Pis'ma v ZhETF, vol. 89, iss. 12, pp. 718 – 724

Coloured noise controlled dynamics of nonlinear polaritons in semiconductor microcavity

N. S. Maslova, R. Johne, N. A. Gippius*†*

Moscow State University, Department. of Physics, 119992, Moscow, Russia

**LASMEA, UMR 6602 CNRS, Université Blaise Pascal, 63177 Aubière, France*

†A.M. Prokhorov General Physics Institute RAS, 119991 Moscow, Russia

Submitted 13 May 2009

The coloured noise induced escape rate from the lower energy stable state of a driven nonlinear microcavity oscillator has been investigated by means of quasi-classical kinetic equations. We show that for coloured, i.e. narrow-band, relatively intense noise, the escape time is controlled by the interplay of two mechanisms: the noise induced drift and adiabatic regular shift of the oscillator state towards unstable saddle point. The crossover between these mechanisms takes place in a particular range of the driving field intensity values, depending on the ratio between the oscillator damping and the coloured noise spectrum width. The dependence of the transition rate on the noise correlation time is analyzed for wide range of correlation time values.

PACS: 02.50.Fz, 42.65.–k, 71.36.+c

1. Introduction. The nonlinear dynamics of exciton-polaritons in semiconductor microcavities have been a topic of intensive research in recent years. This type of polaritons is formed due to strong coupling of a microcavity electromagnetic resonance with the exciton resonance of embedded quantum wells [1].

If the external CW-pump frequency exceeds the lower polariton branch on more than its linewidth, the polariton field amplitude can show bistable [2 – 4] or multistable behaviour [5]. This is illustrated in Fig.1 by

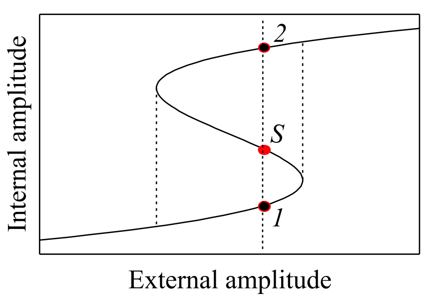

Fig.1. Schematical representation of the bistable region of nonlinear oscillator with two stable points *1* and *2* and one unstable point S. The dotted lines show the bistability jumps in forward and backward direction. The dashed line shows the point of the calculation

the so called S-shape curve. The low(high) energy stable point labelled with "1"("2") are separated by a unstable point "S" in between them. Due to inevitable fluctuations of the reservoir, the system can pass the bistability jump not at the turning point of S-shaped curve (dotted lines) but at intermediate intensities of the pump field (dashed line). After such transition the electromagnetic field inside the resonator grows strongly and significantly modifies all nonlinear effects e.g. polariton parametric scattering.

The physical origin of the fluctuations in a driven polariton system is either "internal" interaction with the polariton and phonon reservoir or the fluctuations of the "external" driving laser field. Being of different origins these fluctuations can show strongly different correlation properties. The correlation time of the "internal" fluctuations can be estimated by the inverse spectral width of polariton distribution, and their intensity depends on the occupation of polaritons reservoir. Both intensity and correlation time of these fluctuations depend on cavity temperature, excitation conditions and can be affected by the additional non-resonant pumping.

On the other hand the statistics of the "external" driving laser field is known to depend upon particular design of a laser and can vary strongly from one experimental setup to another.

For white noise the escape time (i.e. inverse transition rate) is determined by a large number of random force independent fluctuations, which result in pushing the system towards unstable point. The behavior of the system in this case can be treated as Markovian random process and can be obtained by solving the kinetic equation for the system probability distribution function of Fokker–Planck type.

Fluctuation induced transitions between different stable states of driven oscillator system have been addressed by different approaches [6 – 11]. One of the possible methods to analyze this problem is based on solving master equation for density operator [6]. However this approach allows to obtain reasonable results only for zero temperatures. Moreover, as the detailed balance conditions are not valid for driven polariton systems, it is impossible to find exact stationary state solution of master equation. For zero temperatures a stationary state solution has been approximately obtained by calculating numerically eigenfunctions of the master equation [6]. The linearized master equation has been also used to obtain approximate stable state probabilities [7]. The above mentioned methods can be used only for white noise case. Another approach is based on 2D complex Fokker–Planck (FP) equation for density probability function in coherent state representation [7]. But it is not possible to find exact solution of 2D FP equation for the case of bistable nonlinear driven oscillator – polariton systems. Our approach based on Keldysh diagram technique [12] allows analyzing the influence of finite noise correlation time on transition rates of bistable system and revealing different mechanisms governing the transition rates. In the frame of our approach we can reduce the kinetic equation to 1D FP equation in quasienergy space in quasiclassical limit.

