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Extraordinary magnetic field behavior of SIFS Josephson junctions with an inhomogeneous transparency of the FS interface

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Within the framework of the Usadel equations the Josephson effect in a superconductor-insulator-ferromagnet-superconductor (SIFS) structure with spatially heterogeneous transparency of the SF interface has been studied. It is shown that at a certain thickness of the F-layer a step-like variation of the transparency leads to the formation of a region of size $\sim \xi_F$ (coherence length in a ferromagnet) where the Josephson supercurrents of different signs may flow. This may lead to a dependence of the junction critical current on the external magnetic field qualitatively different from the Fraunhofer pattern typically observed in usual Josephson junctions.

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It is well known that the critical current I_c of any traditional uniform Josephson junction is suppressed in an external magnetic field H applied in the plane of the junction independently on whether the junction is in a 0 or π phase ground state. In particular, if the size of a one dimensional (along y) junction is smaller than the Josephson penetration depth λ_J and its critical current density $J_c(y) = \text{const}$, i.e. uniform, then the $I_c(B)$ curve has the form of the Fraunhofer pattern[1, 2]. The uniformity of $J_c(y)$ is an important factor. If $J_c(y) \neq \text{const}$, e.g., changes sign as $J_c(y) = \text{sgn}(y)$, the $I_c(B)$ dependence is strongly modified [3 – 6]. Such a modification has been observed recently in experiments with SFS [4] and SIFS [7 – 9] junctions. In these structures the nonuniformity of $J_c(y)$ had been achieved by making a step-like change in the thickness of the F-layer from d_{F1} to d_{F2}. The thicknesses d_{F1} and d_{F2} were chosen so that one part (if isolated) would be in the ground state with the Josephson phase $\varphi = 0$ and the other part would be in the ground state with $\varphi = \pi$. Both experiment and simple theoretical calculations show that in this case the value of the critical current increases with increasing external magnetic field, approaching a maximum value for fields of the order of a few mT. This scale of magnetic field is typical for the majority of Josephson devices. In previous investigations, which were focused on the properties of arrays of alternating 0 and π Josephson junctions [10, 11], this scale of B comes from the assumption that there are no other peculiarities at the point of the 0-π crossover except for the change in the sign of the critical current density.

In this paper, on the basis of the microscopic theory of superconductivity, we demonstrate that a step-like change of some parameter inside a Josephson junction with ferromagnetic barrier may result in strong and nontrivial changes of $J_c(y)$. This, in turn, leads to the formation of a local conducting region of size $\sim \xi_F$ (the coherence length in the ferromagnet) near the step resulting in an unusual behavior of the junction in an external magnetic field.

As an example, we consider a tunnel SIFS Josephson junction with a step-like change in the conductivity of the SF boundary, as shown in Fig.1. Namely, we assume that the SF interface damping parameter is

$$\gamma_{B1} = R_1 S_1/\rho_F \xi_F, \text{ for } y > 0, \tag{1}$$

$$\gamma_{B2} = R_2 S_2/\rho_F \xi_F, \text{ for } y < 0. \tag{2}$$

Here $R_{1,2}$ are the SF interface resistances at $y > 0$ and $y < 0$, respectively, $S_{1,2}$ are the areas of the each interface and ρ_F is the resistivity of the ferromagnetic film itself, $\xi_F^2 = (D_F/2\pi T_c)$ is the superconducting coherence length in the ferromagnet, D_F is the diffusion coefficient in the F-layer, and T_c is the critical temperature of the superconductors. The conductivity of SF boundaries or their transparencies for the electrons are inverse proportional to the damping parameters $\gamma_{B1,2}$. We assume additionally that all layers of the structure are in the "dirty" limit and that the SF interface resistance is large enough, so that one can neglect the suppression of superconductivity in the S-layers and use the linearized

