

Diquark and triquark correlations in the deconfined phase of QCD

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We use the non-perturbative $Q\bar{Q}$ potential at finite temperatures derived in the Field Correlator Method to obtain binding energies for the lowest eigenstates in the $Q\bar{Q}$ and QQQ systems ($Q = c, b$). The three-quark problem is solved by the hyperspherical method. The solution provides an estimate of the melting temperature and the radii for the different diquark and triquark bound states. In particular we find that J/ψ and ccc ground states survive up to $T \sim 1.3 T_c$, where T_c is the critical temperature, while the corresponding bottomonium states survive even up to higher temperature, $T \sim 2.3 T_c$.

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1. Introduction. Diquark and triquark correlations in the quark–gluon plasma (QGP) are important for understanding the dynamics of heavy–ion collisions, processes in the early Universe and possibly the cores of the neutron stars [1]. Whether hadrons survive in the deconfined QGP is one of the key questions in QCD. In particular, the behavior of charmonia across the deconfinement transition has been suggested¹⁾ as hard probes of the QGP. Above T_c , the critical temperature, most part of static $Q\bar{Q}$ interactions disappear that eventually implies a dissolution of heavy quarkonia bound states into the continuum. A suppression of heavy quarkonia production in heavy ion collisions is usually considered as an observed signal of deconfinement.

On quite general grounds it is expected that the $Q\bar{Q}$ interactions get modified by temperature. As it was first mentioned in Ref. [3] an important part of the static interaction survives at the temperature above transition and can support the bound $Q\bar{Q}$ states even after the deconfinement transition. Later existing of charmonia bound states at $T \geq T_c$ was confirmed on the lattice [4].

Most recently, in line with the study of Ref. [3], a new approach called the Field Correlator Method (FCM), was proposed to study dynamics of QGP, where the main emphasis was done on the nonperturbative vacuum fields [5–7]. Let us summarize the basic formulation of the FCM as applied to finite T. The approach is based on the study of the quadratic field correlators $\langle tr F_{\mu\nu}(x)\Phi(x,0)F_{\lambda\sigma}(0) \rangle$ (x is Euclidian), where $\Phi(x,0)$ is the parallel transporter necessary to maintain gauge invariance. The Gaussian correlator is expressed in terms of two scalar functions, $D(x)$ and

$D_1(x)$, which define the static potential between heavy quarks at $T = 0$, the confinement part of the potential, σr , of is expressed only in terms of $D(x)$:

$$\sigma = 2 \int_0^\infty d\lambda \int_0^\infty d\tau D(\sqrt{\lambda^2 + \tau^2}). \quad (1)$$

At $T \geq T_c$ one should distinguish between electric and magnetic correlators $D^E(x)$, $D^H(x)$, $D_1^E(x)$, and $D_1^H(x)$, and, correspondingly, between σ^E and σ^H . It was argued in [8] and later confirmed on the lattice [9] that above the deconfinement region $D^E(x)$ and, correspondingly, σ^E vanish, while the coloelectric correlator $D_1(x)$ and colomagnetic correlators $D^H(x)$ and $D_1^H(x)$ should stay unchanged at least up to $T \sim 2 T_c$. However, the correlators $D^H(x)$ and $D_1^H(x)$ do not produce static quark–antiquark potentials, they only define the spatial string tension $\sigma_s = \sigma^H$ and the Debye mass $m_D \propto \sqrt{\sigma_s}$ that grows with the temperature in the dimensionally reduced limit [10].

The main source of the quark–antiquark static interaction at $T \geq T_c$ originates from the correlator $D_1^E(x)$ [11]

$$V_{Q\bar{Q}}(r, T) = \int_0^{1/T} d\nu(1 - \nu T) \int_0^r \lambda d\lambda D_1^E(\sqrt{\lambda^2 + \nu^2}), \quad (2)$$

that is responsible for bound states of quarks and gluons in the QGP.

The purpose of this Letter is to report our latest results of low lying charmonium and bottomonium as well as ccc , bbb bound states above T_c , obtained from the use of this approach. The paper is organized as follows. In section 2 we introduce the static $Q\bar{Q}$ potential. In section 3 the numerical results for the ground states of J/ψ

¹⁾This effect was first first investigated in a phenomenological potential model taking into account the Debye screening [2].

and Υ are presented. In section 4 we evaluate the masses of the three-quark ccc and bbb baryons. Our conclusions are given in section 5.

