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Superconductor-insulator transition in thin metallic films induced by interface-roughness scattering

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Disorder due to small random variations of the width L of thin films leads to scattering for superconducting particles. It is shown for the first time that this disorder, interface-roughness scattering for bosons, gives rise to a superconductor-insulator transition, as observed for instance in amorphous Bi films. We present a model calculation of a disordered interacting Bose condensate in a quantum well of finite width L. Films with $L < L_C$ are insulating, with L_C as the critical width, while films with $L > L_C$ are superconducting. Disorder strongly reduces the critical temperature T_C of the superconducting phase and T_C vanishes at L_C. A phenomenological two-fluid model is also discussed.

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Superconductor-insulator transitions (SIT's) have been observed in metallic films (Bi, Pb, InO) and in high-T_C superconductors. For a recent review see Ref.[1]. With decreasing width of films the normal state resistance increases very rapidly for temperatures higher than the critical temperature, see Fig.2 of Ref.[2]. It has been found in experiment that films with a thickness L smaller than a critical (C) thickness L_C behave isolating for temperatures approaching zero temperature, while films with $L > L_C$ behave superconducting for temperatures smaller than a critical temperature T_C [2]. This critical temperature depends on disorder and is smaller than the critical temperature of the bulk material T_{C0}. From experiment it is clear that the width of films is a crucial parameter in describing the SIT and that it is related to disorder.

Fluctuations of the width of thin films lead to strong fluctuations of the confinement energy, which give rise to strong disorder, the interface-roughness scattering (IRS). We suggest that IRS for electrons can explain the strong increase of the normal state resistance with decreasing width observed in these films [2]. In the present work we propose for the first time that IRS also is the source of disorder for superconducting particles and we argue that IRS for bosons might explain the SIT observed at low temperatures.

For quantum particles of mass m in the n-subband of a quantum well (QW) of width L the confinement energy is given by $E_n = n^2\pi^2\hbar^2/2mL^2$ and variations $\delta \mathrm{L} \equiv \Delta < L$ of the well width give rise to fluctuations $\delta E_n \propto n^2\Delta/L^3$. This leads to scattering of quantum particles in thin films and the disorder potential is proportional to $n^4\Delta^2/L^6$. It is well known that IRS is the relevant scattering mechanism for electrons in thin QW's [3 – 5]. In such QW's the conductivity σ increases as $\sigma \propto L^6$, as predicted by theory [6]. It was argued that for electrons in thin QW's a metal-insulator transition (MIT) occurs [6]. Such a MIT was seen in a recent experiment on GaAs/AlGaAs superlattices [5].

In the following we consider IRS for superconducting particles in thin films. The film width is the crucial parameter of our approach while in earlier work on bosons in two dimensions only systems with zero width have been considered. With decreasing width of films one observes a reduction of the critical temperature $T_C < T_{C0}$ with $T_C = 0$ at the SIT. The reduction of T_C can be caused by the reduction of the amplitude of the order parameter. In this case the order parameter should disappears at the critical point and no superconducting pairs are present for $L < L_C$. There exist also the possibility that the condensate becomes localized, and this would mean that in the insulating phase the amplitude of the order parameter is still finite. Here we consider the latter case, when in the insulating phase the Bose condensate is localized due to disorder.

In the present paper we discuss the effects of IRS on the transport properties of an interacting Bose condensate. Some time ago we studied the effects of disorder on a Bose condensate at zero temperature in three [7] and in two [8] dimensions. This model is known as the dirty Bose model [9]. Using a mode-coupling approach we described the transport properties of the condensate in the presence of disorder. The mode-coupling approach corresponds to a self-consistent theory for the current-current relaxation function. For weakly disordered systems one obtains results in agreement with perturbation theory. For strong disorder the theory describes a SIT. We argued that for strong disorder and/or low boson density the Bose condensate is a Bose glass, where the Bose condensate is pinned by disorder [7, 8]. In the present case of IRS we calculate the critical width L_C of the SIT. We present new results concerning the influence of disorder on the critical temperature, as determined by resistance measurements. Finally, we generalise the one-fluid model to a phenomenological two-fluid model in order to discuss the possibility of a superconductor-metal transition (SMT).