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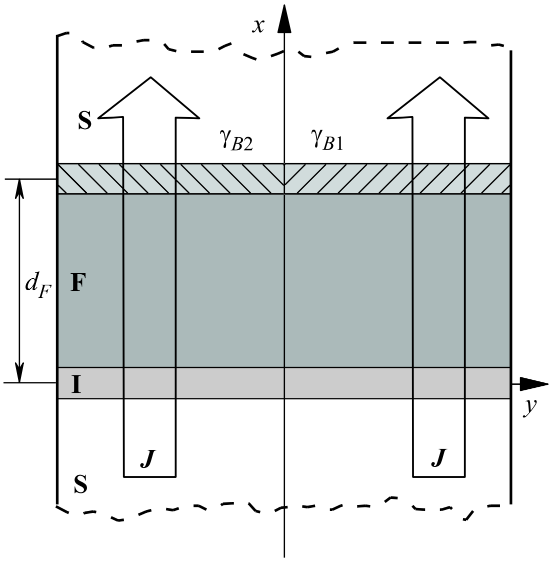

Fig.1. A schematic view of SIFS Josephson junction with the step-like changing of S/F interface transparency

Usadel equations in the F-layer. These equations have the form:

$$\xi_F^2 \left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right\} \Phi_F - \frac{\widetilde{\omega}}{\pi T_c} \Phi_F = 0, \tag{3}$$

where $\Phi_F = \Phi_F(x, y, \omega)$ is the parameterized Usadel function $\Phi_F = \widetilde{\omega} F_F / G_F$, F_F and G_F are the Usadel functions, $\widetilde{\omega} = \omega + iE$, E is the exchange magnetic energy, $\omega = \pi T(2n+1)$ are Matsubara frequencies, T is the temperature, n is an integer. We use the units where the Bolzmann constant and the Plank constant $k_B = \hbar = 1$, so that ω, T and Φ_F all have dimensions of energy. Further on, we introduce other normalization where the order parameter, the exchange magnetic energy E and the Matsubara frequencies ω are normalized to πT_c.

Equation (3) must be supplemented by boundary conditions. At $x = d_F$, $y > 0$ and at $x = d_F$, $y < 0$ they have the form [12, 13]:

$$\gamma_{B1,2} \frac{\xi_F}{\widetilde{\omega}} \frac{\partial}{\partial x} \Phi_F + G_0 \frac{\Phi_F}{\widetilde{\omega}} = G_0 \frac{\Delta \exp\left(i\frac{\varphi}{2}\right)}{\omega}, \tag{4}$$

where Δ is the magnitude of order parameter and φ is the difference of the phases of the order parameter between the superconducting banks, and $G_0 = \omega/\sqrt{\omega^2 + \Delta^2}$ is the Usadel function in the S-electrodes.

At the SI interface ($x = 0$) (we neglect the theckness of the insulator and of the SF boundary) the boundary condition can be written as:

$$\frac{\partial}{\partial x} \Phi_F = 0, \tag{5}$$

and at the left and the right ends of the Josephson junction it reduces to:

$$\frac{\partial}{\partial y} \Phi_F = 0. \tag{6}$$

The solution of the boundary value problem (3)–(6) at $y \geq 0$ can be written in the form:

$$\Phi_F = \frac{\Delta \widetilde{\omega}}{\sqrt{\Delta^2 + \omega^2}} \frac{\exp\left(i\frac{\varphi}{2}\right)}{\gamma_{B1}} \left\{ \frac{\cosh\left(\sqrt{\widetilde{\omega}} x/\xi_F\right)}{\sqrt{\widetilde{\omega}} \sinh\left(\sqrt{\widetilde{\omega}} d/\xi_F\right)} - \right.$$
$$- \frac{\gamma_{B2} - \gamma_{B1}}{\gamma_{B2}} d\xi_F \sum_{k=0}^{\infty}{}' \frac{(-1)^k \cos(\pi k x/d)}{d^2 \widetilde{\omega} + (\pi k \xi_F)^2} \times$$
$$\left. \times \exp\left[-\frac{y}{\xi_F} \sqrt{\widetilde{\omega} + (\pi k \xi_F/d)^2} \right] \right\}, \tag{7}$$

