Pis'ma v ZhETF, vol. 90, iss. 6, pp. 461 – 466 © 2009 September 25

Real-time approach to quark confined systems at finite temperatures

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Submitted 7 July 2009
Resubmitted 20 August 2009

A real-time formalism is proposed to incorporate finite temperatures into confined quark systems at the example of the Generalised Nambu–Jona-Lasinio model for QCD. This approach allows one to study various properties of the system at $T > 0$, such as chiral symmetry breaking and restoration, properties of the bound-state spectrum, and so on.

PACS: 12.38.Aw, 12.39.Ki, 12.39.Pn

1. Introduction. While the property of colour confinement is expected from general considerations and is supported by lattice calculations, no effective theory exists which derives confinement directly from the QCD Lagrangian. Therefore, in the absence of controllable analytic solutions of QCD, one has to resort to models to describe hadronic phenomenology. However, these models must satisfy a very stringent set of minimal requirements. They must be: i) relativistic, ii) chirally symmetric, and iii) able to provide a mechanism of spontaneous breaking of chiral symmetry, iv) they should contain confinement, in order to be able to address the issue of excited states. To satisfy all the above requirements with one and the same model, turns out to be a highly nontrivial matter. There is a model, however, which does incorporate the aforementioned set of requirements. This is the Generalised Nambu–Jona-Lasinio (GNJL) model with the instantaneous vector confining kernel [1, 2] and confinement of quarks guaranteed due to an instantaneous infinitely rising (for example, linear) potential, to which extra ingredients can be added like, for instance, the colour Coulomb potential. Then chiral symmetry breaking can be described by standard summation of valence quark self-interaction loops (the mass–gap equation), while hadrons are obtained from the Bethe–Salpeter equation for the multiquark bound states. Besides, GNJL is known to fulfill the well–known low–energy theorems of Gell-Mann, Oakes, and Renner, Goldberger and Treiman, Adler selfconsistency zero, the Weinberg theorem, and so on – see Ref. [3]. For highly excited hadrons this model predicts effective restoration of chiral symmetry [4] in accordance with general expectations [5]. At this stage, we would like to emphasise the universal nature of the above low–energy theorems as well as of the chiral restoration in excited hadrons regardless of the actual gluonic interactions that would eventually be responsible for chiral symmetry breaking. Thus, the GNJL model is expected to be quite useful to describe all those phenomena which are essentially driven by the same global chiral symmetry.

Incorporation of finite chemical potentials and finite temperatures stand as important forward steps in the development of this class of models which not only help us into solving various puzzles posed by experimental results but, as importantly, they may be of assistance in predicting new phenomena for experimental testing. A number of attempts has been undertaken to include finite quark densities into consideration [6, 7]. By contrast, the incorporation of finite temperature appears to be a slightly more involved problem. Indeed, one may encounter technical problems applying techniques such as the Matsubara finite-temperature approach to the models, where only real-time calculations are possible. Thus we adhere to a different approach, based on the formalism of Bogoliubov–Valatin transformations, which is a natural mathematical tool to deal with GNJL systems. Such an approach to chiral symmetry breaking at $T = 0$ was successfully used in a series of papers [2] and then extended to redefine the theory entirely in terms of the quark–antiquark bound states [8]. The underlying idea of the method is to consider the true thermal vacuum of the system, as a collective (coherent-like) phenomenon of zero-temperature chirally symmetric theory eigenstates. To this end it is necessary to build a pseudounitary, temperature-dependent, transformation relating the thermal vacuum with the zero-temperature unbroken vacuum (such a real-time formalism for not confined systems was suggested in Ref. [9]). For quarks, this transformation leads to the formation of new effective objects – dressed quarks which, when seen from the zero-temperature chirally symmetric Fock space, look like original bare quarks "escorted by" an accompanying cloud of quark–antiquark pairs. The cornerstone of this method, at $T = 0$, amounts to finding, through a variational calculus, the minimal vacuum energy of the theory evaluated as a functional of the order parameter, called the chiral angle. Then in order to include finite temperature effects we have to consider the free energy $F_{\rm vac} = E_{\rm vac} - TS$, with S being the entropy of the vacuum, instead of the usual $E_{\rm vac} = \langle 0|H|0\rangle$. For a given temperature T this minimisation process ensures a proper balance between the vacuum energy and the entropy of the vacuum, so that the resulting vacuum state is stable, with $F_{\rm vac} < F^{(0)}_{\rm vac}$, where $F^{(0)}_{\rm vac}$ is the free energy of the trivial, unbroken, vacuum.