For zero temperature we consider a two-dimensional system of bosons of density N_B in the condensate phase in a QW of width L and with infinite barriers. The random-field approximation is used to take into account interaction effects via the Coulomb interaction [10]. The Fourier transform of the Coulomb interaction potential is given by $V(q) = 2\pi e_B^2 F(q,L)/\varepsilon_L q$. The form factor $F(q,L)$ has been calculated earlier [11] and for an ideal two-dimensional system with $L = 0$ one finds $F(q, L = 0) = 1$. ε_L represents the background dielectric constant and the mass of the bosons is m_B. Screening effects within the random-phase approximation introduce an additional length scale into the system and this scale is given by the screening wave number $q_S = 2/(a^* r_S^{2/3}) \propto N_B^{1/3}$, where $a^* = a_H \varepsilon_L m_e/m_B$ is the effective Bohr radius expressed with the hydrogen Bohr radius $a_H = 0.529$Å. m_e is the free electron mass and r_S is the Wigner-Seitz parameter in two dimensions defined by $r_S^2 = 1/\pi a^{*2} N_B$.

For a QW the IRS for Bose particles in the condensate is described by a random potential with Gaussian form [6]

$$< |U(q)|^2 >= 2\pi^5 \frac{\Delta^2 \Lambda^2}{L^6 m_B^2} \exp(-q^2\Lambda^2/4), \qquad (1)$$

characterized by the fluctuation length Δ of the well width and the length Λ of the fluctuation within the well [12]. The factor 2 in Eq.(1) represents the two interfaces of the film. We stress that only within a model of finite width disorder due to IRS can be discussed. Note that the models discussed in [8, 9] are models where zero width was assumed.

The essential parameter of the theory is the parameter A, which describes the influence of the disorder on the Bose condensate. A is given in terms of the random potential and the compressibility $g_I(q)$ of the Bose condensate and is expressed as [8]

$$A = \frac{1}{4\pi N_B^2} \int_0^\infty dqq < |U(q)|^2 > g_I(q)^2, \qquad (2)$$

with the compressibility of the Bose condensate given by

$$g_I(q) = \frac{4 N_B m_B}{q_S^2} \frac{q/q_S}{F(q,L) + q^3/q_S^3}. \qquad (3)$$

A describes, for zero temperature, the quantum-phase transition from a superconductor for $A < 1$ to an insulator for $A > 1$ [7]. The transition point is defined by $A = 1$. In Fig.1 we show numerical results of the critical well width L_C versus condensate density N_B. For

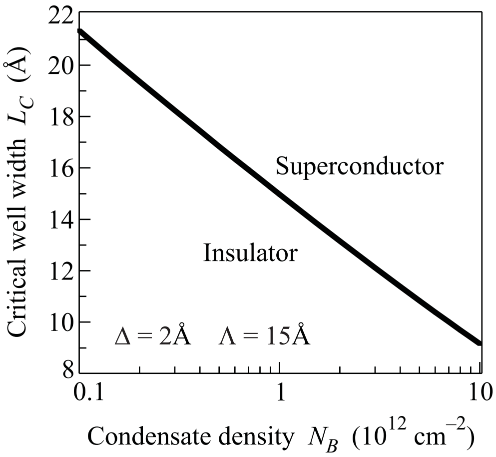

Fig.1. Critical width L_C of the quantum well of the superconductor-insulator transition for interface-roughness scattering as function of the condensate density N_B for $m_B = 0.6 m_e$, $e_B = 2e$, and $\varepsilon_L = 10$

