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Aharonov-Bohm effect in superconducting LOFF state

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We study AB oscillations of transition temperature, paraconductivity and specific heat of thin ring in the regime of inhomogeneous Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) superconducting state. We found that in contrast to uniform superconductivity magnetic flux might increase the critical temperature of LOFF state. Degeneracy of the inhomogeneous superconducting state reveals in double peak structure of AB oscillations.

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A magnetic field destroys superconductivity either by orbital or paramagnetic pair-breaking effects. The Chandrasekhar – Clogston pair breaking limit takes place when paramagnetic energy coincides with the superconducting condensation energy. Larkin and Ovchinnikov [1], Fulde and Ferrel [2] (LOFF) predicted the existence of the nonuniform superconducting state in a ferromagnetic superconductors at low temperatures above the paramagnetic limit (see for a review [3, 4]). This so-called LOFF state is a result of Cooper pairing with nonzero momentum and has a lower energy compared to the uniform superconducting state.

Appearance of the LOFF state is related to change in the sign of coefficient β at the gradient term of the Ginzburg – Landau (GL) free energy functional $\beta|\nabla\Psi|^2$, where Ψ is the order parameter. Coefficient β being a function of Zeeman energy $\mu_B H$, becomes negative at low temperatures $T < 0.56T_c(0)$ and high magnetic fields $H > 1.07T_c(0)/\mu_B$, where $T_c(0)$ is the critical temperature at zero magnetic field, signalling of the formation of nonuniform LOFF state. As a result one has to take into account higher terms in the GL functional expansion $|\nabla^2\Psi|^2$. This effect is very sensitive to impurities [5] and moreover usually orbital pair breaking effect dominates over the paramagnetic limit.

Despite the LOFF state has been theoretically predicted almost 40ty years ago, only recently LOFF phase was found in heavy-fermion compound $CeCoIn_5$ and organic superconductors like $\lambda-(BETS)_2FeCl_4$. The experimental evidence of the LOFF state based on the specific heat measurements [6 – 8] and nuclear magnetic resonance [9] were presented for heavy-fermion superconductor. The signature of phase transition between LOFF state and the homogenous superconducting state was reported for organic superconductors [10 – 13]. These experiments were focused on the identification of the phase transition inferred from a kink of thermal conductivity [10], observation of peculiar properties -dip structures- in the resistance [11] and changes in the rigidity of the vortex system [12]. The thermodynamic evidence of the existence of narrow intermediate state (attributed to LOFF) which separates the uniform superconducting state and normal state based on specific heat measurements was presented in paper [13].

In the paper [14] crossovers between different fluctuational regimes of paraconductivity and specific heat in the vicinity of the LOFF transition were discussed. Authors showed that these fluctuational contributions have specific temperature dependencies compared to the case of uniform superconductivity and could serve as an additional indicator of the LOFF state.

In the present letter we consider the Aharonov-Bohm effect in superconducting ring of radius R, (see inset in fig.1). We examine the superconducting fluctuations in

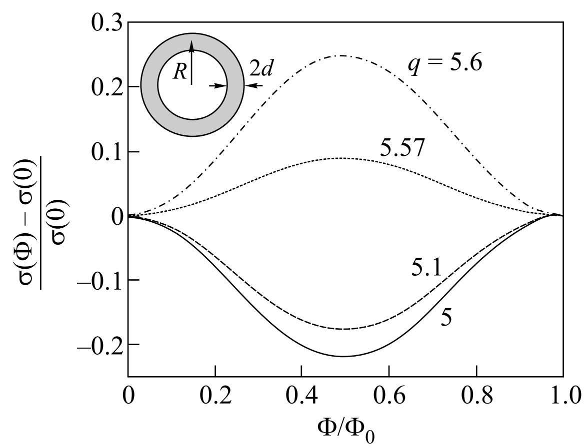

Fig.1. Magnetic flux dependence of paraconductivity in the vicinity of LOFF transition $z = [\beta^2/(4\alpha\delta)]^{1/4} = 20$ for different radiuses of the ring R

this system in the vicinity of the LOFF transition. It will be shown that the transition temperature is an oscillating function of the radius of the ring and can be increased by applied magnetic flux. The detailed analysis of the paraconductivity and specific heat for the superconducting ring will be given.

