Effect of Coulomb scattering on graphene conductivity

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The effect of Coulomb scattering on graphene conductivity in field effect transistor structures is discussed. Inter-particle scattering (electron-electron, hole-hole, and electron-hole) and scattering on charged defects are taken into account in a wide range of gate voltages. It is shown that an intrinsic conductivity of graphene (purely ambipolar system where both electron and hole densities exactly coincide) is defined by strong electron-hole scattering. It has a universal value independent of temperature. We give an explicit derivation based on scaling theory. When there is even a small discrepancy in electron and hole densities caused by applied gate voltage the conductivity is determined by both strong electron-hole scattering and weak external scattering: on defects or phonons. We suggest that a density of charged defects (occupancy of defects) depends on Fermi energy to explain a sub-linear dependence of conductivity on a fairly high gate voltage observed in experiments. We also eliminate contradictions between experimental data obtained in deposited and suspended graphene structures regarding graphene conductivity.

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Different heterostructures on the basis of *Graphene*, i.e., a monolayer of carbon atoms forming a dense honeycomb two-dimensional crystal structure are considered as promising candidates for future micro- and nanoelectronics. (see [1−5] and references therein). The features of the electron and hole energy spectra in graphene provide the exceptional properties of graphene-based heterostructures and devices, in particular, field-effect transistors [4,6−8]. Graphene band structure peculiarities have a substantial impact on Coulomb scattering: inter-particle scattering (electron-electron, hole-hole, and electron-hole) and scattering on charged defects. Here we intend to elucidate the effect of that kind of scattering on graphene conductivity taking into account either deposited and suspended graphene structures.

Experimental studies of graphene have revealed a quite amazing feature of its conductivity. There is almost no dependence of the minimum conductivity of graphene $\sigma_{\min}$ on temperature (between $0.3\,\text{K}$ and $300\,\text{K}$) in deposited graphene structures [3] (Fig.1, upper panel). This is really intriguing because at the same time the carrier density varies in this temperature interval by six orders of magnitude. The minimum conductivity (or intrinsic conductivity) arises at zero gate voltage in gated graphene structures. Hereafter, as usual, we reference the zero gate voltage ($V_G = 0$) just to this point. The conductivity is approximately equal to $\sigma_{\min} \simeq (6\,\text{k}\Omega)^{-1}$. There appears a natural temptation to bind this value to two conductance quanta [3, 9].
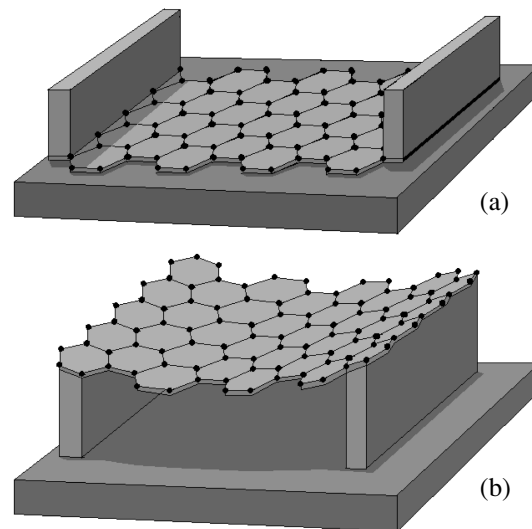

Fig.1. Schematic view of graphene structures with two side contacts and a wafer as a back gate: deposited graphene (upper panel) and suspended graphene(lower panel)

The conductance quantum for spin-unpolarized electrons equals $G_0 = 2e^2/h = (12\,\text{k}\Omega)^{-1}$, where e is the electron elementary charge and h is the Planck constant.

The minimum conductivity of graphene corresponds to an utterly ambipolar system when electron density Σ_e and hole density Σ_h coincide: $\Sigma_e = \Sigma_h$. In the situation, the electron-hole scattering is the strongest scattering mechanism. We propose below a derivation based on a scaling theory approach (see also [10]).

