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Preparation of cluster states of four distant atoms by interference of polarized photons

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We propose a protocol to generate the cluster states of four Λ-type three-level atoms trapped in distant cavities by using interference of polarized photons. The protocol use the effects of quantum statistics of indistinguishable photons emitted by the atoms inside optical cavities. This makes the protocol more realizable in experiments.

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I. Introduction. Entanglement plays a central role in the field of quantum information, leading to ongoing efforts for its quantitative and qualitative characterizations. Entangled states of three or more particles not only provide possibilities to text quantum mechanics against local hidden theory without using inequality [1], but also have practical applications in high-precision frequency measurement [2] and quantum information processing [3]. Most of the research in quantum information processing is based on quantum entanglement of two qubits. Recently, there has been much interest in quantum entanglement of many qubits. It has been shown that there are two inequivalent classes of tripartite entanglement states, the Greenberger-Horne-Zeilinger (GHZ) [1] class states and the W class states [4], under stochastic local operation and classical communication. In Ref.[5], Briegel et al. introduced another class of multiqubit entangled states, the so-called cluster states. Cluster states have many interesting features. It has been shown that cluster states can be regarded as a resource for GHZ class states and are more immune to decoherence than GHZ class states [6]. In Ref.[7], the proof of Bell's theorem without the inequalities was given for cluster states, and a new Bell inequality is considered, which is maximally violated by the four-qubit cluster state and is not violated by the four-qubit GHZ class state. More importantly, it has been shown that the cluster states constitute a universal resource for so-called one-way quantum computation proceeding only by local measurements and feed-forward of their outcomes [8].

Numerous proposals [9 – 12] have been proposed to generate cluster states. For example, Zou et al. have proposed two experimental protocols [9] to generate the cluster states in the context of microwave cavity quantum electrodynamics (QED). In the first protocol to prepare many cavities into the cluster states, they encode the vacuum state and one-photon state of the microwave cavity as the logic zero and one of the qubits. The second protocol is to prepare many atoms into the cluster states, where qubits are represented by the states of Rydberg atoms. Both protocols require the resonant atom-cavity interaction so that the quantum dynamics operates at a high speed, which is important in view of decoherence. Zheng has proposed two protocols [11] for the generation of four-qubit cluster states in ion-trap systems. The first protocol is based on resonant sideband excitation, while the second protocol does not use the vibrational mode as the memory. On the other hand, it has been shown that to entangle distant atoms (or ions) by using the effect of statistics of distinguishable photons is an effective protocol. For example, Deng et al. have proposed a protocol [12] to generate cluster states of four separate qubits in decoherence-free subspace.

In this paper, we present an alternative protocol for generating the cluster states of four distant atoms. Our protocol work in the same way as the proposal in Ref.[13], where the authors presented an idea to entangle Λ-type three-level atoms with two degenerate ground states trapped in two separate cavities. We will show that using different approaches the similar model can also be used to generate cluster states of distant atoms trapped in separate cavities. The protocol is based on the combination of the atom-cavity interaction and linear optics elements. The fidelity is not affected by both the atomic spontaneous emission and cavity decay. The success of the protocol depends upon the detection of a photon leaking out of the cavity, and thus the fidelity is also not affected by the imperfection of the photon detectors.

The paper is organized as follows. In Sec. II, we study the evolution for the Λ-type three-level atom interacting with two cavity modes. In Sec. III, we propose

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a protocol to generate cluster states of four distant atoms based on the combination of the atom-cavity interaction and linear optics elements. Discussions and conclusions appear in Sec. IV.

II. Dispersive Atom-Cavity Interaction. The model we are considering consists of 4 identical Λ-type three-level atoms (Fig.1), with the 4 atoms $(1,2,3,4)$ are trapped in 4 two-mode optical cavities (A,B,C,D), respectively, as shown in Fig.2. All the four atoms have one degenerate excited states $|e\rangle$, two degenerate ground states $|g_l\rangle$ and $|g_r\rangle$, as shown in Fig.1. The quantum information is encoded on the states $|g_l\rangle$ and $|g_r\rangle$. The transitions $|e\rangle\to|g_l\rangle$ and $|e\rangle\to|g_r\rangle$ are strongly coupled to left and right circularly polarizing cavity modes, respectively. The photons leaking out from all cavities (A,B,C,D) transmit through a series of optical elements (PBS, HWP), then the photons are finally detected by photon detectors. We require here that the cavity is one-sided so that the only photon leakage occurs through the side of the cavity facing the beam-splitter.