2. GNJL model at $T = 0$. We start with a short introduction to the GNJL chiral quark model [1, 2] given by the Hamiltonian

$$H = \int d^3x\bar\psi(\mathbf{x},t)\,(-i\boldsymbol{\gamma}\cdot\nabla+m)\,\psi(\mathbf{x},t) + \\ +\frac{1}{2}\int d^3xd^3y\; J^a_\mu(\mathbf{x},t)K_{\mu\nu}(\mathbf{x}-\mathbf{y})J^a_\nu(\mathbf{y},t), \qquad (1)$$

where $J^a_\mu(\mathbf{x},t) = \bar\psi(\mathbf{x},t)\gamma_\mu(\lambda^a/2)\psi(\mathbf{x},t)$. Having in mind that the qualitative results and conclusions do not depend on any particular form of the quark kernel $K_{\mu\nu}(\mathbf{x}-\mathbf{y})$, we only require it to be confining and to introduce a natural scale. For phenomenological applications this scale is to be fixed of order of $300\div400$ MeV. No extra constraints are imposed on the kernel.

Among possible quark–quark interactions described by the Hamiltonian (1) we have quark selfinteractions. It turns out that these self-interactions are removed by the use of an appropriate Bogoliubov–Valatin transformation from bare to dressed quarks, which can be conveniently parametrised by means of the so-called chiral angle φ_p (p being the relative momentum in the dressing quark–antiquark pairs) [1, 2]:

$$\psi^\alpha(\mathbf{x}) = \sum_{p,s} e^{i\mathbf{px}}[b^\alpha_{ps}u_s(\mathbf{p}) + d^{\alpha\dagger}_{ps}v_s(-\mathbf{p})],$$

$$u(\mathbf{p}) = \frac{1}{\sqrt2}\left[\sqrt{1+\sin\varphi_p}+(\boldsymbol{\alpha}\hat{\mathbf{p}})\sqrt{1-\sin\varphi_p}\right]u_0(\mathbf{p}),$$
$$v(-\mathbf{p}) = \frac{1}{\sqrt2}\left[\sqrt{1+\sin\varphi_p}-(\boldsymbol{\alpha}\hat{\mathbf{p}})\sqrt{1-\sin\varphi_p}\right]v_0(-\mathbf{p}).$$

where α is the colour index of N_C colours. It is convenient to define the chiral angle varying in the range $-\pi/2 < \varphi_p \leqslant \pi/2$ with the boundary conditions

$$\varphi_p(p=0) = \pi/2, \quad \varphi_p(p\to\infty)\to 0. \qquad (2)$$

The normal ordered Hamiltonian (1) becomes:

$$H = E_{\rm vac} + :H_2: + :H_4:, \qquad (3)$$

and the usual procedure is to demand the quadratic part $:H_2:$ to be diagonal or, equivalently, that the vacuum energy $E_{\rm vac}$ should become a minimum. Then the corresponding mass-gap equation,

$$\delta E_{\rm vac}[\varphi]/\delta\varphi_p = 0, \quad E_{\rm vac}[\varphi] = \langle 0[\varphi]|H|0[\varphi]\rangle, \qquad (4)$$

ensures the anomalous Bogoliubov terms $b^\dagger d^\dagger$ and db to be absent in $:H_2:$. As soon as the mass-gap equation is solved and a nontrivial chiral angle $\tilde\varphi_p$ is found, the Hamiltonian (3) takes a diagonal form,

