

Baryon resonances in the mean field approach and a simple explanation of the Θ^+ pentaquark

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We suggest to classify baryon resonances as single-quark states in a mean field, and/or as its collective excitations. Identifying the Roper resonance $N(1440, \frac{1}{2}^+)$, the nucleon resonance $N(1535, \frac{1}{2}^-)$, and the singlet hyperon $\Lambda(1405, \frac{1}{2}^-)$ as single-quark excitations, we find that there must be an exotic $S = +1$ baryon resonance Θ^+ (the “pentaquark”) with a mass about $1440 + 1535 - 1405 = 1570$ MeV and spin-parity $\frac{1}{2}^+$. We argue that Θ^+ is an analog of the Gamov–Teller excitation long known in nuclear physics.

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It was argued 30 years ago by Witten [1] that if the number of colors N_c is large, the N_c quarks constituting a baryon can be viewed as moving in a mean field whose fluctuations are suppressed as $1/N_c$. Whether $N_c = 3$ of the real world is large enough for the mean field in baryons to be a working notion is a question to which there is no general answer: it depends on how large are $1/N_c, 1/N_c^2, \dots$ corrections to a particular baryon observable. However, experience in hadron physics tells us that usually the relations between observables found in the large- N_c limit are in satisfactory agreement with reality, unless there are special reasons to expect large $1/N_c$ corrections [1, 2]. In any case, it is helpful to understand how baryons are constructed in the large- N_c limit, before corrections are considered.

The mean field can, in principle, have components with various quantum numbers $J^{PC} = 0^{++}$ (scalar), 0^{-+} (pseudoscalar), 1^{--} (vector), 1^{++} (axial), etc. Since the mean field is created by N_c quarks of the u, d, s flavor it is also characterized by the flavor quantum numbers like isospin T and hypercharge Y . We do not consider baryons with heavy quarks here.

One expects that the mean field inside heavy ground-state baryons has the maximal possible, that is spherical symmetry. There is no problem in writing a spherical-symmetric scalar, flavor-singlet field as $\sigma(\mathbf{x}) = P_1(r)$ where $r = |\mathbf{x}|$ is a distance from the center of a baryon, however we immediately run into a problem of how to write the *Ansatz* for, say, the mean pseudoscalar field. Being pseudoscalar it has to be odd in \mathbf{x} . The minimal extension of spherical symmetry is then the “hedgehog” *Ansatz* “marrying” the isotopic and space axes:

$$\pi^a(\mathbf{x}) = n^a P_2(r), \quad n^a = \frac{x^a}{r}, \quad a = 1, 2, 3; \quad (1)$$

$$\pi^{4,5,6,7,8}(\mathbf{x}) = 0.$$

This *Ansatz* breaks spontaneously the symmetry under independent space and isospin rotations, and only a simultaneous rotation in both spaces remains a symmetry. At the same time it breaks the $SU(3)$ flavor symmetry. One may argue that the $SU(3)$ symmetry is explicitly broken from the start by $m_s \gg m_{u,d}$, however one can as well consider the strange quark mass m_s as a small perturbation²⁾. In the chiral limit, $m_s \rightarrow 0$, the *Ansatz* (1) breaks spontaneously the $SU(3)$ symmetry: the first three component of the pseudoscalar octet are privileged. Full symmetry is restored when one rotates the asymmetric mean field in flavor and ordinary spaces: that produces many baryon states with definite quantum numbers.

The rôle of a small m_s is in fact the same as of the infinitesimal magnetic field in materials with a spontaneous magnetization: it establishes the preferred direction of magnetization. In our case, the *Ansatz* (1) is privileged since $m_s \neq 0$, regardless of whether it is considered sizable or infinitesimal.

If in a baryon there are mean vector fields with the quantum numbers of ω and ϕ (and there are no *a priori* reasons why they should be absent), the *Ansätze* for those fields in correspondence with Eq. (1)

²⁾The lowest $SU(3)$ baryon multiplets satisfy Gell-Mann–Okubo relations with a high accuracy, meaning that the splittings inside the multiplets are linear in m_s , actually $\mathcal{O}(m_s N_c)$. The splitting between the centers of the octet and of the decuplet is $\mathcal{O}(\Lambda/N_c)$ where Λ is the QCD scale parameter. Numerically, the former quantity is about 140 MeV whereas the latter is $1382 - 1152 = 230$ MeV, slightly larger. It means that in baryon physics one can indeed treat m_s as a very small quantity: $m_s = \mathcal{O}(\Lambda/N_c^2)$ or even less.