where the prime means that at $k = 0$ only half of the term is taken. The solution of the boundary value problem (3)–(6) at $y \leq 0$ can be reconstructed from (7) by replacing γ_{B1} by γ_{B2} and by changing the sign of the coordinate y. The second term in (7) describes the perturbation of the Usadel functions nucleated by the change of the SF interface transparency at $x = d$, $y = 0$. Substitution of (7) into the standard expression for the supercurrent J across a Josephson tunnel junctions [14] results in the sinusoidal relation $J(\varphi) = J_c(y)\sin(\varphi)$ with the critical current density $J_c(y)$ given by

$$J_c(y) = \frac{\pi T}{eRS} \sum_{\omega=0}^{\infty} \frac{\Delta}{\sqrt{\Delta^2 + \omega^2}} \mathrm{Re} \left[\frac{\Phi_F(0, y, \omega)}{\widetilde{\omega}} \right], \tag{8}$$

where R is the normal resistance of the Josephson junction, and S is the area of the structure.

From Eqs. (7) and (8) it follows that the critical current density is not homogeneous along the junction. The calculation of the $J_c(y)$ distribution yields the following unexpected result. At values of the ferromagnetic layer thickness d_F for which the junction is near the 0-π crossover in the regions far away from the step, i.e. $J_c(\pm\infty)$ is small, the critical current density $J_c(y)$ oscillates in the vicinity of $y = 0$ (see Fig.2). These oscillations decay on the distance $\sim \xi_F$ as one goes away from the point $y = 0$. The amplitude of these oscillations can exceed the value of $J_c(\pm\infty)$ by several orders of magnitude and the critical current density changes its sign at $y = 0$. An oscillating $J_c(y)$ means that we have alternating 0 and π regions on the scale of the ferromagnetic coherence length ξ_F. Previously, a similar effect has been predicted for Josephson tunnel junctions having alternating normal (N) and ferromagnetic regions located on both sides of a dielectric layer [15], there $J_c(y)$ exhibited damped oscillation with a large amplitude near the

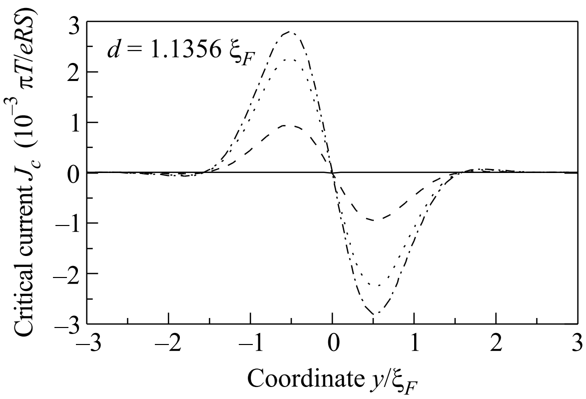

Fig.2. The critical current density distribution (in the units of $\pi T/eRS$) along the nonhomogeneous Josephson junction. The y-coordinate is in the units of ξ_F. The temperature is $T = 0.1T_c$, the exchange magnetic energy is $E = 25\pi T_c$, the ferromagnetic layer thickness is $d = 1.1356\xi_F$, $\gamma_{B1} = 1$, the values γ_{B2} are plotted in the figure

boundary between SFIFS and SNINS junctions. It is qualitatively clear that the presence of such regions in the vicinity of the step at $y = 0$, must lead to peculiarities in dependence of the critical current on the external magnetic field H.

To find $I_c(H)$ we start from the Ferrell-Prange equation for the inhomogeneous Josephson junction:

$$\lambda_J^2 \frac{\partial^2 \varphi(y)}{\partial y} - \frac{J_c(y)}{J_{c0}} \sin\varphi(y) = 0, \qquad (9)$$

where $\lambda_J^2 = \Phi_0/[2\pi\mu_0(d_F + d_I + 2\lambda_L)J_{c0}]$ is the Josephson penetration depth, Φ_0 is the magnetic flux quantum, λ_L is the London penetration depth, d_I is the insulator thickness, and J_{c0} is the maximum critical current density.

