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Hydrodynamical description of a hadron-quark first-order phase transition

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Solutions of hydrodynamical equations are presented for the equation of state of the Van der Waals type allowing for a first-order phase transition. As an example we consider the hadron-quark phase transition in heavy-ion collisions. It is shown that fluctuations dissolve and grow as if the fluid is effectively very viscous. In the vicinity of the critical point even in spinodal region seeds are growing slowly due to viscosity, surface tension and critical slowing down. These non-equilibrium effects prevent enhancement of fluctuations in the near-critical region, which in thermodynamical approach is frequently considered as a signal of the critical endpoint in heavy-ion collisions.

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There are many phenomena, where first-order phase transitions occur between phases with different densities. Description of such phenomena should be similar to that for the gas-liquid phase transition. Thereby it is worthwhile to find corresponding solutions of hydrodynamical equations. Though some simplified analytical [1, 2] and fragmentary two-dimensional numerical solutions [3] have been found, many problems remain unsolved. In nuclear physics different first-order phase transitions (e.g., to pion, kaon condensates and to the quark state) may occur in neutron stars [2, 4] and in heavy-ion collisions [5, 6]. At low energies gas-liquid transition occurs [5]. It is also expected that at finite baryon density the hadron – quark gluon plasma (QGP) phase transition, which might manifest itself in violent nucleus-nucleus collisions, is of the first-order [6]. The hydrodynamical approach is efficient for description of heavy-ion collisions in a broad energy range (e.g. see [7, 8, 6]).

In this letter the dynamics of a first-order phase transition is described by equations of non-ideal non-relativistic hydrodynamics: the Navier-Stokes equation, the continuity equation, and general equation for the heat transport. We solve these equations numerically in two spatial dimensions, $d = 2$, and analytically for arbitrary d in the vicinity of the critical point. Then we perform estimations for the case of the hadron – QGP transition.

The best known example to illustrate principal features of a first-order phase transition is the Van der Waals fluid. The pressure is given by $P_{\rm VW}[n,T] = nT/(1-bn) - n^2 a$, where T is the temperature, n is the density of a conserving charge (e.g., the baryon charge), parameter a governs the strength of a mean field attraction and b controls a short-range repulsion.

We expand the quantities entering EoS and equations of hydrodynamics near a reference point $(\rho_{\rm r}, T_{\rm r})$ chosen somewhere in the vicinity of the critical point on the plane $P(\rho,T)$, where $\rho = mn$ is the mass density, m is the mass of the constituent. Assuming smallness of the velocity $\mathbf{u}(\mathbf{r},\tau)$ of the seed we linearize hydrodynamical equations in u, density $\delta\rho = \rho - \rho_{\rm r}$ and temperature $\delta T = T - T_{\rm r}$. Applying then operator "div" to the Navier-Stokes equation and taking $z = \operatorname{div}\mathbf{u}$ from the continuity equation we obtain [1, 2]:

$$\frac{\partial^2 \delta\rho}{\partial t^2} = \Delta\left[\delta P + \rho_{\rm r}^{-1}\left(\tilde{d}\eta_{\rm r} + \zeta_{\rm r}\right)\frac{\partial \delta\rho}{\partial t}\right], \qquad (1)$$

$\tilde{d} = 2(d-1)/d$, $\delta P = P - P[\rho_{\rm r}, T_{\rm r}] = \rho_{\rm r}\frac{\delta[F(\delta\rho,T)]}{\delta(\delta\rho)}|_T$ is the pressure expressed through the free energy F for slightly inhomogeneous configurations; $\eta_{\rm r}$ and $\zeta_{\rm r}$ are shear and bulk viscosities; $\Delta = \partial^2_{x_1} + ... + \partial^2_{x_d}$.