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are $\omega_0, \phi_0(\mathbf{x}) = P_{3,4}(r)$; $\omega_i, \phi_i(\mathbf{x}) = n_i P_{5,6}(r)$. The Ansatz for the axial a_1 field is $A_0^a = n^a P_7(r)$, $A_i^a = \epsilon_{aij} n_j P_8(r) + \delta_i^a P_9(r) + n^a n_i P_{10}(r)$, and so on. We shall not attempt to construct the profile functions $P_{1-10}(r)$ explicitly (which at present can be done only in a model-dependent way) but rather look at the consequences of the mean field picture, in particular with respect to resonances. To the best of our knowledge, such an analysis has been never done before, except for the bag model [3], which is too crude.

In the mean field approximation, justified at large N_c , one looks for the solutions of the Dirac equation for single quark states in the background mean field. The Dirac Hamiltonian for quarks is, schematically,

$$H = \gamma^0 (i\gamma^i \partial_i + \sigma(\mathbf{x}) + i\gamma^5 \pi(\mathbf{x}) + \gamma^\mu V_\mu(\mathbf{x}) + \gamma^\mu \gamma^5 A_\mu(\mathbf{x}) + \frac{1}{2}[\gamma^\mu \gamma^5 u] T_{\mu\nu}(\mathbf{x})). \quad (2)$$

In fact, the one-particle Dirac Hamiltonian is most probably nonlocal and momentum-dependent (as it would follow e.g. from Fiertz-transforming and then bosonizing color quark interactions), therefore (2) is a symbolic presentation. However, several important statements can be made on general grounds:

- Given the Ansatz for the mean fields σ, π, V, A, T , the Hamiltonian (2) actually splits into two: one for s quarks (H_s) and the other for u, d quarks (H_{ud}). The former commutes with the angular momentum of s quarks, $\mathbf{J} = \mathbf{L} + \mathbf{S}$, and with the inversion of spatial axes, hence all energy levels of s quarks are characterized by half-integer J^P and are $(2J+1)$ -fold degenerate. The latter commutes only with the ‘grand spin’ $\mathbf{K} = \mathbf{T} + \mathbf{J}$ and with inversion, hence the u, d quark levels have definite integer K^P and are $(2K+1)$ -fold degenerate. The energy levels for u, d quarks on the one hand and for s quarks on the other are completely different, even in the chiral limit $m_s \rightarrow 0$.
- The Hamiltonian (2) has mixed symmetry with respect to time reversal, therefore the one-particle spectra for s and u, d quarks are generally not symmetric under the change $E \rightarrow -E$.
- All energy levels, both positive and negative, are probably discrete owing to confinement. Indeed, a continuous spectrum would correspond to a situation when quarks are free at large distances from the center, which contradicts confinement. [One can mimic or model confinement for example by imposing the condition that the effective quark mass $\sigma(\mathbf{x})$ grows at infinity.]

- According to the Dirac theory, all *negative*-energy levels, both for s and u, d quarks, have to be fully occupied, corresponding to the vacuum. It means that there must be exactly N_c quarks antisymmetric in color occupying all (degenerate) levels with J_3 from $-J$ to J , or K_3 from $-K$ to K ; they form closed shells that do not carry quantum numbers.
- Filling in the lowest level with $E > 0$ by N_c quarks makes a baryon.

We shall suppose that the energy levels with minimal $|E|$ are $K^P = 0^+$ for u, d quarks and $J^P = \frac{1}{2}^+$ for s quarks, because it corresponds to the maximal-symmetry wave functions as it is usual for the Dirac equation in smooth external fields. *A priori* the signs of E for those solutions can be any, however experimentally the lowest baryon is a nucleon and not the Ω^- hyperon. It means, that the $\frac{1}{2}^+$ level for s quarks is probably the nearest to $E = 0$ but remains on the negative side. It belongs to the vacuum sector and has to be filled in together with all the rest negative-energy levels. On the contrary, the 0^+ level for u, d quarks must be the lowest at $E > 0$. Filling it in with N_c quarks antisymmetric in color, adds N_c u, d quarks to the vacuum, and makes the nucleon, see Fig.1. We do not know yet how much