well width $L > L_C$ the Bose condensate is superconducting while for well width $L < L_C$ an isolating Bose condensate is found. This is in agreement with experiments where the width of the films was reduced and a SIT was found at L_C [1, 2]. More recently, in experiment one could reduce the density of carriers and a SIT was found at a critical carrier density [13, 14]. In Fig.1 we see that this also is in agreement with our calculation: decreasing the condensate density for fixed well width leads from a superconducting phase to an insulating phase at a critical condensate density $N_{B,C}$. This is essentially a screening effect, at lower density there is less screening and disorder effects are stronger. The parameters used in Fig.1 are adapted to Bi-films studied in [2, 13]. The variation of L_C with boson density is weak, note the logarithmic scale in Fig.1 for the boson density. However, as function of the microscopic parameters Δ and Λ the critical width can strongly be changed, in agreement with experiment [2]. It was noted [13] that disorder varies weakly with density while it varies strongly with width. This is in agreement with our model: disorder due to IRS, see Eq.(1) for $q = q_S$ and $q_S\Lambda \ll 1$.

If the Bose condensate is in the superconducting phase one finds a δ-peak in the real part of the dynamical conductivity given by [7]

$$\sigma(\omega \to 0) = \pi \frac{N_B e_B^2}{m_B} (1 - A)\delta(\omega). \qquad (4)$$

This δ-peak shows that the Bose condensate is superconducting for $A < 1$. A finite value of disorder, described by A, reduces the strength of the δ-peak in the conductivity. This can be interpreted as an effective superconducting density $N_S = N_B(1 - A)$. Note, however, that in our theory the condensate density N_B is assumed to be independent of disorder. At the SIT the static conductivity is finite and given by the critical Boson density $N_{B,C}$ and by the parameter C_C, also determined at the critical point. C is the second parameter of the theory depending on disorder and the compressibility and given by [8]

$$C = \frac{m_B}{4\pi N_B^3} \int_0^\infty dq < |U(q)|^2 > g_I(q)^3/q. \qquad (5)$$

One gets for the conductivity at the critical point $\sigma_C = N_{B,C} e_B^2 C_C^{1/2}/m_B$. With $e_B = 2e$ we describe the conductivity at the transition point by the dimensionless coefficient D_2 as

$$\sigma_C = e^2 D_2/h. \qquad (6)$$

Numerical results for the dimensionless parameter D_2 versus N_B are shown in Fig.2. From our calculation we conclude that with increasing condensate density the critical conductivity increases. This is in agreement with recent experiments [13]. The conductivity at the transition point is smaller than found in experiment where $2 < D_2 < 5$ was reported [1, 2, 13]. However, we mention that our value of D_2 was calculated for a small value of $\Lambda = 15$ Å. For larger values of Λ one also gets larger values for D_2. This should be studied in more detail in the future. Moreover, our calculation is for a one fluid model (bosons), while in real metallic films two fluids are present, bosons and fermions, and in experiments, made at a finite temperature, both fluids might contribute to the conductivity.

For $A = 1$ the conductivity at the transition point is finite and the resistance as a function of temperature will never become zero: $T_C = 0$ for $A = 1$. We conclude that the critical temperature T_C, determined via resistance measurements $R(T)$ by the condition $R(T_C) = 0$, must depend on $N_S = N_B(1 - A)$. Only for clean systems without disorder, where $A = 0$, T_{C0} is determined by N_B. Suppose that the critical temperature of the superconducting transition without disorder is given by $T_{C0} \propto N_B^\nu$ with an exponent ν. We conclude that with disorder the critical temperature is given by

$$T_C = T_{C0}(1 - A)^\nu. \qquad (7)$$

In a simple linear interpolation with $T_C = T_{C0}$ for $A = 0$ and $T_C = 0$ for $A = 1$ one gets $T_C = T_{C0}(1 - A)$. From Eq.(7) we see that the critical temperature decreases with increasing disorder. Near the critical point one gets $T_C = T_{C0}(1 - N_{B,C}/N_B)^\nu$ or $T_C = T_{C0}(1 - L_C/L)^\nu$. Eq.(7) is derived for a Bose condensate. However, we mention that calculations for BCS superconductors also showed a strong decrease of T_C with disorder [15].

