

On the Pion Distribution Amplitude Shape

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We argue that the recent BaBar data on $\gamma \rightarrow \pi$ e.m. transition form factor at large photon virtuality supports the idea that pion distribution amplitude (DA) is close to unity with $\phi'_\pi(0)/6 \gg 1$ at a normalization point of $\mu = 0.6 \div 0.8$ GeV. Such pion DA can be obtained in the effective chiral quark model. The possible flat shape of the pion DA implies that the standard expansion of the DA in Gegenbauer polynomials can be divergent. On basis of chiral models we predict that the two-pion DA should exhibit anomalous endpoint behaviour for pions in the S-wave and that such feature is absent for higher partial waves. The latter implies that the ρ , f_2 , etc. meson DAs have no anomalous endpoint behaviour. Possible implications of such pion DA for other hard exclusive processes are shortly discussed.

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The reaction $\gamma^*(q) + \gamma(q') \rightarrow \pi^0$ at large virtuality of the photon ($-q^2 = Q^2 \gg \Lambda_{\text{QCD}}^2$) is the basic *hard exclusive* process which allows rigorous QCD description [1]. QCD allows to make remarkably nice prediction for the $Q^2 \rightarrow \infty$ limit of the $\gamma\pi$ transition form factor:

$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\gamma\pi}(Q^2) = 2f_\pi. \quad (1)$$

Here $f_\pi \approx 0.0923$ GeV is the pion decay constant. The approach to the asymptotic (1) can be written to the leading order in $\alpha_s(Q^2)$ as follows¹) [1]:

$$Q^2 F_{\gamma\pi}(Q^2) = \frac{2f_\pi}{3} \int_0^1 dz \frac{\phi_\pi(z, Q)}{z + m^2/Q^2} + O(\alpha_s(Q^2)). \quad (2)$$

Here $\phi_\pi(z, \mu)$ is the pion distribution amplitude (DA) at the normalization scale μ . It is defined as the following matrix element:

$$\begin{aligned} & \langle 0 | \bar{d}(n) \gamma_\mu n^\mu \gamma_5 [n, -n] u(-n) | \pi^+(P) \rangle = \\ & = i\sqrt{2} f_\pi (n \cdot P) \int_0^1 dz e^{i(2z-1)P \cdot n} \phi_\pi(z). \end{aligned} \quad (3)$$

Here n^μ is a light-like 4-vector ($n^2 = 0$), and

$$[n, -n] \equiv \text{P exp} \left[\int_{-1}^1 dt n^\mu A_\mu(tn) \right] \quad (4)$$

¹)Note, that in Eq. (2) we modified perturbative quark propagator $1/(z Q^2)$ by $1/(z Q^2 + m^2)$, where m stays for possible non-perturbative contributions to the quark propagator. We stress that we do not derive such a modification, but use it just to mimic non-perturbative contributions. Such simplification is enough for rather qualitative discussion here. Derivation of the modification of the quark propagator due to e.g. instanton non-perturbative contribution will be given elsewhere.

denotes the path-ordered exponential of the gauge field, required by gauge invariance; the path is defined to be along the light-like direction n .

The pion DA can be represented as the series in the eigenfunctions of the leading order evolution equation – Gegenbauer polynomials [1]:

$$\phi_\pi(z, \mu) = 6z(1-z) \left(1 + a_2(\mu) C_2^{\frac{3}{2}}(2z-1) + \dots \right). \quad (5)$$