$$H = E_{\rm vac} + \sum_{p,\alpha,s} E_p[\tilde b^{\alpha\dagger}_{ps}\tilde b^\alpha_{ps} + \tilde d^{\alpha\dagger}_{ps}\tilde d^\alpha_{ps}], \qquad (5)$$

where E_p is the dressed-quark dispersive law and the quark creation and annihilation operators $\tilde b^{\alpha\dagger}_{ps}$, $\tilde b^\alpha_{ps}$, $\tilde d^{\alpha\dagger}_{ps}$, and $\tilde d^\alpha_{ps}$ are consistent with the Fock space $\tilde{\mathcal F}$ built on top of the nontrivial BCS vacuum $|\tilde0\rangle \equiv |0[\tilde\varphi]\rangle$. The contribution of the $:H_4:$ part is suppressed as $1/\sqrt{N_C}$, so this completes the diagonalisation of the Hamiltonian (1) in the quark sector (at the BCS level).

Since the definition of a one-particle state has changed dramatically, the vacuum $|\tilde0\rangle$ annihilated by the dressed single-particle operators is also different from the trivial vacuum $|0\rangle_0 \equiv |0[\varphi=0]\rangle$. Indeed, as it always happens after a Bogoliubov–Valatin transformation, the true vacuum, with the minimal vacuum energy, contains an infinite set of strongly correlated $^3P_0$ quark-antiquark pairs [2],

$$|\tilde0\rangle = e^{Q_0^\dagger - Q_0}|0\rangle_0, \quad Q_0^\dagger = \frac12\sum_p \varphi_p C_p^\dagger, \qquad (6)$$

where $C_p^\dagger = b^{\alpha\dagger}_{ps}[(\boldsymbol{\sigma}\hat{\mathbf{p}})i\sigma_2]_{ss'}d^{\alpha\dagger}_{ps'}$ with σ's being the 2×2 Pauli matrices. The operator $C_p^\dagger$ creates a $^3P_0$ quark-antiquark pair with zero total momentum and the relative three-dimensional momentum $2\mathbf{p}$. The chiral angle $\tilde\varphi_p$ is a solution to the mass-gap equation (4) and it "measures" the weight of the pairs with the given relative momentum, so that the operator $\exp[Q_0^\dagger - Q_0]$ creates a cloud of correlated pairs, and the BCS vacuum $|\tilde0\rangle$ has the form of a coherent-like state when seen from the point of view of the naive Fock space $\mathcal F_0$ (that is, the Fock space built on top of the trivial vacuum $|0\rangle_0$). The quark Fock space $\tilde{\mathcal F}$ is built over $|\tilde0\rangle$ by repeated application of quark/antiquark creation operators.

Using the commutation relations for the quark operators b and d, one can easily find the following representation for the new vacuum [2]:

$$|0[\varphi]\rangle = \prod_p \left[\sqrt{w_{0p}} + \frac{1}{\sqrt{2}}\sqrt{w_{1p}}C_p^\dagger + \frac{1}{2}\sqrt{w_{2p}}C_p^{\dagger 2}\right]|0\rangle_0, \quad (7)$$

where the coefficients

$$w_{0p} = \cos^4\frac{\varphi_p}{2},\ w_{1p} = 2\sin^2\frac{\varphi_p}{2}\cos^2\frac{\varphi_p}{2},\ w_{2p} = \sin^4\frac{\varphi_p}{2}, \quad (8)$$

clearly obey the condition $w_{0p}+w_{1p}+w_{2p}=1$ and thus they admit the following natural interpretation: they represent the corresponding probabilities of having in the new vacuum zero, one, and two quark–antiquark pairs with a given relative momentum. Notice that powers of the operator $C_p^\dagger$ higher than two cannot appear because of the Fermi statistics for quarks. Then it is easy to show that the new vacuum is normalised,

$$\langle 0[\varphi]|0[\varphi]\rangle = \prod_p (w_{0p}+w_{1p}+w_{2p}) = 1, \quad (9)$$

and that it is orthogonal to the trivial vacuum in the limit of infinite volume of the space V:

$$\langle 0[\varphi]|0\rangle_0 = \exp\left[\sum_p \ln\left(\cos^2\frac{\varphi_p}{2}\right)\right] \underset{V\to\infty}{\longrightarrow} 0. \quad (10)$$