Because of the Little-Parks effect [15, 16] the superconducting transition temperature oscillates with the applied magnetic flux Φ/Φ_0 through the ring, where $\Phi_0 = \pi/e$ is the flux quantum. We show that for the case of metal-LOFF transition when parameter $\beta = -|\beta|$ becomes negative, critical temperature is given as

$$T_c^*(\Phi) = T_c(H) + \frac{\beta^2}{4a\delta} - \frac{\delta}{aR^4}\left(\frac{\Phi}{\Phi_0}\right)^2 - - \frac{\delta}{aR^4}\min\left([n - \frac{\Phi}{\Phi_0}]^2 - \frac{R^2|\beta|}{2\delta}\right)^2. \quad (1)$$

Where δ is a coefficient at the term $|\nabla^2\Psi|^2$ of GL functional and where integers n are defined in order to satisfy the minimum of the free energy: $E_n = a(T - T_c^*(\Phi))$. Depending on the ratio Φ/Φ_0 appropriate n which satisfy expression (1) jump between the integer parts of two values

$$\pm\sqrt{\frac{R^2|\beta|}{2\delta}} + \frac{\Phi}{\Phi_0}. \quad (2)$$

This leads to oscillations of transition temperature with magnetic flux or with ring's radius R.

It is seen that oscillations near LOFF behave in qualitatively different way than in the vicinity of uniform superconductor transition. Indeed, firstly, the critical temperature $T_c^*(\Phi)$ is an oscillating function of the radius of the ring R. Secondly, an applied magnetic flux in the regime of LOFF transition can increase the critical temperature depending on the sample size or on the free energy expansion parameters. These effects are shown in fig.1 and fig.2 where the magnetic flux dependencies of paraconductivity and specific heat are presented for a set of rings with different radiuses. From eq.2 it is also seen that magnetic flux shifts the degeneracy of superconducting LOFF state which might result in two peaks in magnetic flux dependency of paraconductivity and specific heat (see fig.3 and fig.4).

Now let us tern to the detailed calculations. Consider free energy density of a thin superconducting ring such that order parameter is constant over the cross section. The spatial dependence of the order parameter over the cross section of the ring will be discussed at the end of the letter. Above the superconducting transition one could use quadratic order parameter approximation

$$F = \Psi^*\{\tilde{\alpha} + \beta\mathbf{D}^2 + \delta[(\mathbf{D}^2)^2 + (\Phi/(R^2\Phi_0))^2]\}\Psi, \quad (3)$$

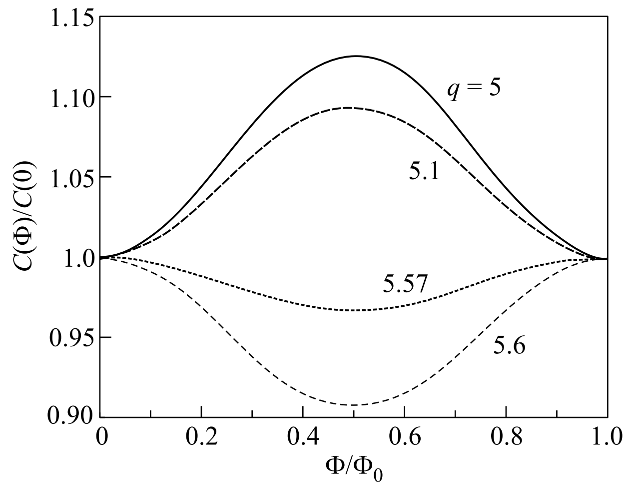

Fig.2. Magnetic flux dependence of specific heat in the vicinity of LOFF transition $z = 20$ for different radiuses of the ring R. Here solid line, dashed line, dash-dot line, dotted line corresponds to $q = [5; 5.1; 5.6; 5.57]$ respectively

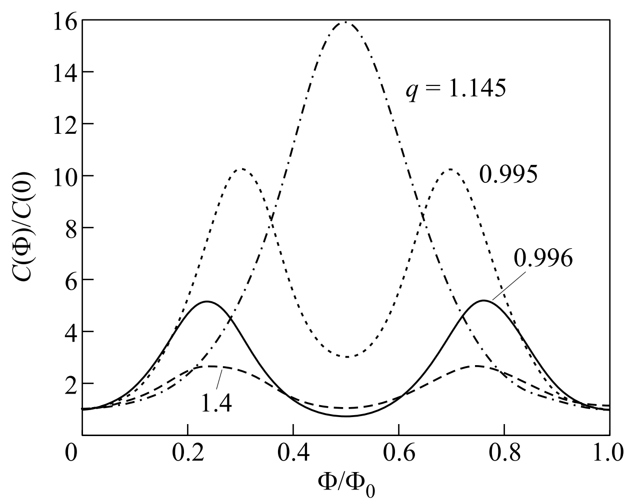

Fig.3. Magnetic flux dependence of specific heat in the vicinity of LOFF transition for $|T - T_c| \ll \beta^2/a\delta$, ($z = 20$) for different radiuses of the ring R. Here solid line, dashed line, dash-dot line, dotted line corresponds to $q = 2\pi R\,(\alpha/2|\beta|)^{1/2} = [0.996; 1.4; 1.145; 0.995]$ respectively