2. The heavy quark-antiquark potential at $T \geq T_c$. In the framework of the FCM, the finite temperature behavior of the static $Q\bar{Q}$ potential $V_{Q\bar{Q}}(r, T)$ at $T \geq T_c$ is given by

$$V_{Q\bar{Q}}(r, T) = V_{Q\bar{Q}}^{\text{pert}}(r, T) + V_{Q\bar{Q}}^{np}(r, T). \quad (3)$$

In Eq. (3) $V_{Q\bar{Q}}^{\text{pert}}(r, T)$ describes the interaction at short distances whereas $V_{Q\bar{Q}}^{np}(r, T)$ is the long-distance potential. The short distance interaction is represented by the perutrative one-gluon exchange potential that undergoes a Debye screening by the color charges of the QGP

$$V_{Q\bar{Q}}^{\text{pert}}(r, T) = -C_F \frac{\alpha_s}{r} \exp(-m_D r), \quad (4)$$

where $C_F = 4/3$ is the color factor, m_D is the inverse of the Debye screening radius. This form of the modified Coulomb potential has been used in earlier works (see *e.g.* Ref. [12]) to specify the in-medium potential between heavy quarks and determine the dissociation points of different quarkonium states. The Debye mass m_D in Eq. (4) can be written as $m_D = 2.06 \sqrt{\sigma_s(T)}$, where $\sigma_s(T)$ is the spatial string tension due to chromomagnetic confinement. In what follows we use the results of Ref. [10] where the quantity $\sqrt{\sigma_s(T)}$ has been computed nonperturbatively up to two loops in the deconfined phase of QCD. For $n_f = 0$ the Debye mass varies between 0.8 GeV and 1.4 GeV, when T varies between $T_c = 275$ MeV and $2T_c$.

The long-distance interaction $V^{np}(r, T)$ requires theoretical assumptions about its shape. We follow Ref. [13], where the long distance nonperturbative $Q\bar{Q}$ potential was derived analytically from the analysis of the non-perturbative part of the correlator function $D_1^E(x)$:

$$D_1^E(x) = B \exp(-M_0 |x|)/|x|. \quad (5)$$

In Eq. (5) the coefficient B must be considered as functions of the physical temperature. In the confinement region

$$B = 2C_F \alpha_s^f \sigma_{adj} M_0 = 6\alpha_s^f \sigma_f M_0, \quad (6)$$

α_s^f being the freezing value of the strong coupling constant in the confinement region, σ_f is zero temperature string tension, and M_0 has the meaning of the lowest gluelump mass [14]. Taking $\alpha_s^f = 0.6$, $\sigma_f = 0.18 \text{ GeV}^2$, $M_0 = 0.9 \text{ GeV}$, we get $B = 0.583 \text{ GeV}^3$. Above the

critical temperature one substitutes $B \rightarrow \xi(T)B$, where the T -dependent constant

$$\xi(T) = \left(1 - 0.36 \frac{M_0}{B} \frac{T - T_c}{T_c}\right) \quad (7)$$

arises from the comparison with lattice data [11]. As the result one obtains

$$V_{Q\bar{Q}}^{np}(r, T) = \xi(T) (V^{np}(\infty, T) - V^{np}(r)), \quad (8)$$

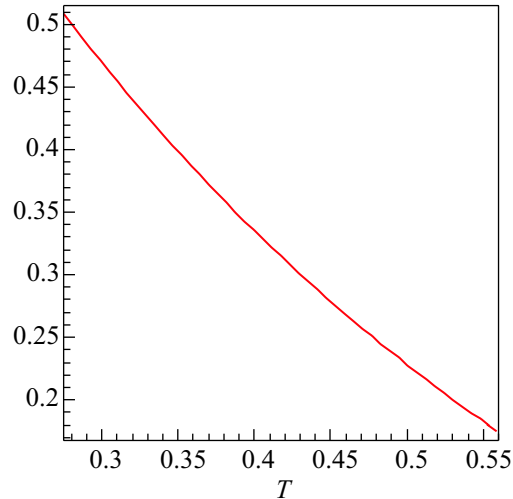
where the continuum threshold (a constant shift in the potential) $V^{np}(\infty, T)$ in Eq. (8) is

$$V^{np}(\infty, T) = \frac{B}{M_0^2} \left[1 - \frac{T}{M_0} \left(1 - \exp\left(-\frac{M_0}{T}\right)\right)\right], \quad (9)$$

and

$$V^{np}(r) = \frac{B}{M_0^2} x K_1(x) + \mathcal{O}\left(\frac{T}{M_0}\right). \quad (10)$$