3. GNJL model at finite temperatures. As it was mentioned before, consideration of finite temperatures amounts to evaluation of both the vacuum energy and the entropy of the system simultaneously and then to the minimisation of the free energy with respect to the order parameter. With the help of Eq. (7) one can easily find for the entropy of the vacuum:

$$S = -N_C N_f \sum_p \sum_{n=0}^{2} w_{np}\ln w_{np}, \quad (11)$$

where the probabilities w_{np} are defined in Eq. (8).

Although we shall be working only with the c-number (11), it is instructive to see how it appears as a result of averaging over the vacuum of a local operator, $S = \langle 0[\varphi]|K|0[\varphi]\rangle$, which should: i) be built entirely in terms of the operators C_p and $C_p^\dagger$ – the only building blocks with the quantum numbers of the vacuum at our disposal; ii) commute with the Hamiltonian (5); iii) comply with the relation (11). The final result reads:

$$K = -\sum_p\left[\ln w_{0p} + \frac{1}{2}C_p^\dagger C_p \ln\frac{w_{1p}}{w_{0p}} + \frac{1}{2}C_p^{\dagger 2}C_p^2\ln\frac{w_{2p}}{w_{1p}}\right].$$

As it was mentioned before, the thermal mass-gap equation guarantees the balance between the energy and the entropy of the vacuum and can be derived by minimising the free energy operator $H - TK$ averaged over the vacuum $|0[\varphi]\rangle$. Then, with the help of Eqs. (4) and (11), one can find for the free energy:

$$F_{\rm vac}[\varphi] = \langle 0[\varphi]|(H-TK)|0[\varphi]\rangle = E_{\rm vac} - TS. \quad (12)$$

Then the mass-gap equation is

$$\delta F_{\rm vac}[\varphi]/\delta\varphi_p = \delta E_{\rm vac}[\varphi]/\delta\varphi_p - T\delta S[\varphi]/\delta\varphi_p = 0, \quad (13)$$

which is nothing but a generalisation of the zero-temperature mass-gap equation (4) to finite temperatures. It is easy to find by a straightforward calculation that

$$\delta S/\delta\varphi_p = -\sin\varphi_p\left[\ln\left(\tan^2\frac{\varphi_p}{2}\right) + \cos\varphi_p\ \ln 2\right]. \quad (14)$$

Mass-gap Eq. (13) with the entropy and its variation given by Eqs. (11) and (14) is the central result of this paper. Its physical interpretation is straightforward: $^3P_0$ quark–antiquark pairs are condensed in the vacuum to lower the vacuum energy and to break chiral symmetry. With temperature, some pairs are "removed" from the vacuum so to absorb the heat that has been added to the system. The chiral condensate decreases (evaporates) accordingly and chiral symmetry gets less broken. The mass-gap equation guarantees the proper balance between these two opposite processes. Concluding this discussion, let us remind the readers that the chiral angle is nothing but the wave function of the Goldstone boson ($\Psi_\pi = \sin\varphi_p$ [1, 2]) responsible for the chiral symmetry breaking. With the proper solution of the thermal mass-gap equation in hands, one is able to study microscopically the process of the Goldstone boson "melting" with the temperature increase.

The extra terms in the mass-gap equation which stem from the temperature are consistent with the boundary conditions (2) imposed on the chiral angle. Indeed, as clearly seen from Eq. (14),

$$(\delta S/\delta\varphi_p)_{|\varphi_p=\pi/2} = (\delta S/\delta\varphi_p)_{|\varphi_p=0} = 0. \quad (15)$$

Since equation $\delta S/\delta\varphi_p = 0$ does not have other solutions, the entropy increases monotonously from its minimum at $\varphi_p = 0$ to its maximum at $\varphi_p = \pi/2$. Then Eq. (13) embodies two natural limits: in the infinite quark mass limit one must have $\varphi_p = \pi/2$ for all p's, and indeed we can immediately see that the temperature dependence of the mass-gap equation vanishes for any finite T and we are left with the zero-temperature mass-gap equation. On the other hand, for very high temperatures and finite bare quark masses, there is only one solution, that is $\varphi_p = 0$, which is the chiral restoration scenario.