In our previous papers [13, 14] we have shown that in the presence of a white noise the bistability transition of polariton nonlinear oscillator becomes allowed in the vicinity of the critical pump intensities. The escape time from the lower to high polariton amplitude state was shown to be an exponential function of the ratio between the driving intensity deviation from its critical value and the noise intensity within the oscillator linewidth. For given amplitude of the noise its spectral intensity is proportional to the noise correlation time t_c. As a result in white nose limit, i.e. for the noise correlation times less then $\hbar/\Delta$ (Δ being the pump detuning from the polariton resonance), the oscillator escape time decreases exponentially with the increase of t_c.

In the other limiting case when noise correlation time is much larger than the polariton damping time one can use the adiabatic approximation. It is based on the assumption that the system completely adapt to the slow variations of the random force and jumps to the upper bistable state as soon as the random force amplitude exceeds its critical value. The probability of this "critical fluctuation" determines the transition rate. Thus, within adiabatic approximation averaging over random force distribution is not necessary for description of the system behavior and for the estimation of the transition rate. The latter can be found from the total probability of appearance of any noise fluctuation above the "critical" one.

What happens, if the narrow band colored noise correlation time becomes comparable with the polariton damping time? In this case the system cannot follow the external noise changes and kinetic description similar to that for white noise limit is necessary. The escape time as in case of white noise is determined by a large number of random force independent fluctuations, which result in pushing the system towards unstable point. How the kinetic equation has to be modified in order to obtain the reasonable values of the fluctuation induced transition rates from different stable states? Our paper partially clarifies these questions.

We consider the influence of coloured noise (random force with finite correlation time t_c) on the fluctuation induced transitions between different stable states of the driven nonlinear polariton oscillator.

2. Model. The behavior of the nonlinear oscillator in quasiclassical limit can be successfully analyzed in quasienergy state representation [15 – 17]. The effective hamiltonian in rotating wave approximation for slow varying amplitude can be written as

$$H = -\Delta a^\dagger a + \frac{\alpha}{4}(a^\dagger a)^2 - f(a^\dagger + a), \qquad (1)$$

where a is the operator of polariton amplitude, $\Delta = \hbar(\omega_l - \omega_{LP})$ is the energy detuning between the driving laser field quanta $\hbar\omega_l$ and the polariton resonance energy $\hbar\omega_{LP}$, α is a polariton nonlinearity constant and f is the effective driving field amplitude. Operators a, $a^\dagger$ correspond to the classical canonical slow variables a, a^* and the eigenvalues of H correspond to the quasienergy E in the classical approach. The hamiltonian (1) results in the following equation of motion for slow varying amplitude:

$$i\hbar\frac{da}{dt} = -\Delta a + \alpha a|a|^2 - f. \qquad (2)$$

Transformation to dimensionless variables $a\,\Delta/f \to a$, introducing damping ϑ and fluctuations $\xi(\tau)$ yields

$$i\frac{da}{d\tau} = -i\vartheta a - a + \beta a|a|^2 - 1 + \xi(t), \quad \tau = t\frac{\Delta}{\hbar}, \qquad (3)$$

and the corresponding dimensionless hamiltonian takes the form

$$H = -a^\dagger a + \frac{\beta}{4}(a^\dagger a)^2 - (a + a^\dagger), \quad \beta = \frac{\alpha f^2}{\Delta^3}. \qquad (4)$$

This dimensionless form of the effective hamiltonian depends on a single parameter β which defines the shape of phase trajectories and the probabilities to find the oscillator in different stable quasienergy states.

In the absence of the fluctuations and damping the quasienergy is conserved. In classical limit behavior of nonlinear driven oscillator can be treated as precession around the stable state with the characteristic period $T(E)$ [18, 19].