where $\mathbf{D} = -i\nabla - 2e\mathbf{A}$, while tangent component of the vector potential is given as $A_\varphi = \Phi/(R\Phi_0)$, Ψ is complex order parameter, Φ is the flux of the magnetic field through the ring. Coefficients $\tilde{\alpha} = a(T - T_c(H))$, β and δ depend on the exchange magnetic field and temperature (see for example [17]). Expanding order parameter in terms of Fourier series so that

$$\Psi = \sum_n \Psi_n e^{i\varphi n}, \quad (4)$$

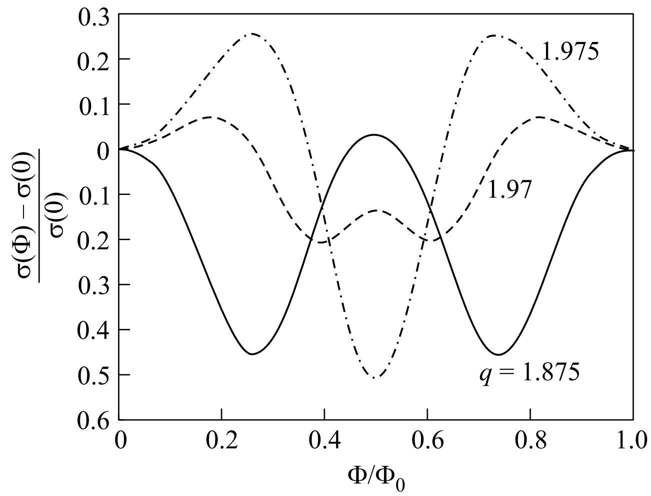

Fig.4. Magnetic flux dependence of paraconductivity in the vicinity of LOFF transition $z = 20$ for different radiuses of the ring R. Here solid line, dashed line, dash-dot line corresponds to $q = [1.875; 1.97; 1.975]$ respectively

we find for the free energy density

$$F = \sum_n E_n |\Psi_n|^2, \tag{5}$$

where spectrum of the fluctuations is given as

$$E_n = \alpha + \frac{\delta}{R^4}\left(\left[n - \frac{\Phi}{\Phi_0}\right]^2 + \frac{R^2\beta}{2\delta}\right)^2. \tag{6}$$

and $\alpha = \tilde{\alpha} - \frac{\beta^2}{4\delta} + \frac{\delta}{R^4}\left(\frac{\Phi}{\Phi_0}\right)^2$. For ring geometry the expressions for the paraconductivity and the specific heat are [18]

$$\sigma = \frac{\pi e^2 a T_c}{R} \sum_n \frac{[\frac{\beta}{R^2}(n - \frac{\Phi}{\Phi_0}) + \frac{2\delta}{R^4}(n - \frac{\Phi}{\Phi_0})^3]^2}{E_n^3} \tag{7}$$

and

$$C = \frac{a^2 T_c^2}{R} \sum_n \frac{1}{E_n^2}. \tag{8}$$

Let us consider the case of metal-LOFF transition which corresponds to negative $\beta = -|\beta|$. Performing Poisson summation we obtain the expression for the paraconductivity

$$\sigma = 2\frac{(2\pi e)^2}{Rp}\frac{aT_c}{\alpha}\sum_k \int_{-\infty}^{\infty} dt \frac{t^2(t^2 - z^2)^2 e^{ik(2\pi\Phi/\Phi_0 + tp)}}{[1 + (t^2 - z^2)^2]^3} \tag{9}$$

and specific heat

$$C = \frac{p}{2\pi R}\frac{(aT_c)^2}{\alpha^2}\sum_k \int_{-\infty}^{\infty} dt \frac{e^{ik(2\pi\Phi/\Phi_0 + tp)}}{[1 + (t^2 - z^2)^2]^2}, \tag{10}$$

where $z = [\beta^2/(4\alpha\delta)]^{1/4}$ and $p = 2\pi R[\alpha/\delta]^{1/4}$. First we will examine the limit $\beta^2 \gg \alpha\delta$ which corresponds to temperatures close to transition $|T - T_c| \ll \beta^2/a\delta$. We also suggest the superconducting ring being relatively large