Note that *derived Eq. (1) differs from the phenomenological Landau equation* for the nonconserving order parameter $\partial_t\phi = -\gamma(\delta F/\delta\phi)$, $\gamma = \text{const}$, and from equations used for the description of the dynamics of first-order phase transitions in heavy-ion collisions [9] and in relativistic astrophysical problems [10]. The difference with the Landau equation disappears, if one sets

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zero the square bracketed term in the r.h.s. of Eq. (1). From the first glance, such a procedure is legitimate, if space-time gradients are small. However for a seed, being prepared in a fluctuation at $t = 0$ with a distribution $\delta\rho(t = 0, \mathbf{r}) = \delta\rho(0, \mathbf{r})$, the condition $\partial\delta\rho(t, \mathbf{r})/\partial t|_{t=0} \simeq 0$ should also be fulfilled (otherwise there appears a kinetic energy term). Two initial conditions cannot be simultaneously fulfilled, if the equation contains time derivatives of the first-order only. Thus, *there exists an initial stage of the dynamics of phase transitions ($t \lesssim t_{\rm init}$), which is not described by the standard Landau equation.*

For low velocities the heat transport is described by the heat conductivity equation $c_V \frac{\partial T}{\partial t} = \kappa \Delta T$, where κ is the heat conductivity and c_V is the specific heat. Time scale of the temperature relaxation is $t_T = R^2(t_T) c_V/\kappa$, where $R(t)$ is the size of the seed. On the other hand, time scale of the density relaxation, following Eq. (1), is $t_\rho \propto R$ (we show below that a seed of rather large size grows with constant velocity). Evolution of the seed is governed by the slowest mode. When sizes of seeds begin to exceed the value $R_{\rm fog}$, where $R_{\rm fog}$ is the size at which $t_T = t_\rho$, the growth is slowed down. Thus number of seeds with the size $R \sim R_{\rm fog}$ grows with time and *there appears a metastable state called the fog.*

We will consider phase transition for the system at fixed values of T and P at the boundary. For further convenience we choose $\rho_{\rm r} = \rho_{cr}$, $T_{\rm r} = T_{cr}$ and expand the Landau free energy in $\delta\rho$ and $\delta\mathcal{T}$:

$$\delta F = \int \frac{d^3x}{\rho_{\rm r}} \left[\frac{c[\nabla(\delta\rho)]^2}{2} + \frac{\lambda(\delta\rho)^4}{4} - \frac{\lambda v^2(\delta\rho)^2}{2} - \epsilon\delta\rho \right], \quad (2)$$

$$\delta F = F_L[\rho, T] - F_L[\rho_{\rm r}, T_{\rm r}].$$

Then

$$a = \frac{9}{8}\frac{T_{cr}}{n_{cr}}, \quad b = \frac{1}{3n_{cr}},$$

$$v^2 = -\frac{3m^2 T_{cr}\delta\mathcal{T}}{2ab} = 4|\delta\mathcal{T}| n_{cr}^2 m^2, \quad \mathcal{T} = (T - T_{cr})/T_{cr},$$

$$\lambda = \frac{3ab}{2m^3} = \frac{9}{16}\frac{T_{cr}}{n_{cr}^2 m^3}, \quad \epsilon = n_{cr}(\mu_1 - \mu_2),$$

where μ_1 and μ_2 are chemical potentials of initial and final configurations (at fixed T and P at the boundary of the system). Maximum value $\epsilon^{\max} = \sqrt{3} T_{cr} n_{cr} |\delta\mathcal{T}|^{3/2}$. In dimensionless variables $\delta\rho = v\psi$, $\xi_i = x_i/l$, $i = 1, \cdots, d$, $\tau = t/t_0$, we arrive at

$$-\beta\frac{\partial^2\psi}{\partial\tau^2} = \Delta_\xi \left(\Delta_\xi\psi + 2\psi(1 - \psi^2) + \tilde{\epsilon} - \frac{\partial\psi}{\partial\tau} \right), \quad (3)$$

$$l = \left(2c/(\lambda v^2)\right)^{1/2}, \; t_0 = 2(\tilde{d}\eta_{\rm r} + \zeta_{\rm r})/(\lambda v^2 \rho_{\rm r}),$$

$$\tilde{\epsilon} = 2\epsilon/(\lambda v^3), \; \tilde{\epsilon}^{\max} = 4/(3\sqrt{3}), \; \beta = c\rho_{\rm r}^2/[\tilde{d}\eta_{\rm r} + \zeta_{\rm r}]^2.$$