In the practically interesting limit $\xi_F \ll 2L < \lambda_J$ (L is the JJ length), the solution of Eq. (9) can be found in the form $\varphi(y) = \varphi_0 + hy/\xi_F$, where $h = H/H_0$ is the normalized applied magnetic field in the junction plane, and $H_0 = \Phi_0/2\pi\xi_F(d_F + 2\lambda_L)$.

To calculate the maximum value of the critical current, we substitute this $\varphi(y)$ into (7) and (8) and integrate (8) over y. This full supercurrent across the junction has to be further maximized with respect to the phase difference φ_0. This procedure finally gives

$$I_{\max}(h) = \frac{\pi T}{eRL\,|h|}\sqrt{\Sigma_1^2 \sin^2\frac{hL}{\xi_F} + \Sigma_2^2(h)}, \qquad (10)$$

where

$$\Sigma_1 = \frac{\gamma_{B1}+\gamma_{B2}}{\gamma_{B1}\gamma_{B2}} \sum_{\omega=0}^{\infty} \frac{\Delta^2}{\Delta^2+\omega^2} \operatorname{Re} \frac{1}{\sqrt{\widetilde{\omega}}\sinh(\frac{d}{\xi_F}\sqrt{\widetilde{\omega}})},$$

$$\Sigma_2 = \frac{\gamma_{B2}-\gamma_{B1}}{\gamma_{B1}\gamma_{B2}} \sum_{\omega=0}^{\infty} \frac{\Delta^2}{\Delta^2+\omega^2} \times$$

$$\times \operatorname{Re}\left[\frac{1}{\sqrt{\widetilde{\omega}+h^2}\sinh\left(\frac{d}{\xi_F}\sqrt{\widetilde{\omega}+h^2}\right)} - \frac{\cos(hL/\xi_F)}{\sqrt{\widetilde{\omega}}\sinh\left(\frac{d}{\xi_F}\sqrt{\widetilde{\omega}}\right)}\right].$$

It is clearly seen from (10) that in the absence of the inhomogeneity ($\gamma_{B1} = \gamma_{B2}$, thus $\Sigma_2 = 0$) the de-