The conductivity, σ_0, of an intrinsic graphene layer determined by the Coulomb scattering of electrons and holes can be expressed as follows:

$$\sigma_0 = \frac{k_{\text{eff}}^2}{e^2} f_0, \tag{1}$$

where k_{eff} is the effective permittivity and f_0 is a so far unknown function of the parameters of the graphene electron-hole system and, generally speaking, of the temperature. The proper dependence (1) on the elementary charge e and the effective permittivity k_{eff} for the Coulomb interaction in the Born approximation is factorized. As it was recently theoretically demonstrated [11 – 14], the screening in a graphene layer, turns out to be determined by a weak function of the dimensionless parameter

$$\alpha = \frac{e^2}{k_{\text{eff}}\, \hbar\, v_F}, \tag{2}$$

where $\hbar$ is the reduced Planck constant. In reality, the screening by highly conducting gates might be substantial were they placed fairly close to a graphene sheet.

The factor f_0 in (1) can solely depend on intrinsic graphene parameters and the Plank constant. In the case when electron and hole densities coincide $\Sigma_e = \Sigma_h = \Sigma_T$, the graphene electron-hole system is fully characterized by the only parameter besides Fermi velocity v_F. It could be the mean momentum p_T, mean wave vector $k_T = p_T/\hbar$, temperature $T \simeq \hbar v_F k_T/k$ (where k is the Boltzmann constant), or density $\Sigma_T \simeq \simeq k_T^2$. All those parameters are directly bound to each other. Therefore, the only possible combination looks like

$$\sigma_0 = \frac{\kappa_{\text{eff}}^2 h v_F^2}{e^2} = \frac{2\kappa_{\text{eff}}^2 v_F^2}{G_0}. \tag{3}$$

Here we have neglected a numerical factor of the order of unity, which could be derived via thorough consideration of electron-hole scattering [15] or determined from experimental data(see, for instance, [3]).

The only temperature-dependent parameter k_T describing the graphene electron-hole system can not be included. Therefore, the graphene conductance appears to be independent of the temperature. Moreover, it is readily seen that the graphene conductivity turned out to be inversely proportional to the conductance quantum $G_0 = 2e^2/h = (13\text{k}\Omega)^{-1}$, in spite of the previous expectations [3].

It is expedient to estimate the value of σ_0 for the typical graphene structures. When a graphene sheet is sandwiched between two dielectrics with permittivity k_1 and k_2, the effective permittivity is

$$\kappa_{\text{eff}} = \frac{\kappa_1 + \kappa_2}{2}. \tag{4}$$

Therefore, in our opinion, the crucial experiment to verify the above speculations lies in changing the permittivity of the adjacent dielectrics. In most of so far made experiments a graphene layer was deposited on a SiO_2 surface. The effective permittivity in this case is equal to $k_{\text{eff}} \simeq 2.5$. The velocity $v_F = 10^8$ cm/s corresponds to the conductance equal to $(3\text{k}\Omega)^{-1}$. Substituting all that into (3), one obtains $\sigma_0 \simeq 4\text{k}\Omega$. This is a very good agreement with the experimental value of $(6\text{k}\Omega)^{-1}$ [3] provided a numerical coefficient in (3) was ignored.

All said above is related to an intrinsic graphene conductivity when both the electron and hole densities are exactly equal to each other, i.e., $\Sigma_e = \Sigma_h = \Sigma_T$, This can also occur in graphene-based structures with a highly conducting gate similar to field-effect transistors. In such a case, the graphene conductivity is determined, in fact, by the friction between the electron and hole subsystems. It appears to be interesting to apprehend what occurs beyond this equality which can be broken by an applied gate voltage. At a moderate gate voltage there is a rather small difference in the electron and hole densities: $|\Sigma_e - \Sigma_h| \ll \Sigma_e,\ \Sigma_h$, so that the electron-hole scattering mechanism is still the strongest one. However, we argue that in this case, the behavior of the conductivity is governed by both the electron-hole as well as other scattering processes, for instance, those on phonons and charged defects. In contarast to an intrinsic graphene, the influence of the electron-hole scattering on the conductivity becomes intermediate. Due to this scattering processes, the majority carriers drag the minority ones. In this case the electron-hole system could be regarded as a unified fluid with a total density equal to $\Sigma = \Sigma_e + \Sigma_h$ and a renormalized elementary charge of composing particles given by the following equation:

$$\bar{e} = e\,\frac{\Sigma_h - \Sigma_e}{\Sigma_h + \Sigma_e}. \tag{5}$$

Taking into account (5), the conventional formula for the conductivity of a two-dimensional electron gas can be adopted to the conductivity of graphene:

$$\sigma_1 = \frac{\bar{e}^2 \Sigma}{\nu_{ex} M} = \frac{e^2}{\nu_{ex} M}\frac{(\Sigma_e - \Sigma_h)^2}{(\Sigma_e + \Sigma_h)}, \tag{6}$$

where ν_{ex} is the rate of scattering on phonons and defects, M is the hydrodynamic mass of particles arising in

the hydrodynamic equations for graphene electron-hole system. The latter can be roughly estimated as

$$M \simeq \frac{\hbar k_T}{v_F} \simeq \frac{\hbar \sqrt{\Sigma_T}}{v_F}. \tag{7}$$

The conductivity (6) implies a quadratic dependence on gate voltage $\sigma_1 \propto V_G^2$ (Fig.2, low voltage) as a sheet
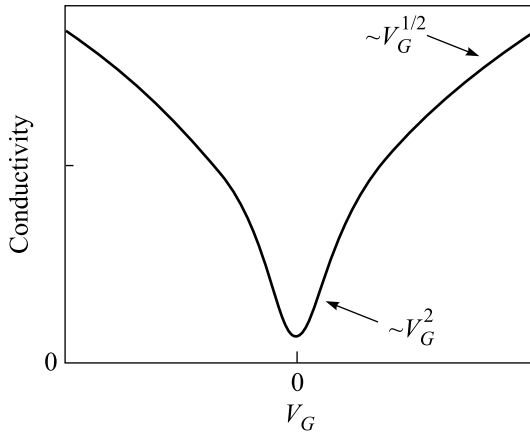

Fig.2. Graphene conductivity vs. gate voltage

charge $\Sigma_e - \Sigma_h \propto V_G$. A crossover from intrinsic graphene conductivity σ_0 given by formula (3) to that given by formula (5) can be very fast because external scattering ν_{ex} is much weaker compared to electron-hole scattering ν_{eh} which could be easily extracted from the expression (3). If among external scattering mechanisms phonon scattering dominates there appears a definite dependence of conductivity on temperature. Obviously, the phonon scattering can predominate and determine conductivity of perfect graphene structures [16]. Then, according to estimations, the electron and hole mobility can achieve values above 200000 cm^2/V·s instead of nowadays 5000–15000 cm^2/V·s at room temperature for deposited graphene structures (Fig.1, upper panel) [17].

Recently a conductivity of suspended graphene (Fig.1, lower panel) was experimentally investigated [18]. This is just a structure where phone scattering can prevail over defect scattering. For the first time the authors observed temperature dependent minimum graphene conductivity. However, we attribute this unusual behavior to the fact that suspended graphene was bent and, therefore, inhomogeneous with respect to charge distribution. The authors have admitted this possibility. In other words, there were regions obeying the relation (3) and that obeying the relation (5) simultaneously. Very high mobility (100 cm^2/V·s) observed close to the point of intrinsic graphene when electron and hole densities almost coincide, really, confirms that phonon scattering is dominant with respect to that on

defects in the case. Nevertheless, below we demonstrate that at high voltage a proportion of phone scattering rate to defect scattering rate can become reciprocal.