Thus $l \propto |\delta\mathcal{T}|^{-1/2}$ and $t_0 \propto |\delta\mathcal{T}|^{-1}$. With $\lambda' = \lambda m^2$, $v' = v/m$, $\eta' = \eta/m$, $\zeta' = \zeta/m$, $\epsilon' = \epsilon/m$, the dependence on the mass m can be excluded from all values in Eq. (3). Note that in (3) $\Delta_\xi\tilde{\epsilon} = 0$. We retained this term for convenience since then solutions (3) yield correct asymptotic for uniform configurations.

There exists an opinion, cf. Ref. [11], that, if at some incident energy the trajectory passes in the vicinity of the critical point, the system may linger longer in this region due to strong thermodynamical fluctuations resulting in the divergence of susceptibilities that may reflect on observables. Contrary, we argue that *fluctuational effects in the vicinity of the critical point in heavy-ion collisions can hardly be pronounced*, since all relevant processes are proved to be frozen for $\delta T \to 0$, while the system passes this region during a finite time.

To describe configurations of different symmetry we search two-phase solution of Eq. (3) in the form [1, 2],

$$\psi = \mp\tanh[\xi - \xi_0(\tau)] + \tilde{\epsilon}/4, \quad (4)$$

$\xi = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}$ for droplets/bubbles ($d_{\rm sol} = 3$), $\xi = \sqrt{\xi_1^2 + \xi_2^2}$ for rods ($d_{\rm sol} = 2$) and $\xi = \xi_1 = x/l$ for kinks ($d_{\rm sol} = 1$) in $d = 3$ space. For $\tilde{\epsilon} > 0$ upper sign solution describes evolution of droplets (or rods and kinks of liquid phase) in a metastable super-cooled vapor medium. The lower sign solution circumscribes then bubbles (or kinks and rods of gas phase) in a stable liquid medium.

The boundary layer has the length $|\xi - \xi_0(\tau)| \sim 1$. Outside this layer corrections to homogeneous solutions are exponentially small. Considering motion of the boundary for $\xi_0(\tau) \gg 1$ we may put $\xi \simeq \xi_0(\tau)$ in (4). Then keeping only linear terms in $\tilde{\epsilon}$ in Eq. (3), we arrive at equation

$$\frac{\beta}{2}\frac{d^2\xi_0}{d\tau^2} = \pm\frac{3}{2}\tilde{\epsilon} - \frac{d_{\rm sol} - 1}{\xi_0(\tau)} - \frac{d\xi_0}{d\tau}. \quad (5)$$

Substituting (4) in (2) we obtain

$$\delta F[\xi_0] = \frac{2\pi^{3/2}\Lambda^{3-d_{\rm sol}}\lambda v^4 l^{d_{\rm sol}}}{\Gamma(d_{\rm sol}/2)\Gamma(1 + (3 - d_{\rm sol})/2)\rho_{\rm r}} \times$$
$$\times \left[\mp\tilde{\epsilon}\xi_0^{d_{\rm sol}}/d_{\rm sol} + 2\xi_0^{d_{\rm sol}-1}/3 \right], \quad (6)$$