$$R \gg (|\beta|/\alpha)^{1/2}. \tag{11}$$

This condition allows one to keep modes with $k = 0$ and $k = 1$ in Eqs. (9) and (10) since higher modes will be exponentially freezed. In this regime we obtain equation for the paraconductivity that is given as

$$\sigma \simeq \frac{e^2}{R}\left(\frac{|\beta|}{R^2 a T_c}\right)^{1/2} |1 - T/T_c|^{-3/2} \times \\ \times \left[1 - 2q^2 \cos(2\pi\Phi/\Phi_0)\cos(\phi)e^{-q}\right], \tag{12}$$

while the specific heat is given by the following expression

$$C \simeq \frac{1}{R}\left(\frac{R^2 a T_c}{|\beta|}\right)^{1/2} |1 - T/T_c|^{-3/2} \times \\ \times \left[1 + 2q \cos(2\pi\Phi/\Phi_0)\cos(\phi)e^{-q}\right], \tag{13}$$

where we introduced parameters $\phi = pz = 2\pi R\,(|\beta|/2\delta)^{1/2}$ and $q = p/2z = 2\pi R\,(\alpha/2|\beta|)^{1/2}$.

Depending on the sign of the $\cos(\phi)$ applied magnetic flux can either increase or decrease the conductivity or specific heat. That is in contrast to the case of normal to uniform superconductor transition in superconducting rings where for example specific heat always decreases with applied magnetic flux. The different sign in expressions (12) and (13) reflects the different origins of specific heat which is associated with the critical temperature behavior and transverse paraconductivity which is a spectral structure dependent quantity [19, 20].

Another interesting feature of the LOFF phase arises for the intermediate values of the radius of the ring when $R \sim (|\beta|/\alpha)^{1/2}$. This situation is illustrated in fig.3 and fig.4 where the magnetic flux dependencies of the specific heat and paraconductivity are presented for different radiuses of the ring.

One sees the crossover between different regimes of the oscillations behavior, in particular, from the one-peak per period into two peaks. Magnetic flux doubles the peaks by removing the degeneracy of the superconducting state Eq.(2). This effect becomes more pronounced in the case of very small rings

$$R \ll (|\beta|/\alpha)^{1/2}, \tag{14}$$

where one will observe strong fluctuations of both paraconductivity and specific heat. The superconducting ring effectively becomes a quasi-zero dimensional system and one yields the following expression for specific heat

$$C \simeq \frac{1}{R}|1-T/T_c|^{-2}\left[f(\phi+\frac{2\pi\Phi}{\Phi_0})+f(\phi-\frac{2\pi\Phi}{\Phi_0})\right], \tag{15}$$

where

$$f(\phi+\frac{2\pi\Phi}{\Phi_0}) \simeq \frac{q^4}{4[1+q^2/2-\cos{(\phi+2\pi\Phi/\Phi_0)}]^2}. \tag{16}$$

Note, that phase shifts $\phi\pm 2\pi\Phi/\Phi_0$ are equal to the values of the phase in equation (2). Magnetic flux removes the degeneracy of the superconducting state which reveals in two-peak oscillations.

Now let us consider high temperatures regime far away from transition where $|T-T_c|\gg\beta^2/a\delta$ or equivalently the regime where coefficient $\beta\to 0$ vanishes. And let the radius of the ring be $R\gg(\delta/\alpha)^{1/4}\gg(|\beta|/\alpha)^{1/2}$. We obtain expressions for paraconductivity

$$\sigma \simeq \frac{e^2}{R}\left(\frac{\delta}{aT_cR^4}\right)^{1/4}|1-T/T_c|^{-5/4}\times$$
$$\times[1-\sqrt{2}p^2e^{-p/\sqrt{2}}\sin{(p/\sqrt{2}+\pi/4)}\cos{(2\pi\Phi/\Phi_0)}] \tag{17}$$

and specific heat

$$C \simeq \frac{1}{R}\left(\frac{aT_cR^4}{\delta}\right)^{1/4}|1-T/T_c|^{-7/4}\times$$
$$\times[1+\frac{2\sqrt{2}p}{3}e^{-p/\sqrt{2}}\sin{(p/\sqrt{2})}\cos{(2\pi\Phi/\Phi_0)}]. \tag{18}$$

Again, one sees that these fluctuational contributions in the high temperatures regime also depend on the random phase. Moreover, different temperature dependencies is an additional property of the LOFF state [14] under these conditions.