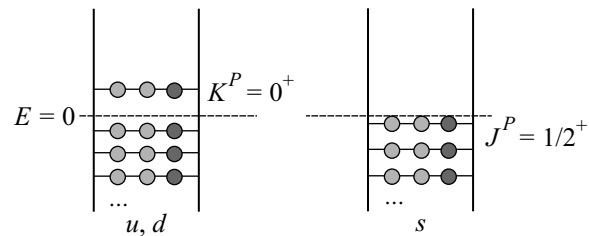


Fig.1. Filled quark levels for the ground-state baryon $N(940, \frac{1}{2}^+)$. The two lightest baryon multiplets $(8, \frac{1}{2}^+)$ and $(10, \frac{3}{2}^+)$ are rotational excitations of the same filling scheme

higher is the highest filled u, d shell than the highest filled s -quark shell but shall determine in shortly from experiment (at $N_c = 3$).

The true quantum numbers of the lightest baryons are determined from the quantization of the rotations of the mean field since the Ansatz discussed above spontaneously breaks symmetry under rotations in ordinary and flavor spaces. However, a simultaneous rotation in ordinary and isospin spaces remains a symmetry. Therefore, if one limits oneself to non-strange baryons, the quantization of rotations produces states with $J = T = \frac{1}{2}$ (for any odd N_c), that is the nucleon, and $J = T = \frac{3}{2}$, the Δ resonance [4]. If m_s is treated as a perturbation, one

has to extend this to $SU(3)$ flavor rotation. Its quantization gives specific $SU(3)$ multiplets that reduce at $N_c = 3$ to the octet with spin $\frac{1}{2}$ and the decuplet with spin $\frac{3}{2}$, see e.g. [5]. Witten's quantization condition $Y' = \frac{N_c}{3}$ [4] follows trivially from the fact that there are N_c u, d valence quarks each with the hypercharge $\frac{1}{3}$ [6]. Therefore, the ground state shown in Fig.1 entails in fact 56 rotational states. It is the same **56**-plet as the ground shell of the nonrelativistic quark model but its interpretation is different. The splitting between the centers of the multiplets $(\mathbf{8}, \frac{1}{2}^+)$ and $(\mathbf{10}, \frac{3}{2}^+)$ is $\mathcal{O}(1/N_c)$, and the splittings inside multiplets can be determined as a linear perturbation in m_s [6].

The picture has a similarity in nuclear physics where at large atomic numbers A , Z protons and $A - Z$ neutrons are considered in different self-consistent mean fields and have a different system of one-particle levels. One fills proton and neutron levels separately up to the common Fermi surface. Contrary to the quark case, the negative-energy levels for nucleons can be neglected because of the large nucleon mass. There are collective excitations of heavy nuclei, e.g. rotation whose energy scale is $\mathcal{O}(1/A)$ (in the baryon case it scales as $1/N_c$). However, there are also one-particle and particle-hole excitations that are of the order of unity in A . Similarly, one should expect $\mathcal{O}(N_c^0)$, that is large, one-particle and particle-hole excitations. Let us try to identify them.

The lowest baryon resonance beyond the rotational excitations of the ground state is the singlet $\Lambda(1405, \frac{1}{2}^-)$. Apparently, it can be obtained only as an excitation of the s quark, and its quantum numbers must be $J^P = \frac{1}{2}^-$. The resonance $\Lambda(1405, \frac{1}{2}^-)$ is excited if one of the u, d quarks from the valence 0^+ level jumps, under the action of a $S = -1$ force, to the first excited state for s quarks, see Fig.2, top. It is predominantly a 3-quark state (at $N_c = 3$). This excited state generates a rotational band of $SU(3)$ multiplets of its own, but we do not consider them here.