In Eq. (10) $K_1(x)$ is the modified Bessel function and $x = M_0 r$. At $T = T_c$ one obtains $V_{Q\bar{Q}}^{np}(\infty, T_c) \approx 0.5 \text{ GeV}$ that agrees with estimate obtained in Ref. [5] from lattice data. The large positive value of the continuum threshold is a consequence of non perturbative vacuum fields and can not be explained in perturbation theory. The behavior of the potential $V_{Q\bar{Q}}(\infty, T)$ is shown in Figure.



The dependence of the $V_{Q\bar{Q}}^{np}(\infty, T)$ on the temperature T given by Eq. (9)

To find out whether the non perturbative interaction can support the bound states at $T \sim T_c$ we calculate the Bargmann condition

$$\begin{aligned} n \leq I &= \mu_Q \int_0^\infty |V^{np}(r) + V_{Q\bar{Q}}^{\text{pert}}(r, T)| r dr = \\ &= \mu_Q \left(\frac{4\alpha_s}{3m_d} + \frac{B}{M_0^4} \right), \end{aligned} \quad (11)$$

where n is the number of the S -wave bound states. Since $m_d \sim 1$ GeV, $B/M_0^2 \sim 0.5$ GeV, and $M_0 \sim 1$ GeV, we conclude that to support at least one bound state one needs $\mu_Q \geq 1$ GeV, *i.e.* there is no bound states of light quarks, but $c\bar{c}$ and especially $b\bar{b}$ binding is possible. Moreover, the bottomonium spectrum should display a much larger number of bound states above T_c .

3. Quarkonia at finite temperature. Having specified the static $Q\bar{Q}$ potential, we can now exploit the relativistic Hamiltonian technique, developed in [15] to calculate the masses of the $Q\bar{Q}$ states as a function of the temperature. This technique does not take into account chiral degrees of freedom and is applicable when spin-dependent interaction can be treated as perturbation. Therefore below consider heavy quarkonia and baryons. The masses of heavy quarkonia are defined as

$$M_{Q\bar{Q}} = \frac{m_Q^2}{\mu_Q} + m_Q + E_0(m_Q, \mu_Q), \quad (12)$$

where $E_0(m_Q, \mu_Q)$ is an eigenvalue of the Hamiltonian $H = H_0 + V_{Q\bar{Q}}$, where $V_{Q\bar{Q}}$ is given by Eq. (3), m_Q are the bare quark masse, and μ_Q are the constant auxiliary fields (AF) that were introduced in [4] to treat the kinematics of the relativistic particles. These parameters have to be found from the variational condition

$$\frac{\partial M_{Q\bar{Q}}}{\partial \mu} = 0. \quad (13)$$

The eigenvalue problem is solved for each set of μ_Q ; then one has to minimize $M_{Q\bar{Q}}$ with respect to μ_Q . Such an approach allows for a very transparent interpretation of AF: starting from bare quark masses m_Q , we naturally arrive at the dynamical masses μ_Q that appear due to the interaction. The AF are treated as c -number variational parameters. The bound state exists if $E_0(m_Q, \mu_Q) \leq V^{np}(\infty, T)$.

The solutions for the binding energy for the $1S$ states of charmonium and bottomonium are shown in Tables 1, 2. In these Tables we present the constituent quark masses μ_Q for $c\bar{c}$ and $b\bar{b}$ ²⁾, the differences $\varepsilon_Q = E_0 - V_{Q\bar{Q}}(\infty)$, the mean squared radii $\sqrt{\langle r^2 \rangle}$, and the masses $M_{Q\bar{Q}}$ for the $c\bar{c}$ and $b\bar{b}$ mesons. We employ $m_c = 1.4$ GeV, $m_b = 4.8$ GeV, and $\alpha_s = 0.35$. As expected, we obtain the weakly bound state at $T = T_c$ that disappears at $T \sim 1.3T_c$. The charmonium masses lie in the interval 3.2-3.3 GeV, that agrees with the results of Ref. [11]. Note that immediately above T_c the mass of the $c\bar{c}$ state is about 0.2 GeV higher than that of J/ψ .