Usually it is tacitly assumed that the series (5) is convergent, that is why in analyses of experimental data (see e.g. [2–4]) only a finite number of terms in this series is considered. Actually, the assumption about the convergence of the series (5) does not follow from any principle. Moreover, there are counterexamples for such convergency. First counterexample results from the effective chiral quark model calculations of the photon DA in [5]. It was shown that the photon DA is not zero at the endpoints $z = 0, 1$, which explicitly demonstrated that the series (5) is divergent. The result for the pion DA in the same model [5, 6] is $\phi_\pi(z, \mu_0) = 1$ at the normalization point $\mu_0 = 1/\rho \sim 0.6$ GeV determined by the average size of the instanton. The normalization scale in the chiral effective quark model is inherited from the theory of instanton vacuum [7, 8] from which the effective quark model has been derived. However, it was noted in [9] that the effective quark model should be extended to the higher orders in instanton packing fraction $\rho^2/R^2 \sim 1/10$ when one considers pion DA at $z \sim \rho^2/R^2$. In Refs. [5, 6, 10–12] it was suggested *ad hoc* modification of the model beyond the leading order in the instanton packing fraction which led to the pion

DA only slightly wider than the asymptotic one (however rather strong sensitivity to additional *ad hoc* parameters was demonstrated). The proposed in Refs. [5, 6, 10–12] extensions of the effective chiral quark model were based essentially on modelling the momentum dependence of the dynamical quark mass $M(p)$ by the rational function of the momentum. A drawback of such extensions is that the endpoint nullification of the pion DA rely on contribution of the artificial (non-physical) poles in the used Ansatz for $M(p)$. Position of that poles is far from the Euclidean domain where one can trust the result of the instanton liquid model. Another problem of the *ad hoc* modification of the model beyond the leading order in the instanton packing fraction used in Refs. [5, 6, 10–12] is that the axial current is not conserved to the order ρ^2/R^2 . Possible solution of the problem with the axial current conservation was suggested in Ref. [13]. The calculation of the pion DA with improved axial current in Ref. [13] gave the function which is non-zero at the endpoints.

We can summarize that the theory of the instanton vacuum in the leading order of the instanton packing fraction predicts $\phi_\pi(z, \sim 1/\rho) = 1$, however in the vicinity of the endpoint of order ρ^2/R^2 theory should be modified. Precise form of the modification is not strictly derived, that is very interesting problem to study. Given such state of art, we can only state that the pion DA in the instanton theory of QCD vacuum is expected to be rather flat – meaning that it is close to unity with $\phi_\pi'(0)/6 \gg 1$.

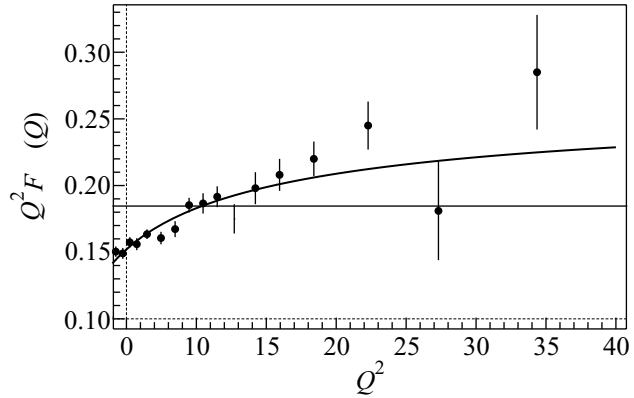
Calculation of the pion DA in the Nambu–Jona-Lasinio model [14] gave the same result as in the leading order effective chiral quark model – $\phi_\pi(z, \mu_0) = 1$, however it was attributed to a very low normalization point of $\mu_0 = 0.313$ GeV. The same result is obtained in the large- N_c Regge model [15].

We see that the wide class of chiral models predict the pion DA which is flat and even non-zero at the endpoints. We note that the possibility of the pion DA $\phi_\pi(z) = 1$ was considered almost three decades ago in Ref. [16]. Recent studies of the hadronic wave function in AdS/QCD [17] also suggests the wide pion DA $\phi_\pi(z) \sim \sqrt{z(1-z)}$ with anomalous behaviour at the endpoints, supporting the idea that the series (5) is divergent.