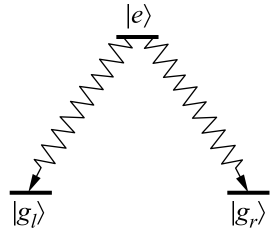

Fig.1. Atomic level structure.

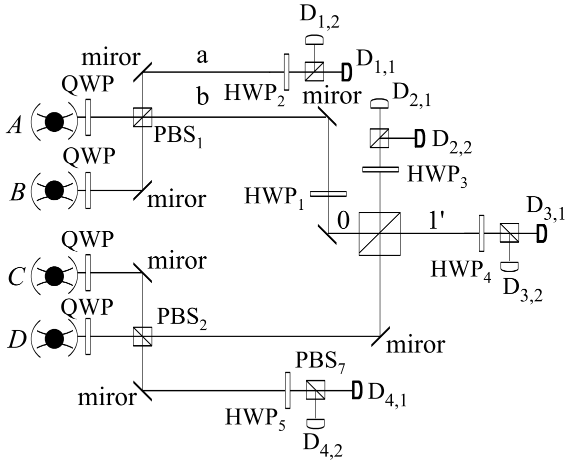

Fig.2. Experimental setup for generating cluster states of four distant atoms. The PBS transmits the horizontal polarization and reflects the vertical polarization, and the HWP's action is given by transformation $|H\rangle\to(1/\sqrt{2})(|H\rangle+|V\rangle)$ and $|V\rangle\to(1/\sqrt{2})(|H\rangle-|V\rangle)$

The interaction Hamiltonian governing the interaction between the trapped Λ-type atoms and cavities is given by

$$H_I=\hbar\Sigma_{k=l,r}\lambda_k(a_k|e\rangle\langle g_k|+a_k^\dagger|g_k\rangle\langle e|), \qquad (1)$$

where l, r denoted the left-polarized and right-polarized cavity modes. a_k and $a_k^\dagger$ are the annihilation and creation operators of photons in the k mode, and λ_k is the coupling constant between the k mode and the atom. We suppose λ_k to be real, and for the sake of generality we allow the coupling between th atom and the cavity modes to be different, i.e., $\lambda_l\neq\lambda_r$. If the atom and the cavity are prepared initially in its excited state $|e\rangle$ and vacuum states $|00\rangle_{lr}$, respectively, after interacting time t, the system of atom and cavity will evolve to the state

$$|\varpi(t)\rangle=\cos\Omega t\,|e\rangle\,|0_l\rangle\,|0_r\rangle-i\sin\Omega t\,|\phi(t)\rangle\,, \qquad (2)$$

with $|\phi(t)\rangle=\frac{1}{\Omega}(\lambda_l\,|g_l\rangle\,|1_l\rangle\,|0_r\rangle+\lambda_r\,|g_r\rangle\,|0_l\rangle\,|1_r\rangle)$ and $\Omega=\sqrt{\lambda_l^2+\lambda_r^2}$ supposed to be a given constant.

Analogous to Ref. [13], we suppose when photons are passing through the quarter wave plate (QWP), left-polarized and right-polarized photons become vertically (V) and horizontally (H) polarized photons, respectively. That is, $|1\rangle_l|0\rangle_r\to|V\rangle$ and $|0_l\rangle|1_r\rangle\to|H\rangle$. For the sake of simplicity, we have ignored the vacuum modes in the above notation because they have no contribution to the click of the photon detectors D. For the same reason, the term $|e\rangle|0_l\rangle|0_r\rangle$ in Eq.(2) can be safely neglected for simplification. Therefore, when the photon is passing through the QWP, the total state of the photon and the atoms can be written by

$$|\Psi(t)\rangle=\frac{1}{\Omega}(\lambda_l|g_l\rangle|V\rangle+\lambda_r|g_r\rangle|H\rangle), \qquad (3)$$

with a probability $P_1=\sin^2\Omega t$. Later it implies that photons have passed through QWP if we say photons leak out of cavities.