The interaction of the nonlinear oscillator with coloured noise is described by an operator

$$\hat{V}(\tau) = \hat{f}(\tau)\hat{a}^\dagger + \hat{f}^\dagger(\tau)\hat{a}, \tag{5}$$

with a symmetrized correlation function of $\hat{f}(\tau)$ given by

$$\langle \hat{f}^+(\tau)\hat{f}(\tau') + \hat{f}(\tau)\hat{f}^+(\tau')\rangle = Q\tau_c^{-1}\exp(-|\tau-\tau'|/\tau_c). \tag{6}$$

Therein τ_c denotes the coloured noise dimensionless correlation time.

The dimensional values of noise spectral density (σ^2), pump amplitude (f), pump detuning (Δ), oscillator dephasing (γ) and nonlinearity constant (α) and escape time t_1 can be obtained by the relations

$$\beta = \frac{\alpha f^2}{\Delta^3}, \quad \vartheta = \frac{\gamma}{\Delta}, \quad Q = \frac{\sigma^2}{f^2}\frac{\Delta}{\hbar}, \quad t_1 = \tau_1\frac{\hbar}{\Delta}. \tag{7}$$

In the following, we will use the notation τ and Γ for dimensionless variables and t and γ for dimensional values.

3. Adiabatic noise. As we have mentioned above in the limit $\tau_c \gg T(E)$, i.e. very slow variation of the noise compared to oscillator precession around the stable state, the noise induced transitions between these stable states of the nonlinear driven oscillator is described by adiabatic approximation. Within this approximation the adiabatic modification of quasienergy states with changes of parameter $\beta = \alpha f^2/\Delta^3$ takes place. If $\beta = \beta_c$, the stable state "1" with quasienergy $E_1(\beta_c)$ and the unstable state "S" with quasienergy $E_S(\beta_c)$ coincide and the system transfers to the state "2" – the only one stable state for β exceeding its critical value β_c.

For Gaussian noise, the probability of the fluctuation with amplitude δf is

$$P(\delta f) \propto \exp\left[-\frac{(\delta f)^2}{\langle(\delta f)^2\rangle}\right], \quad \langle\delta f\rangle = 0. \tag{8}$$

Taking into account that for fixed detuning and nonlinearity

$$\frac{(\delta f)_c^2}{f^2} = \frac{(f_c - f)^2}{f^2} = \frac{(\sqrt{\beta}-\sqrt{\beta_c})^2}{\beta} \tag{9}$$

the probability of the critical fluctuation of β from its initial to the critical value β_c is

$$P((\delta f)_c) \propto \exp\left[-\frac{f^2}{\langle(\delta f)^2\rangle}\frac{(\sqrt{\beta}-\sqrt{\beta_c})^2}{\beta}\right]. \tag{10}$$

This means that transition time t_1^{ad} in the adiabatic limit reads as follows

$$t_1^{ad} \propto t_c \exp\left[\frac{f^2}{\langle(\delta f)^2\rangle}A_{ad}\right], \tag{11}$$

where $A_{ad}(\beta) = (\sqrt{\beta}-\sqrt{\beta_c})^2/\beta$.

We want to stress once again that this linear dependence of the transition time on noise correlation time τ_c is based on the assumption that the system completely adapt the slow variations of the random force. With decreasing τ_c the system less and less adapts the changes of the driving field and it can show qualitatively different dependence of transition time on noise correlation time. The crossover between these dependencies is discussed in the next sections of the paper.

4. Finite noise correlation time. For finite correlation time of the noise one should deal with the kinetic equation for the nonlinear oscillator probability distribution function.

For coloured noise described as a set of harmonic modes linearly connected with the driven oscillator [18, 13] by the coupling constant $\tilde{\lambda}$, the correlation function Eq.(6) corresponds to the changes of the noise oscillator density of states $\nu \to \nu\Gamma_c^2/\omega^2 + \Gamma_c^2$, where ν is the constant density of states for the case of white noise and $\Gamma_c = 1/\tau_c$ is the spectral width of the coloured noise.