In these notes we consider an extreme possibility that the pion DA is $\phi_\pi(z, \mu_0) = 1$ at a normalization point of $\mu_0 \approx 0.6 - 0.8$ GeV. The same shape as in the instanton liquid model in the leading order in the instanton packing fraction ρ^2/R^2 [5, 6]. Such assumption about pion DA would imply that the scaled form factor $Q^2 F_{\gamma\pi}(Q)$

overcomes the asymptotic value given by Eq. (1) and then very slowly approaches it from above²⁾.

Recently the BaBar collaboration reported [18] results for the scaled form factor $Q^2 F_{\gamma\pi}(Q)$ for $4 < Q^2 < 40$ GeV². Despite common expectations [2, 3, 4] the scaled form factor crosses the asymptotic line of $2f_\pi$ around $Q^2 = 10$ GeV² and continues to grow slowly at higher Q^2 . The BaBar data [18] are shown in Figure.



The scaled form factor $Q^2 F_{\gamma\pi}(Q)$ as a function of Q^2 . Points with error bars are BaBar data [18], thick solid line our fit with pion DA (6), horizontal line is the QCD asymptotic value (1)

Now we make a following simple exercise. We assume that the shape of pion DA at the normalization point $\mu_0 = 0.6 \div 0.8$ GeV has the following form:

$$\phi_\pi(z, \mu_0) = N + (1 - N)6z(1 - z), \quad (6)$$

where N is a free constant. We evolve the DA (6) to the scale of Q^2 and then vary parameter N in Eq. (6) and mass parameter m in Eq. (2) to fit the BaBar data [18]. The obtained values of the parameters are:

$$N = 1.3 \pm 0.2, \quad m = 0.65 \pm 0.05 \text{ GeV}, \quad (7)$$

indicating that the BaBar data favour the flat pion distribution amplitude.

Notably the resulting value of the mass parameter m is close to the inverse instanton size, which sets the scale for non-perturbative effects in quark propagator. The contribution of the pion DA (6) with the central values of parameters (7) to the scaled form factor is shown by the thick solid line in Figure.

²⁾Interestingly, such enhancement for the similar to $\gamma^*\gamma \rightarrow \pi^0$ process – DVCS amplitude – was discussed in Ref. [9] where the generalized parton distributions were computed in the effective chiral quark model and the anomalous endpoint behaviour was found.

We note that our “back of envelope” analysis is rather oversimplified as it is limited to the leading order in $\alpha_s(Q^2)$ and we used rather simple model for the higher twists (2). We believe that such model for the higher twist contributions, in the case of a flat pion DA, grasps the most essential contributions related to the non-perturbative contributions to the quark propagator.

Qualitatively speaking, the BaBar data show that the rise of the scaled form factor with Q^2 for $Q^2 > 10 \text{ GeV}^2$ is rather slow, indicating that the Q^2 dependence is governed by rather large mass parameter of order of several GeV. The way to obtain such large mass scale from rather low non-perturbative mass scales of order of hundreds MeV is to enhance the latter due to the endpoint contribution of the flat pion DA. Such “transmutation” of mass scales is taken into account by our simple formula (2) in which the low non-perturbative mass scale $m \sim 0.65 \text{ GeV}$ is transformed into the large mass scale characteristic for the Q^2 dependence of the scaled form factor observed by the BaBar collaboration.

We are also limited ourselves to the leading order in $\alpha_s(Q^2)$, for the flat pion DA the next-to-leading contributions can be large. Possible large NLO corrections indicates that for the complete QCD analysis one needs some kind of resummation of the higher order contributions or modifications of the NLO coefficient function. That is very interesting problem one can study in future.