If there is a $\frac{1}{2}^-$ level for s quarks, it can be excited also by an s quark jumping from its highest filled level $\frac{1}{2}^+$, see Fig.2, bottom. This is a particle-hole excitation which does not change the nucleon quantum numbers (except for parity), as we are just adding an $s\bar{s}$ pair to the valence u, d level that determines the quantum numbers. We therefore identify this excitation with $N(1535, \frac{1}{2}^-)$. We see that at $N_c = 3$ it is predominantly a pentaquark state $u(d)uds\bar{s}$. That explains its large branching ratio in the ηN decay [7], a long-time mystery. We also see that, since the highest filled level for s quarks is lower than the highest filled level for u, d quarks, $N(1535, \frac{1}{2}^-)$ must be heavier than

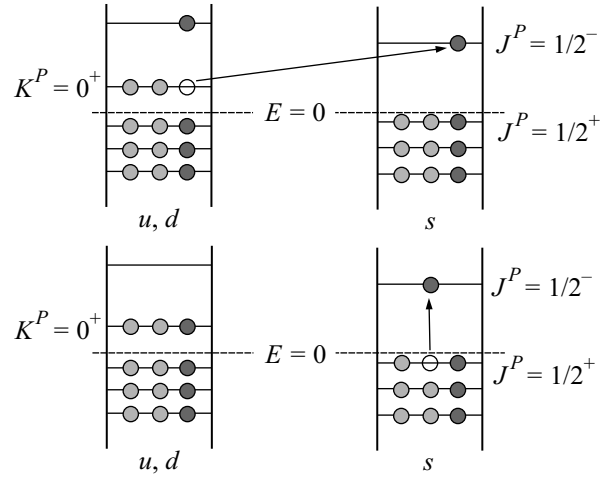


Fig.2. One excited level for s quark gives rise to two resonances: $\Lambda(1405, \frac{1}{2}^-)$ (top) and $N(1535, \frac{1}{2}^-)$ (bottom)

$\Lambda(1405, \frac{1}{2}^-)$: the opposite prediction of the nonrelativistic quark model has been always of some concern. We stress that in our picture the existence of an unusual nucleon resonance $N(1535, \frac{1}{2}^-)$ is a *consequence* of the $\Lambda(1405, \frac{1}{2}^-)$ existence. The transition shown in Fig. 2, bottom, also entails its own rotational band. Subtracting $1535 - 1405 = 130$, we find that the $\frac{1}{2}^+$ s -quark level is approximately 130 MeV lower in energy than the valence 0^+ level for u, d quarks.

There is also a low-lying Roper resonance $N(1440, \frac{1}{2}^+)$. It requires that there is an excited one-particle u, d state with $K^P = 0^+$, see Fig.3, top.

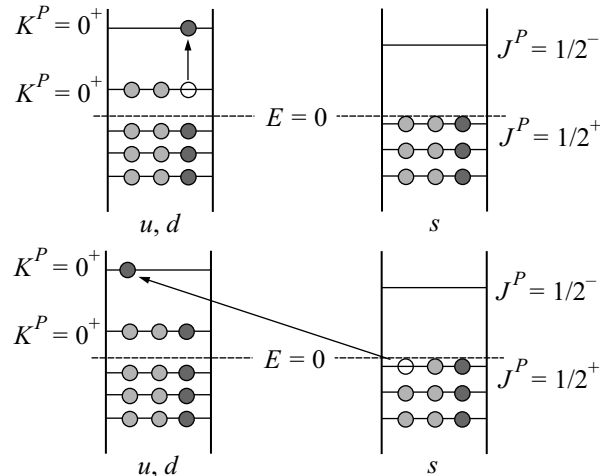


Fig.3. One excited level for u, d quarks gives rise to two resonances: $N(1440, \frac{1}{2}^+)$ (top) and $\Theta^+(\frac{1}{2}^+)$ (bottom)

Just as the ground state nucleon, it is part of the excited $(\mathbf{8}', \frac{1}{2}^+)$ and $(\mathbf{10}', \frac{3}{2}^+)$ split as $1/N_c$. In fact the first excited state could be also $K^P = 1^+, 2^+$ which would

generate more $SU(3)$ multiplets including one with the Roper resonance; $K^P = 0^+$ is a minimal hypothesis. The identification of the nature of the Roper resonance solves another problem of the nonrelativistic model where $N(1440, \frac{1}{2}^+)$ must be heavier than $N(1535, \frac{1}{2}^-)$. In our approach they are simply unrelated.