Using the Keldysh diagram technique [12] we obtain the kinetic equation for the probability distribution function of the oscillator in quasi-energy representation. The interaction with the thermal bath is supposed to be weak enough, i.e. within the quasi-classical limit $E_k - E_{k-1} = \omega(E_k) \gg \vartheta$, where E_k are the quasienergy levels characterized by large integer quantum numbers k, and ϑ is the width of the quasienergy levels.

The contribution of the the coloured noise to the kinetic equation depends on the functions $\tilde{D}^<_{kk'}(\omega)$, $\tilde{D}^>_{kk'}(\omega)$, which are determined as

$$\tilde{D}^{\lessgtr}_{kk'}(\omega) = D^{\lessgtr}_{kk'}(\omega)\frac{\tilde{\lambda}^2}{\lambda^2}\frac{\Gamma_c^2}{\omega^2+\Gamma_c^2}. \tag{12}$$

The functions $D^{\lessgtr}_{kk'}(\omega)$ correspond to the case of white noise ($\tau_c = 0$) with a coupling constant λ and have been derived in our previous paper [13]:

$$D^<_{k,k'}(\omega) = \lambda^2\nu\left\{|a_{k,k'}|^2(N_{\omega_l-\omega}+1) + |a_{k',k}|^2 N_{\omega_l+\omega}\right\},$$

$$D^>_{k,k'}(\omega) = \lambda^2\nu\left\{|a_{k,k'}|^2 N_{\omega_l-\omega} + |a_{k',k}|^2(N_{\omega_l+\omega}+1)\right\}.$$

The quantity N_ω is the filling number of noise modes with frequency ω. After integrating over ω the total kinetic equation can be written as

$$\frac{\partial n_k}{\partial \tau} = \sum_{k'} \left(\overline{D}^{<}_{k,k'} + D^{<}_{k,k'} \right) n_{k'} - \left(\overline{D}^{>}_{k,k'} + D^{>}_{k,k'} \right) n_k, \tag{13}$$

with

$$\overline{D}^{\lessgtr}_{kk'}(E_k - E_{k'}) = \tag{14}$$

$$= D^{\lessgtr}_{kk'}(E_k - E_{k'}) \frac{\tilde{\lambda}^2}{\lambda^2} \frac{\Gamma_c(\Gamma_c + \vartheta)}{(E_k - E_{k'})^2 + (\Gamma_c + \vartheta)^2}.$$

The source of the coloured and white noise in a polariton system can be different resulting in essentially different effective temperatures. If one neglect the changes of quasienergy states white noise case ($\Gamma \to \infty$) the equation is similar to the master equation for density operator obtained in [6]. Assuming that the intensity of coloured noise strongly exceeds the intensity of white noise, the kinetic equation can be transformed to a Fokker-Planck equation in the quasi-classical limit for each region of phase space

$$\frac{\partial n_i(E)}{\partial \tau} = \frac{\partial}{\partial E} \left((\tilde{\vartheta} \tilde{K} + \vartheta K) n_i(E) + Q \tilde{D} \frac{\partial n_i(E)}{\partial E} \right), \tag{15}$$

where

$$Q = \frac{\eta}{2}(2N_{\omega_l} + 1)\vartheta, \quad \vartheta = \frac{\lambda^2 \nu}{\Delta}, \quad \tilde{\vartheta} = \frac{\tilde{\lambda}^2 \nu}{\Delta}. \tag{16}$$

The coefficient $\eta = \Delta^2/f^2$ arises from the transition to dimensionless variables (a, a^*).

The kinetic equation (13) describes the relaxation of the nonlinear driven microcavity oscillator at times much greater then Γ_c^{-1} and ϑ^{-1}, and the value of the escape time from the lower energy stable state of the nonlinear driven microcavity oscillator t_1 can be determined by means of the equation (15) if $\tau_1(Q) \gg \Gamma_c^{-1}, \Gamma^{-1}$. This condition leads to the restriction that the dimensionless noise intensity should not be too strong.