An attractive feature of the GNJL model is that the qualitative predictions of the model are robust against variations of the quark kernel. We therefore consider, as a paradigmatic example, the simplest quark kernel compatible with the requirements of confinement,

$$K_{\mu\nu}(\mathbf{x}-\mathbf{y}) = g_{\mu 0} g_{\nu 0} V_0(\mathbf{x}-\mathbf{y}), \quad V_0(r) = K_0^3 r^2, \quad (16)$$

and study the properties of the thermal mass-gap Eq. (13) numerically. The harmonic oscillator confining potential is known to lead to the simplest nontrivial mass-gap equation. Indeed, since the Fourier transform of the quadratic potential (16) is given by the Laplacian of the 3D delta-function, the mass-gap equation is differential [2]. Switching from the harmonic oscillator potential to another form of the potential amounts to simple modifications of the quantitative results, while all qualitative conclusions remain intact (see, for example, Ref. [10] for a detailed analysis).

Then the vacuum energy is given by the functional

$$E_{\rm vac} = -\frac{g}{2}\int \frac{d^3p}{(2\pi)^3}\Bigl(A_p \sin\varphi_p + B_p \cos\varphi_p\Bigr), \quad (17)$$

$$A_p = m + \frac{1}{2}C_F \int \frac{d^3k}{(2\pi)^3} V_0(\mathbf{p}-\mathbf{k}) \sin\varphi_k, \quad (18)$$

$$B_p = p + \frac{1}{2}C_F \int \frac{d^3k}{(2\pi)^3}\, (\hat{\mathbf{p}}\hat{\mathbf{k}}) V_0(\mathbf{p}-\mathbf{k}) \cos\varphi_k. \quad (19)$$

Here $C_F = \frac{1}{2}(N_C - 1/N_C)$ is the $SU(N_C)_C$ Casimir operator in the fundamental representation and the degeneracy factor g counts the number of independent quark degrees of freedom, $g = (2s+1)N_C N_f = 12$. Also the substitution $\sum_p \to V\int[d^3p/(2\pi)^3]$ was made for the sake of convenience. The mass-gap equation takes the form:

$$[A_p \cos\varphi_p - B_p \sin\varphi_p] - (T/2)(\delta S/\delta\varphi_p) = 0, \quad (20)$$

with $\delta S/\delta\varphi_p$ given in Eq. (14) or, in the explicit form,

$$K_0^3\left[\varphi_p'' + \frac{2\varphi_p'}{p} + \frac{\sin 2\varphi_p}{p^2}\right] = 2p\sin\varphi_p - 2m\cos\varphi_p - $$
$$-\frac{T}{2}\sin\varphi_p\left[2\ln\left(\tan^2\frac{\varphi_p}{2}\right) + \cos\varphi_p \ln 2\right]. \quad (21)$$

In what follows we consider the chiral limit and set the quark current mass $m = 0$.

In Fig.1 we plot solutions of Eq. (21) for various values of the temperature. From this figure one can see that, as was anticipated before, with the increase of the temperature, the chiral angle approaches chirally symmetric trivial solution. In addition, in Fig.2, we plot the

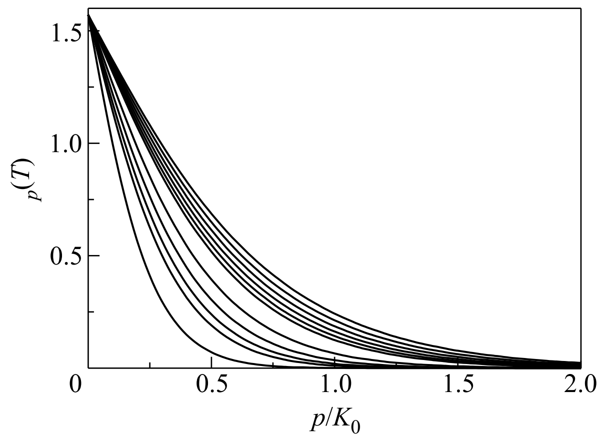

Fig.1. Solutions to the mass-gap equation (21) for the temperatures $T = 0$ (the upper curve), $0.2K_0$, $0.4K_0$, $0.6K_0$, $0.8K_0$, K_0, $2K_0$, $3K_0$, $4K_0$, $5K_0$, and $10K_0$ (the lower curve)