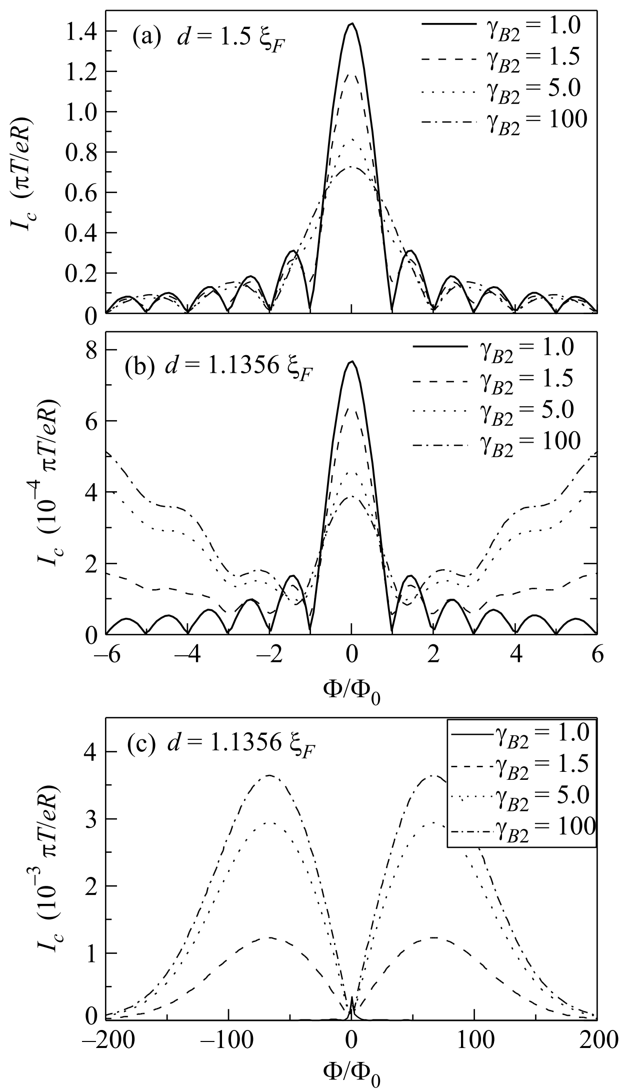

Fig.3. The junction critical current (in the units of $\pi T/eRS$) as a function of the magnetic flux through the junction (in the units Φ_0) at two ferromagnetic layer thicknesses $d = 1.5\xi_F$ (a) and $d = 1.1356\xi_F$ (b). Figure (c) shows the the same dependencies as in (b), but in the scale of large magnetic field. All curves are calculated for $L = 100\xi_F$, the temperature is $T = 0.1T_c$, the exchange magnetic energy is $E = 25\pi T_c$, $\gamma_{B1} = 1$, the values $\gamma_{B2} = 1$, 1.5, 5, 100 (solid, dashed dotted and dashed-dotted lines respectively) are plotted in the figure

pendence of $I_{\max}(h)$ reduces to the well known Fraunhofer form (see Fig.3a). An increase of the difference $\eta = \gamma_{B2} - \gamma_{B1}$ results in two completely different situations. First, if for $|y| \gg \xi_F$ the junction is either in the 0 or in the π state, then an increase of η (increase of γ_{B2}) blocks the left part of the structure for the current, so the lenght of the junction effectively decreases from L to $L/2$, see Figs.2 and 3a. This conclusion correlates well with the data of Fig.5 from [7], when the current density of one part of the junction vanishes. Second, the situation completely changes, if at $|y| \gg \xi_F$ the system is close to a 0-π transition and $J_c(y)$ is small. In this case the second term under the square root in (10) starts to dominate, resulting in the $I_{\max}(h)$ dependence presented in Fig.3b, c. This dependence has an extraordinary form. It reminds a $[\sin^2(\pi\Phi/2\Phi_0)]/(\pi\Phi/2\Phi_0)$ dependence typical for long 0-π Josephson junctions (see, e.g., Fig.3 in Ref. [9]) – the critical current goes up with increasing h, up to a certain value. But in our case this value certainly exceeds not only the scale typical for Josephson devices, but even the critical magnetic field of the S-electrodes. Such a field scale is associated with the small scale (of the order ξ_F) of the nonuniformity in $J_c(y)$. Therefore, raising $I_{\max}(h)$ should be interrupted at some point before reaching its (theoretical) maximum just because the superconducting electrodes will switch to the normal state. In experiment, one will probably observe just a linear part of $I_{\max}(h)$ close to $H = 0$ and then some effects related to the suppresion of superconductivity close to H_{c1} and H_{c2}.

The physics of this almost linear increase of $I_{\max}(h)$ is rather transparent. The external field destroys the initial antisymmetric distribution of the supercurrent around the point $y = 0$ (see Fig.2). Therefore the larger the field the smaller is the asymmetry of $J_c(y)$ and the larger is the net current across the junction.

Based on the theory presented above, we may speculate that any sharp (on the scale of ξ_F) inhomogeneities inside the Josephson junctions with ferromagnetic material may lead to an extraordinary behavior of $I_{\max}(h)$ in the vicinity of the 0-π transition. This behavior results from the emegence of several 0 and π regions around each inhomogeneity having the size $\sim \xi_F$. This effect must be certainly taken into account for interpretation of experimental data. This effects is similar to the field induced superconductivity: at zero applied field the junction can carry only tiny supercurrent, but when one applies field, JJ can carry the supercurrent several orders greater. It also may be used for measurement of the absolute value of the external magnetic field.

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