The expressions for effective drift $K(E)$ and diffusion $D(E)$ coefficients of nonlinear driven oscillator in the presence of white noise were derived in our previous paper Ref.[13]:

$$K(E) = \frac{1}{2iT(E)} \oint_{C(E)} (a da^* - a^* da), \tag{17}$$

$$D(E) = \frac{1}{2iT(E)} \oint_{C(E)} \left(\frac{\partial H}{\partial a} da - \frac{\partial H}{\partial a^*} da^* \right).$$

The functions $\tilde{K}$ and $\tilde{D}$ in Eq.(15) differ from the case of interaction with white noise and depend on the correlation time τ_c. To determine $\tilde{K}$ and $\tilde{D}$ one has to derive the quasi-classical limit of the following expressions:

$$\tilde{K} = \omega(E_k) \sum_q |a_{k,k+q}|^2 q \frac{\Gamma_c(\Gamma_c + \vartheta)}{\omega^2(E_k) q^2 + (\Gamma_c + \vartheta)^2}, \tag{18}$$

$$\tilde{D} = \omega^2(E_k) \sum_q |a_{k,k+q}|^2 q^2 \frac{\Gamma_c(\Gamma_c + \vartheta)}{\omega^2(E_k) q^2 + (\Gamma_c + \vartheta)^2}. \tag{19}$$

We can distinguish two limiting cases: narrow band colored noise ($\Gamma_c \ll \omega(E_k)$) and nearly white noise ($\Gamma_c \gg \omega(E_k)$), where $\omega(E_k)$ is the classical frequency of the oscillator precession along the phase trajectory with quasienergy E_k.

5. Colored noise. For the first case with $\Gamma_c \ll \ll \omega(E_k)$ that should be compared with adiabatic approximation the formula for $\tilde{K}$ can be transformed to

$$\tilde{K}/\omega(E_k) = \tag{20}$$

$$= \sum_q |a_{k,k+q}|^2 q \frac{\Gamma_c(\Gamma_c + \vartheta)}{\omega^2(E_k) q^2 + (\Gamma_c + \vartheta)^2} +$$

$$= \sum_q \frac{q \Gamma_c(\Gamma_c + \vartheta)(2\pi\omega(E_k))^2}{\omega^2(E_k) q^2 + (\Gamma_c + \vartheta)^2} \left| \int_0^{T(E_k)} a(\tau) e^{i\omega(E_k) q \tau} d\tau \right|^2.$$

Let us define the function $F(\tau)$ by the following mathematical conditions:

$$i\dot{F} = a(\tau) - \langle a \rangle, \quad \int_0^{T(E_k)} F(\tau) d\tau = 0, \tag{21}$$

where

$$\langle a \rangle = \frac{1}{T(E_k)} \int_0^{T(E_k)} a(\tau) d\tau.$$

Then, in the quasiclassical limit, Eq.(20) can be transformed into

$$\tilde{K} = \frac{\Gamma_c(\Gamma_c + \vartheta)}{2iT(E_k)} \times \tag{22}$$

$$\times \int_0^{T(E_k)} \{(a(\tau) - \langle a \rangle) F^*(\tau) - (a^*(\tau) - \langle a^* \rangle) F(\tau)\} d\tau.$$

Using the same approximations, the function $\tilde{D}(E_k)$ can be determined in a similar way as

$$\tilde{D} = \frac{\Gamma_c(\Gamma_c + \vartheta)}{T(E_k)} \int_0^{T(E_k)} |a(\tau) - \langle a \rangle|^2 d\tau. \tag{23}$$

Finally, substituting $D \to \tilde{D}$ and $K \to K + \tilde{K}\tilde{\vartheta}/\vartheta$ in Eq. (32) of Ref.[13] we obtain the fluctuations induced escape time in the presence of a narrow band coloured noise:

$$\tau_1 \propto \vartheta^{-1} \left| \exp \frac{\vartheta}{2Q} \int_{E_1}^{E_S} dE \frac{K + \tilde{K}\,\tilde{\vartheta}/\vartheta}{\tilde{D}} \right|^2 . \quad (24)$$

For the considered system of cavity polaritons with a considerable damping due to escape through the cavity mirrors it is reasonable to assume that $\tilde{\vartheta}/\vartheta \ll 1$. This means that the second term in the numerator within the integral can be neglected.

Let us introduce the factor A_{cn} depending only on the parameter β

$$A_{cn}(\beta) = \Gamma_c(\Gamma_c + \vartheta) \int_{E_1}^{E_S} \frac{K}{\tilde{D}} dE. \quad (25)$$

For given detuning and nonlinearity parameter β is determined by the intensity of external force f^2 (see Eq.(7)).