The pion DA (6) should not be taken literary, its form is just a handy way to parametrize a class of flat functions (close to unity and with $\phi'_\pi(0)/6 \gg 1$). In principle one can try the functional form $\phi_\pi(z) \sim [z(1-z)]^\alpha$ with $\alpha \ll 1$. Let us point out that the pion DA (6) has the following coefficients in the Gegenbauer expansion (5):

$$a_2(2 \text{ GeV}) \approx 0.3, \quad a_4(2 \text{ GeV}) \approx 0.15. \quad (8)$$

These values are compatible with estimates by various non-perturbative methods, see survey of the results in [4]. The main difference of our analysis with the commonly accepted that (see e.g. [2–4]) is that we *do not assume* that the Gegenbauer series (5) is convergent.

We were motivated to try the flat pion DA against the new BaBar data by the results of the instanton liquid model of QCD vacuum, which predicts in the leading order of the instanton packing fraction $\phi_\pi(z) = 1$ at the normalization point of $\mu_0 \sim 1/\rho \sim 0.6 \text{ GeV}$. Such unusual picture of pion DA has direct implications for other hadronic DA. In particular, the two-pion DA [19, 20] (entering QCD description of hard $\gamma^*\gamma \rightarrow 2\pi$ processes) computed in the instanton liquid model of QCD vac-

uum has the following form (in the leading order in the instanton packing fraction and at $m_{\pi\pi} = 0$) [21]:

$$\begin{aligned} \phi_{2\pi}(z, \zeta) = & -(2z - 1) + [2z\theta(1 < z < \zeta) + (2z - 1) \times \\ & \times \theta(\zeta < z < 1 - \zeta) + 2(z - 1)\theta(1 - \zeta < z < 1)]. \quad (9) \end{aligned}$$

Note that the first term in the above equation originates from the *contact* two pion couplings to quarks required by the spontaneously broken chiral symmetry. The remarkable feature the first term is that it is non-zero at the endpoints. The following terms in Eq. (9) are zero at the endpoints. The presence of the first term implies that the scaled amplitude of the $\gamma^*\gamma \rightarrow 2\pi$ given by the formula similar to Eq. (2) should exhibit the Q^2 rise as for the $Q^2 F_{\gamma\pi}(Q)$. However, there is an important difference – the term with endpoint singularities in Eq. (9) is ζ independent. That means that the raise of the amplitude with Q^2 is expected only for two pions in the S-wave. The two pion DA $\phi_{2\pi}(z, \zeta)$ in other partial waves is expected to be free from the endpoint singularities. Using the connection of two pion DA with the DAs of the resonances [22] we predict that DAs of mesons with non-zero spin (ρ, f_2 , etc.) are not anomalously flat as the pion DA.

Physics picture behind the flat pion DA can be traced back to Nambu–Goldstone nature of the pion. Due to the spontaneous breakdown of the chiral symmetry in QCD the quark acquires sizable mass and in the hadron spectrum contains (almost) massless Nambu–Goldstone bosons (pions). The broken chiral symmetry dictates that the $\pi q\bar{q}$ coupling is proportional to dynamical quark mass and is rather large ($g_{\pi q\bar{q}} \sim M/f_\pi$). The instanton mechanism for chiral symmetry breaking [7, 8] predicts that this coupling is almost *point-like* meaning that it is rather sizeable for k_\perp of quark up to momenta $\sim 1 - 2 \text{ GeV}$. Presence of such “point-like” component in the pion is the reason for the flat pion DA. Possible existence of the “point-like” component can have consequences for other hard processes, for instance, it can contribute considerably to hard exclusive pion production off nucleon at Q^2 of order several GeV^2 . Such contribution is obviously sensitive to the chirally odd generalized quark distributions in the nucleon – transversity distributions. We note also the point-like coupling of Nambu–Goldstone bosons to quarks appears also in V.N. Gribov theory of quark confinement [23].

Finally we note that the phenomena, similar to the anomalous endpoint behaviour of the pion DA has been discussed in the chiral models for nucleon GPDs [9] and nucleon DAs [24].

Due to big importance of the phenomenon observed by the BaBar collaboration, these results should be cross checked by other independent experimental groups.

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