At fairly high gate voltages when $\Sigma_e \gg \Sigma_h$ or vice versa, the graphene electron-hole system is virtually unipolar and the electron-hole scattering processes are not essential. In that case, the conductivity of the graphene structures fabricated so far is likely governed by the scattering on charged defects [3, 14, 16], as there is no temperature dependence of mobility evidenced by experiments. Nevertheless, theoretical attempts to investigate an interplay of phonon scattering and Coulomb scattering among particles are made [19].

The conductivity σ_2 limited by the scattering on charged defects can be obtained almost similarly as that for the case of the dominant electron-hole scattering. Following (1), the conductivity in this situation can be presented as

$$\sigma_2 = \frac{k_{\text{eff}}^2}{e^2 \Sigma_i} f_2, \tag{8}$$

where Σ_i is the density of charged defects. If the thermal energy kT is much smaller than the Fermi energy ε_F, the factor f_2 in (8) can depend only on the majority carrier density $\Sigma \simeq \Sigma_e$ (or $\Sigma \simeq \Sigma_h$), the Fermi velocity v_F, and the Plank constant h, so that

$$\sigma_2 = \frac{k_{\text{eff}}^2 h v_F^2 \Sigma}{e^2 \Sigma_i}. \tag{9}$$

For an invariable density of defects, (9) results in a linear dependence of the conductivity on the gate voltage. On the contrary, it seems plausible to suggest that the charged defect concentration can depend on position of the Fermi level in graphene, i.e., on the Fermi energy ε_F (Fig.3).
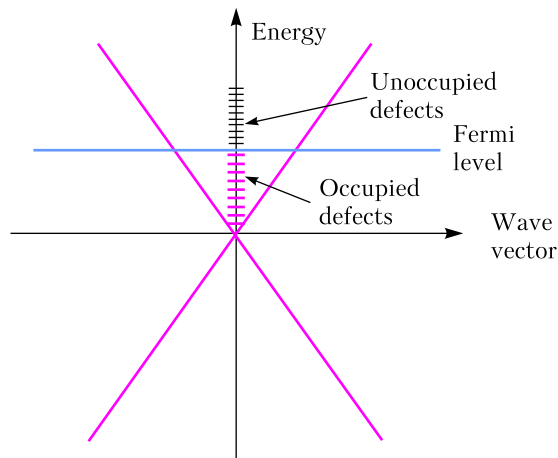

Fig.3. Charged defect states under the Fermi level

For instance, such defects may arise at the graphene-SiO_2 interface. If the number of defects per unit energy and area is constant and equals n_i, their sheet density is

$$\Sigma_i = n_i \varepsilon_F \simeq n_i v_F \hbar \sqrt{\Sigma}. \tag{10}$$

After substituting (10) into (9), one can arrive at a square-root dependence of the conductivity on the gate voltage $\sigma_2 \propto \sqrt{V_G}$ (Fig.2, high voltage). The experimental data show that the observed voltage dependences are somewhat in between the linear and square-root dependences [17]. In the case of an intrinsic graphene, the Fermi energy in (10) should be replaced by the thermal energy kT. Since according to (10), the density of charged defects can be smaller than that of carriers, the electron-hole scattering mechanism can really dominate even at low temperatures.

In conclusion, we have asserted that an intrinsic graphene conductivity (when both electron and hole densities exactly equal to each other) is defined by electron-hole scattering. It has a universal value independent of temperature. We have given an explicit derivation based on scaling theory. This value could be manipulated by varying permittivity of surrounding dielectrics. When there is even a small deviation in electron and hole densities caused by applied gate voltage the situation becomes utterly different. In the case the conductivity is determined by both strong electron-hole scattering and weak external scattering: on defects or phonons. If phonon scattering prevails the conductivity depends on temperature. We also suggest that density of charged defects (occupancy of defects) depends on Fermi energy to explain a sub-linear dependence of conductivity on gate voltage observed in experiments.

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