We would like to stress that the modifications of quasienergy states caused by the coloured noise are ignored in this equation. Within this approximation we obtain the escape time, depending on dimensionless parameters β and the ratio $Q(\Gamma_c + \vartheta)\Gamma_c/\vartheta$. Returning to dimensional values we obtain

$$\frac{Q}{\vartheta} = \frac{\langle(\delta f)^2\rangle t_c \Delta^2}{f^2 \hbar\gamma}, \quad \tau_c = t_c\Delta/\hbar, \quad \sigma^2 = \langle(\delta f)^2\rangle t_c, \quad (26)$$

and

$$\frac{Q(\Gamma_c + \vartheta)\Gamma_c}{\vartheta} = \frac{\langle(\delta f)^2\rangle}{f^2} \frac{\gamma_c + \gamma}{\gamma}, \quad \gamma_c = \hbar/t_c, \quad (27)$$

where δf is the fluctuation amplitude of coloured noise.

Thus the dimensional escape time within narrow band colored noise approximation is given by

$$t_1^{cn} \propto \hbar\gamma^{-1} \exp\left[\frac{f^2}{\langle(\delta f)^2\rangle} \frac{\gamma}{\gamma_c + \gamma} A_{cn}(\beta)\right]. \quad (28)$$

6. Crossover. Let us now compare the results obtained from the adiabatic approximation and the narrow band colored noise approximation for estimations of escape time. If $t_1^{cn} < t_1^{ad}$, the colored noise approximation should be used to determine the escape time of nonlinear driven oscillator from the state "1". In the opposite case $t_1^{cn} > t_1^{ad}$ the adiabatic approach describes the system escape from the stable state "1". The crossover between these two approximations occurs when

$$\frac{\gamma}{\gamma + \gamma_c} A_{cn}(\beta) \propto A_{ad}(\beta) = (\sqrt{\beta} - \sqrt{\beta_c})^2/\beta \quad (29)$$

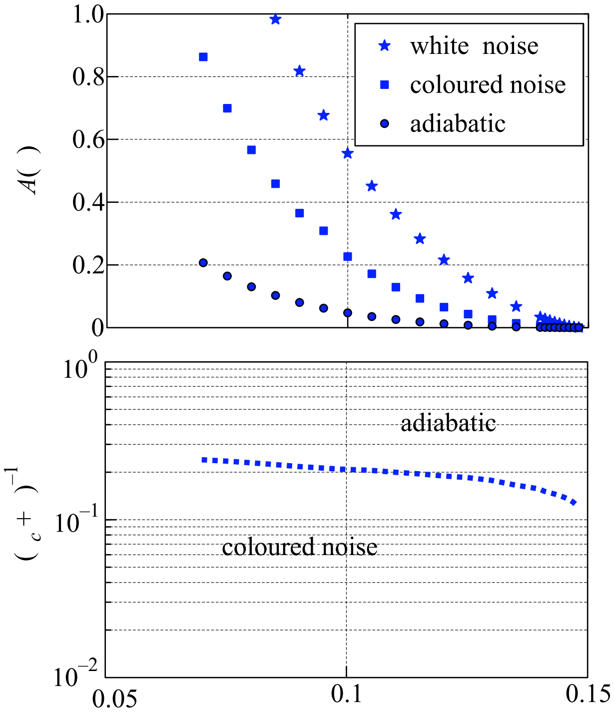

Fig.2. Top panel: β-dependence of escape time exponential factors for three different approximations of fluctuation induced transitions: white noise (Eq. (30)), narrow-band colored noise (Eq. (25)) and adiabatic (Eq. (11)). Bottom panel: schematic phase diagram for the crossover between colored noise and adiabatic approximations