2Λ is the diameter, height of cylinder and the length of the squared plate for $d_{\rm sol} = 3, 2$ and 1, respectively; Γ is the Euler Γ-function. The first term in (6) is the volume term and the second one is the surface contribution, $\delta F_{\rm surf}$. At fixed volume in $d = 3$ space, the surface contribution for droplets/bubbles is smaller than for rods and slabs. Thereby if a seed prepared in a fluctuation is initially nonspherical it acquires spherical form with passage of time. Surface term is $\delta F_{\rm surf} \equiv \sigma S$, S is the surface of the seed, σ is the surface tension, and the gradient term in (2) is then $\delta F^{\rm grad}_{\rm surf} = \frac{2v^2c}{3l\rho_r}S = \frac{1}{2}\delta F_{\rm surf}$. Thus we are able to find relations: $\sigma = \sigma_0|\delta\mathcal{T}|^{3/2}$, $\sigma_0^2 = 32mn_{cr}^2T_{cr}c$; $l = \frac{\sigma_0}{6T_{cr}n_{cr}|\delta\mathcal{T}|^{1/2}}$. There are two dimensionless parameters in (3) and (5): $\widetilde{\epsilon}$ and β. The value $\widetilde{\epsilon}$ distinguishes metastable and stable state minima in the Landau free energy, $\beta = (32T_{cr})^{-1}[\tilde{d}\eta_{\rm r}+\zeta_{\rm r}]^{-2}\sigma_0^2 m$ controls dynamics. For $\beta \ll 1$ one deals with effectively viscous fluid and at $\beta \gg 1$, with perfect fluid. *Thus the smaller surface tension, the effectively more viscous is the fluidity of seeds.*

Using Eq. (5) we can consider analytically several typical solutions for evolution of seeds of stable phase in metastable matter.

1) *Short time evolution of a seed.* For small τ (initial stage) using Taylor expansion in τ and assuming zero initial velocity, $\frac{d\xi_0}{d\tau}|_{\tau=0} \simeq 0$, we obtain

$$R(t) \simeq R_0 + (wt^2/2)\,[1 - 2t/(3t_0\beta)]$$

valid for $t \ll \left(\frac{R_0}{w}\right)^{1/2}$, $t \ll t_{\rm init} = \frac{2(\tilde{d}\eta_{\rm r}+\zeta_{\rm r})\beta}{\lambda v^2 m n_{cr}} \propto \propto \frac{\sigma_0^2}{(\tilde{d}\eta_{\rm r}+\zeta_{\rm r})|\delta\mathcal{T}|}$. Initial stage of the process proceeds with acceleration

$$w = (d_{\rm sol}-1)\lambda v^2\,(R_0 - R_{cr})\,/(R_0R_{cr}),$$

which changes sign at the initial size $R_0 = R_{cr}$, where

$$R_{cr} = (d_{\rm sol}-1)v^2\sqrt{2c\lambda}/(3|\epsilon|),\ R_{cr}(\epsilon^{max}) \propto 1/|\delta\mathcal{T}|^{1/2},$$

is the critical size. Seeds with $R_0 < R_{cr}$ shrink, while seeds with $R_0 > R_{cr}$ grow. For seeds with $|R_0 - R_{cr}| \ll R_{cr}$ the size changes very slowly ($w \propto \propto |\delta\mathcal{T}|(R_0 - R_{cr})/R_{cr}^2$). For undercritical seeds of a small size, $w \propto -|\delta\mathcal{T}|/R_0$. Slabs of stable phase, being placed in a metastable medium, grow independently of what was their initial size. Note that the same value R_{cr} follows from minimization of (6).

2) *Long time evolution of a large seed.* For $t \gg t_{\rm init}$, we may drop the term $\partial^2\xi_0/\partial\tau^2$ in the l.h.s of Eq. (5). For $R(t) \gg R_{cr}$, surface effects become unimportant and we arrive at the solution

$$R(t) \simeq R_0 + u_{\rm asymp}t,\ u_{\rm asymp} = 3|\epsilon|\sqrt{\beta}/\sqrt{2\lambda v^4}.$$

Seeds grow with constant velocity. The time scale for the growth of the seed with size $R \gg R_{cr}$ is $t_\rho = R/u_{\rm asymp}$, $t_\rho(\epsilon^{\max}) = (2m/T_{cr})^{1/2}R/((3\beta)^{1/2}|\delta\mathcal{T}|^{1/2})$. Asymptotic regime is reached at very large values of time, provided the system is near the critical point.