We now come to the crucial point: Given that there is an unoccupied level for u, d quarks, one can put there an s quark as well, taking it from one of the filled s -quark shells. The minimal-energy excitation is from the highest occupied shell for s quarks, to the lowest unoccupied level for u, d quark, that is to the would-be Roper level, see Fig.3, bottom. It is a particle-hole excitation with the valence level left untouched, its quantum numbers being apparently $S = +1, T = 0, J^P = \frac{1}{2}^+$. At $N_c = 3$ this excitation is a pentaquark state $uudd\bar{s}$, precisely the exotic Θ^+ baryon predicted in Ref. [8] from other considerations. The quantization of its rotations in flavor and ordinary spaces produces the antidecuplet $(\overline{10}, \frac{1}{2}^+)$ and higher multiplets $(27, \frac{3}{2}^+)$ and $(27, \frac{1}{2}^+)$ [9].

Since the relative position of all four levels involved are already known from the masses of the well-established resonances $N(1440, \frac{1}{2}^+)$, $N(1535, \frac{1}{2}^-)$ and $\Lambda(1405, \frac{1}{2}^-)$, it is a matter of trivial arithmetics to find the energy difference between the s -quark shell and the first excited u, d -quark level. We obtain an estimate for the Θ^+ mass: $m_\Theta \approx 1440 + 1535 - 1405 = 1570$ MeV. Of course, one should not understand the number literally. First, the masses of the resonances exploited here are known with an uncertainty of a few tens MeV. Second, the masses of physical resonances are not fixed exactly by the one-particle levels but get $\mathcal{O}(1/N_c)$ and $\mathcal{O}(m_s)$ corrections (which in principle are calculable). Nevertheless, the most interesting prediction that the exotic pentaquark is a consequence of the three well-known resonances and must be light, is an unambiguous feature of the picture.

In nuclear physics, the charge-exchange excitations generated by the axial current $j_{\mu 5}^\pm$, when a neutron from the last occupied shell is sent to an unoccupied proton level or vice versa are known as Gamov–Teller transitions [10]; they have been extensively studied both experimentally and theoretically. Thus our interpretation of the Θ^+ is that it is a Gamov–Teller-type resonance long known in nuclear physics.

However, the calculation of the Θ^+ width should be different from that in nuclear physics. For the Gamov–Teller transition in a nuclei, it is sufficient to calculate the matrix element between the initial and final states of the transition axial current. To make it more accurate, one can take into account a correction from the ad-

mixture of particle-hole states to the one-particle states. This approximation is often called the RPA; it goes beyond the mean field.

In the baryon case, even a one-particle state of the leading mean-field approximation, shown in Fig.1, is in fact a Fock state with additional quark-antiquark pairs. These arise when one decomposes the filled negative-energy levels in the plane wave basis [11, 12]. Therefore, unlike its nuclear physics counterpart, the Θ^+ decay amplitude has, already in the mean field approximation, two contributions: one is from the conventional “fall-apart” process whereas the other is the “5-to-5” transition of the Θ^+ to the 5-quark component of the nucleon [13, 5]. The two amplitudes are separately not Lorentz invariant – only their sum is. In the lab frame there is a tendency for the two amplitudes to cancel each other [14]. In the infinite momentum frame, however, the “fall-apart” amplitude (the simple one) is zero in the chiral limit, and only the “5-to-5” amplitude survives. The one-particle Hamiltonian (2) is covariant, such that there is no problem in transforming the mean field to an infinite momentum frame, which is the shortest way to evaluate the Θ^+ width. [It may seem somewhat unusual but we do not have much experience from the past in computing pentaquarks widths!] The program has been carried out in the Chiral Quark Soliton Model with the result $\Gamma_\Theta \sim 1$ MeV with no parameter fitting [12, 15].

It should be stressed that the small width of Θ^+ has no relation to and no influence from the large width of the Roper resonance. When Θ^+ decays, the valence level (to which the Roper resonance decays) is Pauli-blocked, and the fact that an ordinary “3-to-3” decay width of the Roper resonance is large does not effect the width of the Θ^+ which is very narrow owing to physics completely different from that governing the Roper decay.

There can be additional one-particle and/or particle-hole excitations, however just the two excited levels suggested here (0^+ for u, d quarks and $\frac{1}{2}^-$ for s quarks) are sufficient to explain the majority of baryon resonances up to 2 GeV, if the rotational states generated by each of the excitations are taken into account. A detailed study of the ensuing baryon spectrum will be published separately. For high-spin resonances (actually for $J \geq N_c$) it may become energetically favorable to depart from a spherically-symmetric mean field.

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