The escape time exponential factor dependence on parameter β are shown in Fig.2 for the three different approximations of fluctuation induced transitions: white noise [13] (and Eq. (30)), narrow-band colored noise (Eq. (25)) and adiabatic (Eq. (11)). The bottom panel in Fig.2 depicts the boundary of the system parameters, where the escape time obtained by narrow band colored noise approximation exceeds the one obtained from adiabatic approach. Below the line the colored noise has shorter escape times, while above the adiabatic escape time is the shorter one. One can see that in wide range of β this boundary is almost constant at $\gamma/(\gamma + \gamma_c) = 0.2$. The escape times dependencies on the noise correlation time t_c for $\beta = 0.09$ are shown in Fig.3. As it can be seen from Fig.3, for large enough noise correlation time the oscillator escape time from the initial state to the higher energy stable state due to narrow band colored noise mechanism (squares) becomes larger than the expectation time of the external noise large fluctuation (circles) that will course the adiabatic escape. This crossover is calculated for Gaussian-like distribution of fluctuation amplitudes. In case of more narrow than Gaussian dis-

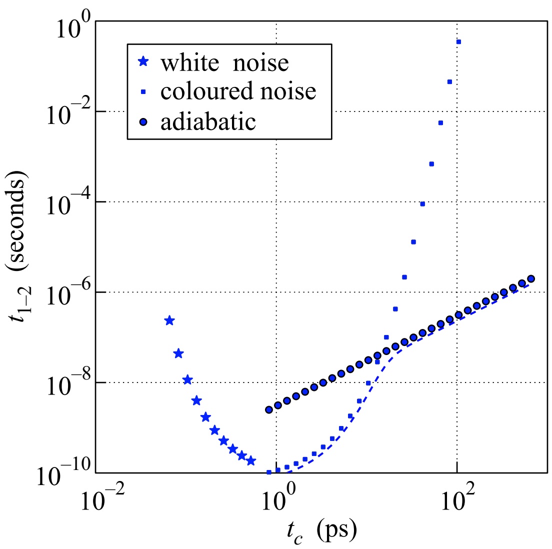

Fig.3. Escape time for three different noise regimes: stars – white noise, squares – coloured noise and circles – adiabatic regime. All data is calculated for relative noise amplitude $\delta f/f = 0.1$, oscillator linewidth $\gamma = 0.01$ meV and parameter $\beta = 0.09$, corresponding to $I/I_c = \beta/\beta_c = 0.61$. The dashed line is a guide for the eye of the crossover discussed in the text

tribution, the crossing point moves towards the larger times.

7. Quasi-white noise. With further decrease of t_c below precession period $T(E)$ we should use the quasi-white noise approximation for description of the oscillator transitions between different stable states. The escape time in the case of white noise is given by [13]:

$$t_1^{wn} \propto \hbar\gamma^{-1} \exp\left[\frac{f^2}{\sigma^2\Delta^2/\hbar\gamma} A_{wn}(\beta)\right], \quad \sigma^2 = \langle(\delta f)^2\rangle t_c, \tag{30}$$

where

$$A_{wn}(\beta) = \int_{E_1}^{E_S} \frac{K}{D} dE,$$

with K and D defined by Eq.(17).

The stars in Fig.3 are plotted assuming the white noise approximation, i.e. very small noise correlation times compared to detuning. It can be shown that accounting for finite noise correlation time does not modify significantly these estimations. Indeed, in the limiting case $\Gamma_c \gg \omega(E_k)$ the functions K and D acquire small corrections connected with the noise finite correlation time:

$$\delta K(E_k) \propto \frac{\omega^2(E_k)}{\Gamma_c^2} K(E_k), \qquad \delta D(E_k) \propto \frac{\omega^2(E_k)}{\Gamma_c^2} D(E_k).$$

The correction to the exponential factor of escape time for quasi-white noise can be obtained from the above formulas and Eq. (30) replacing σ^2 by $\langle(\delta f)^2\rangle t_c$ and $A_{wn}(\beta)$ by $A_{wn}(\beta) + \delta A_{wn}(\beta)$. This correction is expressed as $\delta A_{wn}(\beta) = \delta \int_{E_1}^{E_S} K/DdE$ and can be estimated as $A_{wn}\Delta^2/\gamma_c^2$. Thus the correction to the exponential factor of escape time is of the order

$$\delta \ln(t_1^{wn}) \propto \frac{\gamma}{\gamma_c} \frac{f^2}{\langle(\delta f)^2\rangle} A_{wn}. \tag{31}$$

For quasi-white noise ($\gamma_c \gg \gamma, \Delta$) this correction is not essential.