3) *Long time evolution of a small seed.* Describing seeds of a small size ($l \ll R \ll R_{cr}$, $d_{\rm sol} \neq 1$) for $t \gg t_{\rm init}$, we can drop the term $\propto \widetilde{\epsilon}$ in (5). Then we find

$$R(t) \simeq \sqrt{R_0^2 - 2(d_{\rm sol}-1)tl^2/t_0}.$$

The time scale at which the initial seed of a small size dissolves is, $t_{\rm dis} = \frac{16n_{cr}T_{cr}(\tilde{d}\eta_{\rm r}+\zeta_{\rm r})R_0^2}{(d_{\rm sol}-1)\sigma_0^2}$, and is $\propto R_0^2$. Thus, fluctuations of sufficiently small sizes are easily produced and dissolve rapidly.

4) *Fluctuations in spinodal region.* Let the system be driven to a spinodal region where fluctuations of even infinitesimally small amplitudes and sizes may grow into a new phase. To demonstrate this we take the free energy δF to be close to its maximum ($\delta F \simeq 0$). Then we linearize Eq. (3) dropping ψ^3 term. Setting $\psi = -\frac{\widetilde{\epsilon}}{2} + {\rm Re}\{\psi_0 e^{\gamma_\psi\tau + i\mathbf{k}\boldsymbol{\xi}}\}$, ψ_0 is an arbitrary but small real constant, we find two solutions,

$$\gamma_\psi(k) = (-k^2 \pm \sqrt{k^4 + 8\beta k^2 - 4\beta k^4}\,)/(2\beta). \qquad (7)$$

Growing modes correspond to the choice of "+"-sign and $k^2 < 2$. The time scale at which an aerosol of seeds develops is $t_{\rm aer} = t_0/\gamma_\psi(k_m)$, k_m corresponds to $\max\{\gamma_\psi(k)\}$. For an effectively large viscosity ($\beta \ll 1$) there are two solutions: the damped one, and the growing one for $k < \sqrt{2}$. The most rapidly growing mode is $\gamma_\psi(k_m) \simeq 2$, $k_m = 2\beta^{1/4} \ll 1$. The time scale characterizing growth of this mode is $t^\eta_{\rm aer} \sim \frac{1}{2}t_0 = \frac{4(\tilde{d}\eta_0+\zeta_0)}{9n_{cr}T_{cr}|\delta\mathcal{T}|}$. The typical size of seeds, $R^\eta_{\rm aer} \simeq l/(2\beta^{1/4})$, increases with an increase of the viscosity. For $k^2 > 2$ both modes are damped. In the case of an effectively small viscosity ($\beta \gg 1$) we get $\gamma_\psi(k) \simeq \pm k\sqrt{2/\beta}\sqrt{1-k^2/2}$, and $\gamma_\psi^{\max}(k_m = 1) = \beta^{-1/2}$. The time scale characterizing growing modes, $t^{\rm id}_{\rm aer} \sim t_0/\gamma_\psi = 2c^{1/2}/(\lambda v^2) \propto \delta\mathcal{T}^{-1}$, does not depend on the viscosity in this limit. The size scale of seeds is $R^{\rm id}_{\rm aer} \simeq l$. Modes with $k^2 > 2$ oscillate and do not grow into a stable phase.

For the description of the hadron–QGP first-order transition we take values $T_{cr} \simeq 162$ MeV, $n_{cr}/n_{\rm sat} \simeq \simeq 1.3$, as they follow from lattice calculations, see [12]. Parameters of the EoS are then as follows: $a \simeq \simeq 8.76\cdot 10^2({\rm MeV}\cdot{\rm fm}^3)$, $b \simeq 1.60$ fm^{-3}, $\lambda \simeq 7.80\cdot \cdot 10^{-5}q^{-3}({\rm fm}^6/{\rm MeV}^2)$, $v^2 \simeq 1.56\cdot 10^4 q^2|\delta\mathcal{T}|({\rm MeV}^2/{\rm fm}^6)$, $\epsilon^{\max} \simeq 58.4(\delta\mathcal{T})^{3/2}({\rm MeV}/{\rm fm}^3)$, m is the effective quark mass, $q = (m/300{\rm MeV})$. Further we obtain $l(T=0) \simeq \simeq 0.2$ fm (radius of confinement) for $\sigma_0 \simeq 40$ MeV/fm^2. If one used $\sigma_0 \simeq 100$ MeV/fm^2, one would estimate $l(T=0) \simeq 0.5$ fm.