Now, we discuss possible effects coming from spatial dependence of the order parameter over the cross section of the ring $2d$. Complex order parameter written in cylindrical coordinates takes the form $\Psi(\phi,\rho)= e^{in\phi}f(\rho)$ where $f(\rho)$ is the Landau wave function. Then again the critical temperature should be found by taking the minimum with respect to n of the free energy and we obtain

$$E_n=\alpha+\frac{\delta}{R^4}\min{[(n-\Phi/\Phi_0)^2+\beta R^2/2\delta+g]^2}, \tag{19}$$

where g is a function of the ring thickness

$$g=\frac{R^2}{d^2}\left(\frac{\varphi}{\Phi_0}\right)^2+\frac{d^2n^2}{3R^2} \tag{20}$$

and where φ is the magnetic flux over the cross section of the ring. The critical temperature below which LOFF modulation appears is defined by the condition

$$\beta+\frac{2\delta}{3d^2}\left(\frac{\varphi}{\Phi_0}\right)^2=0. \tag{21}$$

From this equation one sees that due to the orbital effect LOFF critical temperature decreases since now it is not enough for β to change sign but $\beta<-\frac{2\delta}{3d^2}\left(\frac{\varphi}{\Phi_0}\right)^2$.

Finally, we focus on the validity of the Gaussian approximation used in Aharonov-Bohm (AB) effect. Indeed, Brazovskii [21] showed that the critical fluctuations could be essential in LOFF like systems and could lead to the first-order type transition. The width of this critical fluctuations region (given by the Levanyuk-Ginzburg parameter) increases [14] compared to the uniform superconductor – metal transition. However, our main result – the increase of the superconducting transition temperature and double peaks in oscillations- is the effect of the magnetic field that removes the degeneracy of the superconducting state above transition. We note, that critical fluctuations do not change the way of removing this degeneracy.

In conclusion, we have shown that an applied magnetic flux through the thin ring in the vicinity of the LOFF transition can increase the critical temperature. We have calculated the expressions for the paraconductivity and specific heat in this regime. Both values exhibit double peak oscillations in contrast to usual Little – Parks effect in the normal to uniform superconducting transition.

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1. A.I. Larkin and Yu.N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **47**, 1136 (1964) [Sov. Phys JETP **20**, 762 (1965)].
2. P. Fulde and R. A. Ferrel, Phys. Rev. A **135**, 550 (1964).
3. Y. Matsuda and H. J. Shimahara, Phys. Soc. Jpn. **76**, 051005 (2007).
4. R. Casalbuoni and G. Nardulli Rev. Mod. Phys. **76**, 263 (2004).
5. L. G. Aslamazov, Zh. Eksp. Teor. Fiz. **55**, 1477 (1968), [Sov. Phys. JETP**28**, 773 (1969)].
6. H. A. Radovan et. al., Nature (London) **425**, 51 (2003).
7. A. Bianchi et. al., Phys. Rev. Lett **91**, 187004 (2003).
8. C.F. Miclea et. al., Phys.Rev.Lett **96**, 117001 (2006).
9. K. Kumagai et. al., Phys. Rev. Lett **97**, 227002 (2006).
10. M. A. Tantar et. al., Phys. Rev. B **66**, 134503 (2002).
11. S. Uji et. al., Phys. Rev. Lett **97**, 157001 (2006).

12. J. Singleton et. al., J. Phys. Condens. Matter **12**, L641 (2000).
13. R. Lortz et. al., Phys. Rev. Lett **99**, 187002 (2007).
14. F. Konschelle, J. Cayssol and A. Buzdin, Europhys. Lett. **79**, 67001 (2007).
15. W. A. Little and R. D. Parks, Phys. Rev. Lett. **9**, 9 (1962).
16. R. D. Parks and W. A. Little, Phys. Rev. A **133**, 97 (1964).
17. A. I. Buzdin and H. Kachkachi, Phys. Lett. A **225**, 341 (1997).
18. A. I. Larkin and A. Varlamov, *Fluctuation phenomena in superconductors*, Superconductivity, v.1, chp. 10.3, chp.10.4, eds. K.H. Bennemann and J.B. Ketterson, Springer-Verlag, 2008.
19. L. I. Glazman, F. W. J. Hekking, and A. Zyuzin, Phys. Rev. B **46**, 9074 (1992).
20. A. I. Buzdin and A. A. Varlamov, Phys. Rev. Lett. **89**, 076601 (2002).
21. S. A. Brazovskii, Sov. Phys. JETP **41**, 85 (1975) [Zh. Exp.&Teor. Fiz. **68**, 175 (1975)].