8. Conclusion. To conclude we would like to stress that the switching between the stable states of driven polariton systems can be caused not only by the "control" signals but also by fluctuations in the polariton reservoir and pump beam. We have analyzed the fluctuation induced transitions of nonlinear driven oscillator for different types of noise spectra. We have shown that for coloured, i.e. narrow-band, relatively intense noise the escape time is controlled by the interplay of two mechanisms. The adiabatic mechanisms is based on the assumption that the system follows the slow variations of the random force. It is realised if the correlation time of random force is much greater than the inverse dumping constant. In such situation the estimation of the escape time can be obtained from the distribution function of the random force. No averaging over noise distribution function is needed.

With decreasing of noise correlation time, the system can not follow the changes of random force and the adiabatic mechanism is not valid. One should deal with kinetic equations to determine the distribution function of the system after averaging over noise fluctuations. The crossover of these mechanisms takes place in particular range of the driving field intensity values, depending on the ratio between the oscillator damping and the coloured noise spectrum width.

Keldysh diagram technique allows calculating both limits (white noise and adiabatic approximation). For adiabatic approximation we have to calculate the changes of quasi-energy spectrum (i.e. the poles of the retarded Green functions) caused by the external noise, considered as external regular force. When the noise correlation time is comparable with the oscillator damping, it is possible to use kinetic equation without vertex corrections and to obtain the reasonable results if $\gamma \ll \Delta$. The dependence of the escape time on the noise correlation time (τ_c) differs strongly for two limiting cases of small and large τ_c. For narrow-band coloured noise, the escape time grows with the increase of the noise correlation time contrary to the quasi-white noise case, for which it decreases. For a given noise intensity it is reasonable to expect that the minimal value of escape times can be achieved for noise correlation times comparable with the inverse detuning of the pump frequency from the oscillator resonance.

This work was financially supported in part by the ANR Chair of Excellence Program, the Russian Foundation for basic research and Russian Ministry of Science. The authors thank S.G. Tikhodeev, S.S. Gavrilov, G. Malpuech and D. Solnyshkov for fruitful discussions.

---

1. C. Weisbuch, M. Nishioka, A. Ishikawa, and Y. Arakawa, Phys. Rev. Lett. **69**, 3314 (1992).
2. V. F. Elesin and Y. V. Kopaev, Sov. Phys. JETP **36**, 767 (1973).
3. A. Baas, J. P. Karr, H. Eleuch, and E. Giacobino, Phys. Rev. A **69**, 023809 (2004).
4. D. Whittaker, Superlattices and Microstructures **41**, 297 (2007).
5. N. A. Gippius, I. A. Shelykh, D. D. Solnyshkov et al., Phys. Rev. Lett. **98**, 236401 (2007).
6. H. Risken, C. Savage, F. Haake, and D. F. Walls, Phys. Rev. A **35**, 1729 (1987).
7. P. D .Drummond and D. F. Walls, J. Phys. A. **13**, 725 (1980).
8. K. Vogel and H. Risken, Phys. Rev. A **42**, 627 (1990).
9. H. Risken and K. Vogel, Phys. Rev. A **38**, 1349 (1988).
10. K. Vogel and H. Risken, Phys. Rev. A **39**, 4675 (1989).
11. K. Vogel and H. Risken, Phys. Rev. A **38**, 2409 (1988).
12. L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)].
13. N. S. Maslova, R. Johne, and N. A. Gippius, Pis'ma v ZhETF **86**, 135 (2007).
14. R. Johne, N. S. Maslova, and N. A. Gippius, Solid State Commun. **149**, 496 (2009).
15. V. Sazonov and S. Zatsepin, Teor. Mat. Fiz. **41**, 111 (1979).
16. M. Kuzmin and V. Sazonov, Zh. Exp. Theor. Fiz. **79**, 1759 (1980) [Sov. Phys. JETP **52**, 889 (1980)].
17. P. V. Elyutin, Soviet Physics Doklady **29**, 936 (1984).
18. N. Maslova, Sov. Phys. JETP **64**, 537 (1986).
19. M.I. Dykman, Sov. Phys. JETP **64**, 927 (1986).