Next we use $s \simeq 7T^3(T/T_{cr})$ at T near T_{cr}, $c_V \simeq \simeq 28T^3(T/T_{cr})$, as it follows from the lattice data [12]. Assuming the minimal value of the viscosity

$\eta_{\rm min} = s/4\pi \simeq 60\,{\rm MeV/fm}^2$, $\zeta_{\rm min} = 0$ we evaluate $\beta^{\rm max}_{\rm QGP} \simeq 0.015q$ for $\sigma_0 \simeq 40$ MeV/fm^2, that corresponds to the limit of *effectively very large viscosity.* Even for $\sigma_0 \simeq 100$ MeV/fm^2, $m = 600$ MeV we would get $\beta^{\rm max}_{\rm QGP} \simeq 0.2 \ll 1$. Note that following [13] the bulk viscosity diverges in the critical point. If were so ($\beta \to 0$), the quark-hadron system would behave as absolutely viscous fluid, like glass, in near critical region. Contrary, Ref. [14] argues for a smooth behavior of the bulk viscosity.

With $\beta = 0.015$ we further estimate $t_0 \simeq \simeq 2|\delta\mathcal{T}|^{-1}$ fm, $t_\rho(\epsilon^{\rm max}) \simeq 9.1Rq^{1/2}|\delta\mathcal{T}|^{-1/2}$, and $t_{\rm dis} \simeq 14qR_0\,(R_0/{\rm fm})$. Typical time for the formation of the aerosol is $t^\eta_{\rm aer} \simeq |\delta\mathcal{T}|^{-1}$ fm. Typical size of seeds in aerosol is $R^\eta_{\rm aer} \simeq 0.24|\delta\mathcal{T}|^{-1/2}$ fm. Only $t_{\rm init} \simeq 0.03q|\delta\mathcal{T}|^{-1}$ fm proves to be small (excluding quite small $\delta\mathcal{T}$). Critical slowing down that limits growing of the σ meson correlation length was discussed in [15].

For the thermal conductivity we use an estimation $\kappa_{\rm QGP} \simeq \alpha_0\eta/m$ taking $\alpha_0 \simeq 3$ to recover the relation between values of κ and η for nuclear gas-liquid phase transition at low energies [16]. The scale of the heat transport time is $t_T \simeq 26q\,(R/{\rm fm})^2$ fm. Using that $R_{cr} \simeq \simeq 0.1|\delta\mathcal{T}|^{-1}$ fm, we obtain $R_{\rm fog} \simeq 0.1q^{-1/2}|\delta\mathcal{T}|^{-1}$ fm $\lesssim \lesssim R_{cr}$. The value $R_{\rm fog}$ proved to be very small ($\lesssim 0.1 \div 1$ fm). However, time scale t_T is rather long. Therefore, the system most probably would have no time to fully develop a fog-like state in a hadron-quark phase transition in heavy-ion collisions.

Thus for the system near the critical point all estimated time scales (except $t_{\rm init}$) are very large. If the system trajectory paths rather far from the critical point (T_{cr}, ρ_{cr}), all time scales, except t_T, become less than the typical life-time of the fireball (~ 10 fm at RHIC conditions).

We solved numerically the general system of equations of nonideal hydrodynamics for $d = 2$. To illustrate the results we consider dynamics of overcritical and undercritical seeds (disks) in infinite matter taking initial density profile as $\rho(x, y; t = 0) = \rho_{\rm out} + (\rho_{\rm in} - \rho_{\rm out})\Theta(R_0 - r)$, $r = \sqrt{x^2 + y^2}$, $\rho_{\rm in}$ and $\rho_{\rm out}$ are densities in stable and metastable homogeneous phases, respectively.