²⁾These masses are computed solely in terms of the bare quark masses m_c and m_b , respectively. Note that, as in the confinement region [16], the constituent masses μ_Q only slightly exceed bare quark masses m_Q that reflect smallness of the kinetic energies of heavy quarks.

Table 1

Masses of the $c\bar{c}$ states as a function of the temperature above the deconfinement region. $M_0 = 0.9$ GeV, $B = 0.583$ GeV³, $m_c = 1.4$ GeV, $\alpha_s = 0.35$. Masses and energies are given in units GeV, the mean squared radius in units of GeV⁻¹

T/T_c	μ_c	ε_c	$\sqrt{\langle r^2 \rangle}$	$M_{c\bar{c}}$
1	1.451	-0.019	7.53	3.291
1.3	1.419	+0.006	10.50	3.186

Table 2

Masses of the $b\bar{b}$ states as a function of the temperature above the deconfinement region. The notations are the same as in Table 1, $m_b = 4.8$ GeV

T/T_c	μ_b	$E_0 - V_{Q\bar{Q}}(\infty)$	$\sqrt{\langle r^2 \rangle}$	$M_{b\bar{b}}$
1	4.984	-0.300	1.27	9.815
1.3	4.950	-0.183	1.55	9.802
1.6	4.915	-0.095	2.06	9.783
2.0	4.863	-0.021	4.25	9.742
2.2	4.832	-0.003	7.62	9.712
2.3	4.818	+0.001	9.52	9.694

As expected, the $b\bar{b}$ bound states remain intact up to the larger temperatures, $T \sim 2.3T_c$, see Table 2. For $T \sim T_c$ the masses of the $L = 0$ bottomonium lie in the interval 9.7–9.8 GeV, about 0.2–0.3 GeV higher than 9.460 GeV, the mass of $\Upsilon(1S)$.

Our $c\bar{c}$ and $b\bar{b}$ bound states found from the MFC can be compared to the calculations based on phenomenological $Q\bar{Q}$ potentials identified with the free energy measured on the lattice [17, 18]. Our results for $1S(J/\psi)$ are qualitatively agree with those of Refs. [17, 18] while our melting temperature for $1S(\Upsilon)$ is much smaller than $T \sim (4-6)T_c$ found in Ref. [18].

4. QQQ baryons at $T \geq T_c$. The three quark potential is given by

$$V_{QQQ} = \frac{1}{2} \sum_{i < j} V_{Q\bar{Q}}(r_{ij}, T), \quad (14)$$

where $1/2$ is the color factor. We solve the three quark Schrödinger equation using the hyperspherical method. The wave function of a QQQ baryon depends on the three-body Jacobi coordinates

$$\rho_{ij} = \sqrt{\frac{\mu_Q}{2}}(\mathbf{r}_i - \mathbf{r}_j), \quad \lambda_{ij} = \sqrt{\frac{2}{3}}\mu_Q \left(\frac{\mathbf{r}_i + \mathbf{r}_j}{2} - \mathbf{r}_k \right), \quad (15)$$

(i, j, k cyclic). There are three equivalent ways of introducing the Jacobi coordinates, which are related to each other by linear transformations with the Jacobian equal to unity. In what follows we omit the indices i and j .

In terms of the Jacobi coordinates the three-quark kinetic energy operator H_0 is written as

$$H_0 = -\frac{1}{2} \left(\frac{\partial^2}{\partial \rho^2} + \frac{\partial^2}{\partial \lambda^2} \right) = -\frac{1}{2} \left(\frac{\partial^2}{\partial R^2} + \frac{5}{R} \frac{\partial}{\partial R} + \frac{\mathbf{L}^2(\Omega)}{R^2} \right), \quad (16)$$

where R is the six-dimensional hyperradius that is invariant under quark permutations,

$$R^2 = \rho^2 + \lambda^2 = \frac{\mu_Q}{3} (r_{12}^2 + r_{23}^2 + r_{31}^2), \quad (17)$$

$$\rho = R \sin \theta, \quad \lambda = R \cos \theta, \quad 0 \leq \theta \leq \pi/2, \quad (18)$$

Ω denotes five residuary angular coordinates, and $\mathbf{L}^2(\Omega)$ is an angular operator

$$\mathbf{L}^2 = \frac{\partial^2}{\partial \theta^2} + 4 \cot \theta \frac{\partial}{\partial \theta} - \frac{\mathbf{l}_\rho^2}{\sin^2 \theta} - \frac{\mathbf{l}_\lambda^2}{\cos^2 \theta}, \quad (19)$$

whose eigenfunctions (the hyperspherical harmonics) satisfy

$$\mathbf{L}^2(\Omega) Y_{[K]}(\theta, \mathbf{n}_\rho, \mathbf{n}_\lambda) = -K(K+4) Y_{[K]}(\theta, \mathbf{n}_\rho, \mathbf{n}_\lambda), \quad (20)$$

with K being the grand orbital momentum.