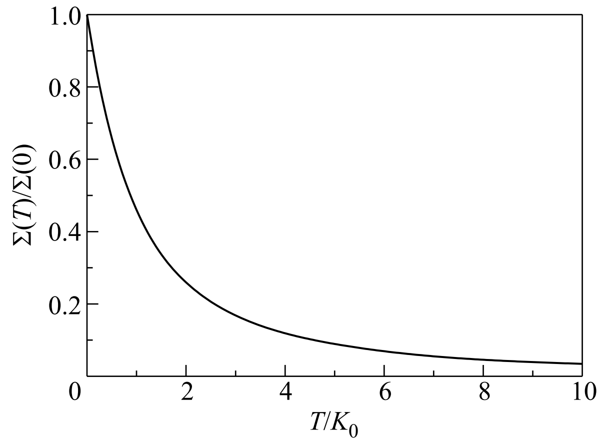

Fig.2. The chiral condensate $\Sigma(T)$ (normalised to the that at $T = 0$) as a function of T (measured in the units of the K_0)

temperature dependence of the chiral condensate

$$\Sigma(T) = \langle\bar{q}q\rangle = -\frac{3}{\pi^2}\int_0^\infty dp\, p^2 \sin\varphi_p(T), \quad (22)$$

which also demonstrates the same pattern of chiral symmetry restoration.

4. Discussion. In this paper we studied the GNJL model at finite temperatures. The effect of pair creation affected by both quark selfinteractions and temperature, is conveniently described in the formalism of the Bogoliubov–Valatin transformations, parametrised by the order parameter: the thermal chiral angle $\varphi_p(T)$. At $T = 0$ spontaneous breaking of chiral symmetry happens as a result of such $^3P_0$ pairs condensation in the vacuum. The resulting true (BCS) vacuum possesses a lower vacuum energy than the trivial unbroken vacuum.

At finite temperatures, two processes proceed in opposite directions: confining interaction tends to condense more $^3P_0$ quark–antiquark pairs in the vacuum, that is to increase the chiral angle, while the effect of the temperature is a suppression of the latter. The balance of these two tendencies is encoded in the thermal mass-gap Eq. (13). Notice that, since the mass-gap equation is a nonlinear equation, then the interplay of these two effects is essentially nonlinear. For low T's the full thermal mass-gap equation can be split, however, into two parts: the nonlinear mass-gap equation which describes chiral symmetry breaking and the linear thermal equation which describes its partial restoration due to the temperature. The resulting chiral angle is therefore just a sum of the zero-T angle $\varphi_p(T=0)$ and the thermal correction $\delta\varphi_p(T)$. Let us stress once more that such a disentanglement is only possible at $T \ll K_0$, where K_0 is the natural energy scale introduced to the theory by the confining interaction. For high temperatures ($T \gg K_0$), chiral symmetry restoration is expected to take place, so the thermal mass-gap equation can be simplified under the assumption $\varphi_p(T) \ll 1$. However, for $T \sim K_0$, the full nonlinear thermal mass-gap equation has to be considered and solved.

As soon as the thermal mass-gap equation is solved and the chiral angle is found, it can be used in Bethe–Salpeter equations for hadrons or, alternatively, a second, generalised Bogoliubov-like transformation can be performed in order to diagonalise, for the hadronic sector, the Hamiltonian of the theory therefore enabling us to build bound–state equations for mesons [8]. The proposed real-time approach is general and can be applied to any microscopic model for confined systems. It allows one to investigate microscopically the proper balance between chiral symmetry breaking due to confinement and chiral symmetry restoration due to the temperature.