In Fig.1 we show the time evolution of a liquid disk (upper panel) and a gas disk (lower panel) for $T/T_{cr} = 0.85$. In the middle column we show dynamics of an initially overcritical seed with $R_0 = 0.3L > R_{cr} \simeq 0.16L$ and in the right column, of undercritical seed $R_0 = 0.1L$, $L = 5$ fm. The time snapshots are shown in Figure in units L. The configuration is computed for values of kinetic parameters $\eta \simeq 45$ MeV/fm^2 and $\beta \simeq 0.2$. We

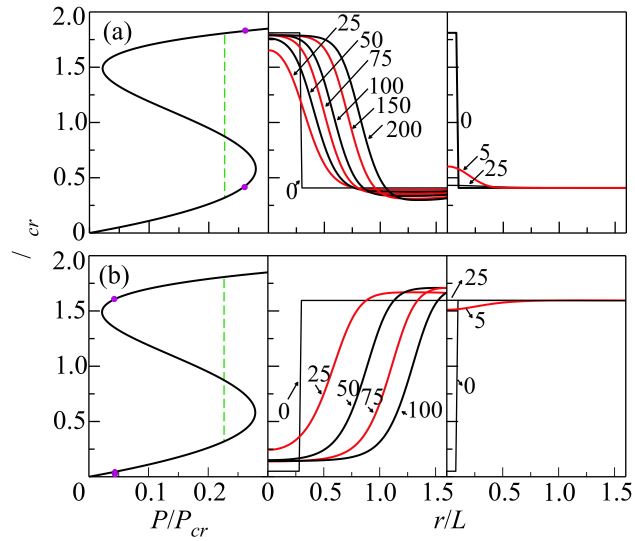

Fig.1. Isotherm for the pressure as function of the density with initial and final states shown by dots (left column). Dash vertical line shows the Maxwell construction, MC. In the upper panel the initial state corresponds to the stable liquid phase disk in the metastable super-cooled gas, and in the lower panel, to a stable gas phase disk in the metastable super-heated liquid. Middle column shows time evolution of density profiles for the overcritical liquid disk (upper panel) and gas disk (lower panel). Numbers near curves (in L) are time snapshots. Right column, the same for initially undercritical liquid or gas disk

see that in case $R_0 > R_{cr}$ (middle column) disks slowly grow with the time passage. For overcritical discs the initially selected distribution acquires the tanh-like shape, see (4), only for $t \gtrsim (50 \div 100)L$. Initial disks of a small size practically disappear for $t \gtrsim 10L = 50$ fm. Due to the matter supply to the disk surface and the shape reconstruction, the density decreases in the liquid disk neighborhood below the value of the density in the homogeneous metastable matter and it increases in the gas disk surrounding above the value of the density in the homogeneous metastable matter (see the middle column).

In Fig.2 we demonstrate time evolution of the wave amplitudes, $\rho(t) = \bar{\rho} + A_0 f(t)\sin(\mathbf{kr})$, for an undercritical value of the wave number k (left panel) and for an overcritical value (right panel). In case of the overcritical k and effectively small viscosity ($\beta = 20$) we demonstrate change of the amplitude in the 3/2-periods of the oscillation. Such a behavior fully agrees with that follows from our analytical treatment of the problem.

Concluding, even in the spinodal region seeds are growing slowly, if the system is somewhere in the vicinity of the critical point. Thus in heavy-ion collisions the expanding fireball may linger in the QGP state, until $T(t)$