III. Generating cluster states. Photons leaking out of cavities 1 and 2 (3 and 4) will first meat polarizing beam splitters PBS_1 (PBS_2), which always transmits H polarizing photons and reflects V polarizing photons (all PBS in the paper work in this way). Thus the joint state including cavities A,B,C and D, and atoms 1, 2, 3 and 4 will become (see Fig.2)

$$
\begin{aligned}
|\Psi\rangle = {} & \frac{1}{4}[(|g_r\rangle_1|H\rangle_A + |g_l\rangle_1|V\rangle_A) \otimes \\
& \otimes (|g_r\rangle_2|H\rangle_B + |g_l\rangle_2|V\rangle_B)] \otimes \\
& \otimes [(|g_r\rangle_3|H\rangle_C + |g_l\rangle_3|V\rangle_C) \otimes \\
& \otimes (|g_r\rangle_4|H\rangle_D + |g_l\rangle_4|V\rangle_D)] \xrightarrow{\mathrm{PBS_1,PBS_2}} \\
& \xrightarrow{\mathrm{PBS_1,PBS_2}} \frac{1}{4}[|g_r\rangle_1|g_r\rangle_2|H\rangle_b|H\rangle_a + \\
& + |g_r\rangle_1|g_l\rangle_2|H\rangle_b|V\rangle_b + \\
& + |g_l\rangle_1|g_r\rangle_2|V\rangle_a|H\rangle_a + |g_l\rangle_1|g_l\rangle_2|V\rangle_a|V\rangle_b] \otimes \\
& \otimes [|g_r\rangle_3|g_r\rangle_4|H\rangle_d|H\rangle_c + |g_r\rangle_3|g_l\rangle_4|H\rangle_d|V\rangle_d + \\
& + |g_l\rangle_3|g_r\rangle_4|V\rangle_c|H\rangle_c + |g_l\rangle_3|g_l\rangle_4|V\rangle_c|V\rangle_d], \qquad (4)
\end{aligned}
$$

where we suppose $\lambda_l = \lambda_r$ for the cluster state. We consider only those terms that contain one photon in each of the modes a, b, c and d, hence we have discarded the bunching outcomes in Eq.(4). Then Eq.(4) changes into

$$
\begin{aligned}
& \frac{1}{2}[|g_r\rangle_1|g_r\rangle_2|H\rangle_b|H\rangle_a + |g_l\rangle_1|g_l\rangle_2|V\rangle_a|V\rangle_b] \otimes \\
& \otimes [|g_r\rangle_3|g_r\rangle_4|H\rangle_d|H\rangle_c + |g_l\rangle_3|g_l\rangle_4|V\rangle_c|V\rangle_d], \qquad (5)
\end{aligned}
$$

with the probability $P_2 = 25\%$.

Then the photon of mode b will meet HWP$_1$, which can lead to the following transformation between input modes and output modes: $|H\rangle \to \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ and $|V\rangle \to \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$. The joint state of the whole system (four atoms and four cavity modes) follows that

$$
\begin{aligned}
& \frac{1}{2\sqrt{2}}[|g_r\rangle_1|g_r\rangle_2|H\rangle_a(|H\rangle_0 + |V\rangle_0) + \\
& + |g_l\rangle_1|g_l\rangle_2|V\rangle_a(|H\rangle_0 - |V\rangle_0)] \otimes \\
& \otimes [|g_r\rangle_3|g_r\rangle_4|H\rangle_d|H\rangle_c + |g_l\rangle_3|g_l\rangle_4|V\rangle_c|V\rangle_d]. \qquad (6)
\end{aligned}
$$