A brief comment on the temperature behaviour of the model is in order. An intrinsic feature of the GNJL model, with confinement provided by an infinitely rising potential, is that chiral symmetry breaking happens for any temperature T, thus the chiral symmetry restoration process being asymptotical. This is related to the fact that temperature effectively reduces the confining interaction strength, while even quite weak confinement is able to break chiral symmetry. Notice however that any realistic interquark interaction is expected to be temperature-dependent. In particular, lattice calculations [11] support the conjecture [12] that, while colour–magnetic vacuum fields do not change across the deconfinement phase transition, the QCD vacuum loses its confining colour–electric part. If this feature is incorporated into the quark kernel, the resulting mass-gap equation will describe chiral phase transition.

The small-T behaviour of the model also deserves special attention. From general considerations (see Ref. [13] for a review) one expects the following behaviour of the chiral condensate at small T's:

$$\Sigma(T)/\Sigma(0) = 1 - T^2/(8f_\pi^2) + \ldots \qquad (23)$$

However, since $f_\pi \sim \sqrt{N_C}$, such corrections $O(T^2)$ appear only when the theory is considered in the order $1/N_C$ or higher, that is beyond the mean-field approximation – see, for example, Ref. [14]. In the GNJL model the mean-field approximation corresponds to the BCS level, considered in this paper, while proceeding beyond the BCS approximation corresponds to the full diagonalisation of the Hamiltonian (1), including the $:H_4:$ part – see Eq. (3). The details of this diagonalisation procedure can be found in Ref. [8]. The correction to the chiral condensate at small temperatures found in this paper is linear in T which differs from the behaviour obtained in other models (see, for example, the discussion of the NJL model in Ref. [14]). This should not come as a surprise however since the BCS vacuum of the model behaves like an infinite medium which is disturbed by the temperature, so that the response of this medium, in the leading order, is linear in the perturbation, that is in T. The medium has to possess rather peculiar features in order to respond quadratically or in higher powers of the temperature, while we do not observe such properties of the BCS vacuum. However, beyond the BCS approximation, the GNJL model is subject to a severe realignment, so that it finally acquires a description entirely in terms of mesonic states [8], with the massless pions being the lowest states in the spectrum. Then, with the free energy calculated in terms of a pion gas, the standard approach can be applied to recover the usual behaviour (23). Given a complicated form of the Bogoliubov-like transformation which diagonalises the Hamiltonian (1) up to the order $1/N_C$ (to be confronted to the $1/\sqrt{N_C}$ diagonalisation at the BCS level), the cancellation of the linear in the temperature term in the chiral condensate does not look unnatural. Indeed, in the mesonic vacuum, only properly correlated pairs of mesons can be created or annihilated which appears to be a more involved process than $^3P_0$ quark–antiquark pairs creation/annihilation in the BCS vacuum. Thus the mesonic vacuum is expected to respond weaker to the temperature increase, as compared to the quark vacuum.

Notice that the approach suggested in this paper is very different from other, commonly adopted approaches to investigation of the thermal properties of QCD. Indeed, while the "standard" procedure amounts to indirect tests of the vacuum through the properties of the hadrons created on top of the vacuum, in our approach the vacuum is probed directly. This makes important further investigations of the interplay between our real-time approach and the standard imaginary-time approach to finite temperatures in confined systems. We leave this problem for future publications.

The authors would like to thank A. Abrikosov Jr., L. Glozman, Yu. Simonov, and V. Vieira for useful discussions. A.N. would like to thank the staff of the Centro de Física das Interacç oes Fundamentais (CFIF-IST) for cordial hospitality during his stay in Lisbon, where this work was originated and to acknowledge the support of the State Corporation of Russian Federation "Rosatom" as well as of the grants # RFFI-09-02-00629a, # RFFI-09-02-91342-NNIOa, # DFG-436 RUS 113/991/0-1(R), # NSh-843.2006.2, # PTDC/FIS/70843/2006-Fisica, and of the non-profit "Dynasty" foundation and ICFPM.

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