005); A.M. Badalian, I.V. Danilkin (in preparation).