The wave function $\psi(\rho, \lambda)$ is written in a symbolical shorthand as

$$\psi(\rho, \lambda) = \sum_{[K]} \psi_{[K]}(R) Y_{[K]}(\Omega), \quad (21)$$

where the set $[K]$ is defined by the orbital momentum of the state and the symmetry properties. We truncate this set using the hypercentral approximation $K = K_{\min} = 0$. Introducing the reduced function $u(R)$

$$\Psi(\rho, \lambda, T) = \frac{1}{\sqrt{\pi^3}} \frac{u(R, T)}{R^{5/2}}, \quad (22)$$

and averaging the three-quark potential (14) over the six-dimensional sphere one obtains the one-dimensional Schrödinger equation for $u(R, T)$

$$\frac{d^2 u(R, T)}{dR^2} + 2 \left[E_0 - \frac{15}{8R^2} - \frac{3}{2} \left(\mathcal{V}_{QQQ}^{\text{pert}}(R, T) + \mathcal{V}_{QQQ}^{np}(R, T) \right) \right] u(R, T) = 0, \quad (23)$$

where

$$\mathcal{V}^{\text{pert}}(R, T) = -\frac{4}{3} \alpha_s \frac{a_C(R)}{R}, \quad (24)$$

$$a_C(R) = \frac{16}{\pi} \sqrt{\frac{\mu_Q}{2}} \int_0^{\pi/2} \exp \left(\frac{-m_D R \sin \theta}{\sqrt{\mu_Q/2}} \right) \sin \theta \cos^2 \theta d\theta, \quad (25)$$

and

$$\mathcal{V}_{QQQ}^{np}(R, T) = V_{QQQ}^{np}(\infty, T) - \frac{16B}{\pi M_0} \sqrt{\frac{2}{\mu_Q}} \times \left(\int_0^{\pi/2} K_1 \left(\frac{M_0 R \sin \theta}{\sqrt{\mu_Q/2}} \right) \sin^3 \theta \cos^2 \theta d\theta \right) R. \quad (26)$$

The temperature dependent mass of the colorless QQQ states is defined as

$$M_{QQQ} = \frac{3}{2} \frac{m_Q^2}{\mu_Q} + \frac{3}{2} m_Q + E_0(m_Q, \mu_Q). \quad (27)$$

The bound QQQ state exists if $E_0(m_Q, \mu_Q) \leq V_{QQQ}(\infty, T)$, where

$$V_{QQQ}(\infty, T) = \frac{3}{2} V^{np}(\infty, T). \quad (28)$$

In Tables 3 and 4 we show the masses of the ground ccc and bbb states as a function of the temperature.

Table 3

Masses of the ccc states as a function of the temperature above the deconfinement region

T/T_c	μ_c	$E_0 - V_{QQQ}(\infty)$	$\sqrt{R^2}$	M_{bbb}
1	1.474	-0.046	6.60	4.922
1.3	1.434	-0.002	9.96	4.769

Table 4

Masses of the bbb states as a function of the temperature above the deconfinement region

T/T_c	μ_b	$E_0 - V_{QQQ}(\infty)$	$\sqrt{R^2}$	M_{bbb}
1	4.977	-0.506	3.12	14.665
1.3	4.927	-0.314	3.86	14.661
1.6	4.885	-0.171	5.04	14.645
2.0	4.840	-0.046	8.21	14.598
2.2	4.822	-0.011	10.59	14.562
2.3	4.815	+0.001	11.62	14.541

5. Conclusions. Based on the non-perturbative dynamics driven by the field correlators at finite temperatures we have calculated binding energies for the lowest eigenstates in the $c\bar{c}$, ccc , $b\bar{b}$, and bbb systems. For

what concerns the charmonium we find that J/ψ (and ccc ground states) survive up to $T \sim 1.3T_c$. On the other hand, the $b\bar{b}$ and bbb states survive even up to higher temperature, $T \sim 2.3T_c$. This suggests that the systems are strongly interacting above T_c .

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