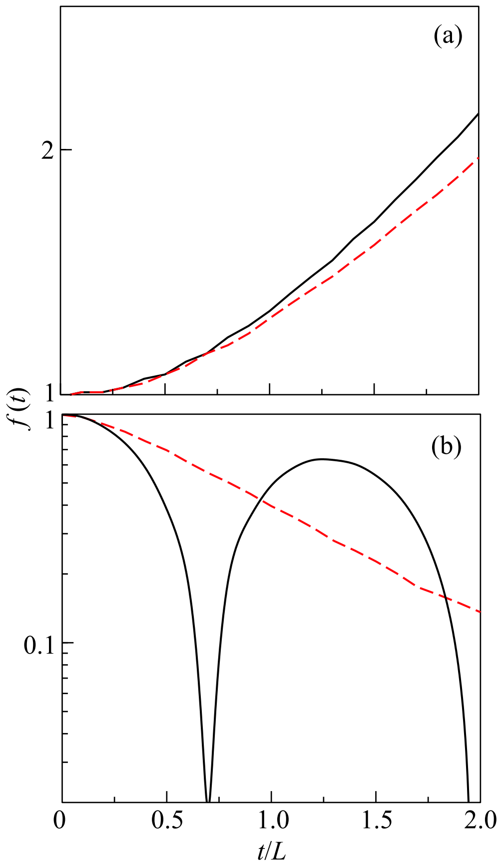

Fig.2. Time evolution of wave amplitudes $f(t)$ in aerosol for effectively small ($\beta = 20$, solid line) and large ($\beta = 0.2$, dash line) viscosity. Left panel: $k = 2l/L$ (growing modes). Right: $k = 8l/L$ (oscillation modes for large β and damping modes for small β). Other parameters are the same as in Fig.1

decreases below the corresponding equilibrium value of the temperature of the phase transition. There exists a belief that strongly coupled QGP state, represents almost perfect fluid [8]. We demonstrate the essential role of viscosity and surface tension in dynamics of first-order phase transitions, including the hadron-QGP one. Fluctuations in QGP (at a finite baryon density) grow and dissolve as if the fluid were very viscous. Although variation of parameters in broad limits does not change our conclusions, further investigations of the phase transition within dynamical simulations of heavy-ion collisions with a realistic equation of state are obviously required.

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1. A. Z. Patashinsky and B. I. Shumilo, JETP **50**, 712 (1979); preprint IYaF SO USSR (1984).
2. A. B. Migdal, E. E. Saperstein, M. A. Troitsky, and D. N. Voskresensky, Phys. Rep. **192**, 179 (1990); D. N. Voskresensky, Phys. Scripta **47**, 333 (1993).
3. A. Onuki, Phys. Rev. E **7**, 036304 (2007).
4. N. K. Glendenning, Phys. Rep. **342**, 393 (2001).
5. P. Chomaz, M. Colonna, and J. Randrup, Phys. Rep. **389**, 263 (2004).
6. E. Shuryak, arXiv:0807.3033 [hep-ph].
7. H. Stoecker and W. Greiner, Phys. Rep. **137**, 277 (1986); Yu. B. Ivanov, V. N. Russkikh, and V. D. Toneev, Phys. Rev. C **73**, 044904 (2006).
8. P. Romatschke and U. Romatschke, Phys. Rev. Lett. **99**, 172301 (2007).
9. D. Bower and S. Gavin, Phys. Rev. C **64**, 051902 (2001); K. Paech, H. Stoecker, and A. Dumitru, Phys. Rev. C **68**, 044907 (2003); C. Nonaka and M. Asakawa, Phys. Rev. C **71**, 044904 (2005); T. Koide, G. Krein, and R. Ramos, Phys. Lett. B **636**, 96 (2006).
10. E. J. Copeland, M. Gleiser, and H.-R. Muller, Phys. Rev. D **52**, 1920 (1995).
11. M. A. Stephanov, K. Rajagopal, and E. V. Shuryak, Phys. Rev. Lett. **81**, 4816 (1998).
12. Z. Fodor and S. D. Katz, JHEP **0404**, 050 (2004); Y. Aokiz, Z. Fodor, S. D. Katz, and K. K. Szabo, JHEP **0601**, 089 (2006).
13. D. Kharzeev and K. Tuchin, arXiv:0705.4280 [hep-ph].
14. C. Sasaki and K. Redlich, arXiv: 0806.4745 [hep-ph].
15. B. Berdnikov and K. Rajagopal, Phys. Rev. D **61**, 105017 (2000).
16. V. M. Galitsky, Yu.B. Ivanov, and V. A. Khangulian, Sov. J. Nucl. Phys. **30**, 401 (1979); L. Shi and P. Danielewicz, Phys. Rev. C **68**, 064604 (2003).