As a result, after modes 0 and c passing through the polarization beam splitter PBS$_3$, the state of the system becomes

$$
\begin{aligned}
& \frac{1}{2}[|g_r\rangle_1|g_r\rangle_2|g_r\rangle_3|g_r\rangle_4|H\rangle_a|H\rangle_{1'}|H\rangle_{0'}|H\rangle_d + \\
& + |g_r\rangle_1|g_r\rangle_2|g_l\rangle_3|g_l\rangle_4|H\rangle_a|V\rangle_{0'}|V\rangle_{1'}|V\rangle_d + \\
& + |g_l\rangle_1|g_l\rangle_2|g_r\rangle_3|g_r\rangle_4|V\rangle_a|H\rangle_{1'}|H\rangle_{0'}|H\rangle_d - \\
& - |g_l\rangle_1|g_l\rangle_2|g_l\rangle_3|g_l\rangle_4|V\rangle_a|V\rangle_{0'}|V\rangle_{1'}|V\rangle_d], \qquad (7)
\end{aligned}
$$

where we only preserve the antibunching outcome with the probability $P_3 = 50\%$. At last, the four photons with different modes will, respectively, meet four HWPs (HWP$_2$, HWP$_3$, HWP$_4$ and HWP$_5$) four PBSs (PBS$_4$, PBS$_5$, PBS$_6$ and PBS$_7$). After passing through the four HWPs and four PBSs, if photon detectors $D_{m,1}$ ($m = 1, 2, 3, 4$) detect photons, we can obtain the cluster state as follows:

$$
\begin{aligned}
& \frac{1}{2}[|g_r\rangle_1|g_r\rangle_2|g_r\rangle_3|g_r\rangle_4 + |g_r\rangle_1|g_r\rangle_2|g_l\rangle_3|g_l\rangle_4 + \\
& + |g_l\rangle_1|g_l\rangle_2|g_r\rangle_3|g_r\rangle_4 - |g_l\rangle_1|g_l\rangle_2|g_l\rangle_3|g_l\rangle_4], \qquad (8)
\end{aligned}
$$

with the probability $P_4 = 1/2^4$. The maximal probability of getting the state is given by

$$
P = P_1^4 P_2 P_3 P_4 = \sin^8 \Omega t \times 25\% \times 50\% \times (50\%)^4 = \frac{1}{2^7}, \qquad (9)
$$

with $\sin^8 \Omega t = 1$.

IV. Discussions and Conclusions. The protocol given here is similar in spirit to that of Yu et al. [14]. Both protocols are based on the interference effect of light to generate entangled states rather than on the effective interaction between the atoms. The realization of our protocol is appealing due to the fact that photons are ideal carriers for transmitting quantum information over long distances, and the atoms are good memorizers for storing information long time. So we believe that the combination of the atom-cavity interaction and linear optics elements is a good idea to generate cluster states.

In an experimental scenario, our atomic-level structure can be achieved by Zeeman sublevels [15, 16] and has been realized to entangle two atoms [17]. What we used also consists of linear optical elements, and photon detectors, which have been widely used to entangled photons [18]. In particular, the similar optical setups have been used to successfully prepare W (GHZ) states of photons in experiment [19] ([20]). Experimental techniques for single-photon detection have made tremendous progress [21]. A photon detector based on a visible light photon counter has been reported, which can distinguish between single-photon incidence and two-photon incidence with high quantum efficiency, good time resolution, and low bit-error rate [22]. Therefore our protocol is feasible by current technologies.

Now, let us briefly discuss the robustness of our protocol. (1) From above derivation for generating cluster states of four distant atoms, one can find that different λ_{lj} and λ_{rj} only reduce the efficiency, while the fidelity is not influenced et all. (2) The success of the protocol depends upon the detection of a photon leaking out of the cavity, and thus the fidelity is also not affected by the imperfection of the photon detectors. (3) Considering the cavity decay and atomic spontaneous emission, which reduce the efficiency, the same discussions as those in Ref.[13] are valid, which is not repeated here. So our protocol with the error below the threshold under realistic conditions.

In summary, by using single-photon interference, we have shown how to generate cluster states with the aid of cavity-assisted photon scattering. Our protocol are robust against the asynchronous emission of photons. With the rapid development of single photon experimental technology, we hope our protocol can be achieved in the near future.

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