Eq.(7) should also be applicable to the SIT in three-dimensional systems, for instance for thick films and bulk systems, and to other forms of disorder, for instance impurity scattering or alloy scattering. It is well known that for a free Bose gas in three dimensions one has $T_{C0} \propto N_B^{2/3}$. From this we predict $T_C \propto (N_B - N_{B,C})^{2/3}$ if disorder is present. This relation was found experimentally for superfluid Helium disordered by nanopores [16, 17]. In doped semiconductors the superconductivity is influenced by disorder, which can be modified by doping and annealing. Recently, superconductivity has been found in Gallium-doped Germanium and a huge difference has been found between the superconducting charge carrier density $N_S = 1 \cdot 10^{14}\mathrm{cm}^{-3}$ and the normal conducting carrier density $N = 1 \cdot 10^{21}\mathrm{cm}^{-3}$ [18]. We believe that the difference between N_S and N is due to disorder. This implies that the studied samples are near the SIT with $A \approx 1$.

For weak disorder and for a short-range random potential we can characterize the parameter $A \propto \langle|U(q)|^2\rangle$ by the conductivity $\sigma_m \equiv \sigma(T \approx T_{C0})$ in thin film for $T > T_C$ in the metallic (m) phase. Here we assume that $\sigma(T \approx T_{C0}) \propto L^6/\Delta^2\Lambda^2$ is rather independent of temperature for $T_C < T < 2T_{C0}$, characteristic of a metallic system. For $\sigma_m \gg \sigma_C$ we conclude that the variation of the critical temperature with the normal state conductivity is given by

$$T_C - T_{C0} \propto 1/\sigma_m^\nu. \qquad (8)$$

Eq.(8) is only valid for a short-range random potential. For $\nu = 1$ Eq.(8) can explain the nearly linear reduction of the critical temperature with the resistance often found in experiments.

A real superconducting system consists of a mixture of electrons and bosons. Including disorder we expect the SIT for the Bose condensate (B) at a critical disorder (D) potential $U_{BD,C}$. For fermions (F) one expects a MIT at $U_{FD,C}$. Within a two-fluid model the conductivity of the two fluids will be additive and on has to discriminate two cases. In the first case, where $U_{BD,C} > U_{FD,C}$, there exists only a SIT at $U_{BD,C}$, see the left-hand side of the schematic Fig.3.

It appears that in most materials such a situation is realized [2, 13, 14]. In the second case, where $U_{BD,C} < U_{FD,C}$, two phase-transitions occur: with increasing disorder first a SMT occurs at $U_{BD,C}$ and later a MIT

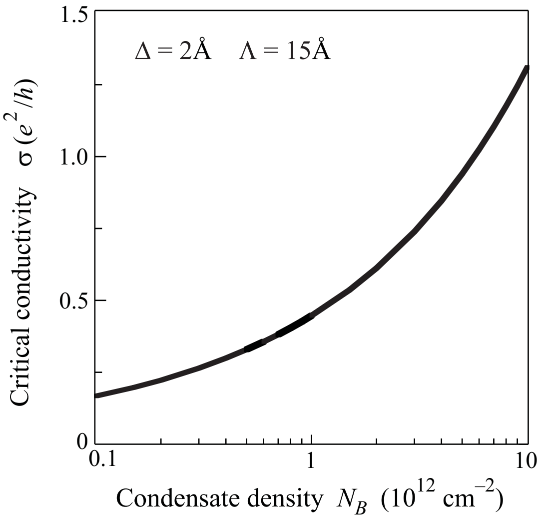

Fig.2. Critical conductivity $\sigma = e^2 D_2/h$ at the superconductor-insulator transition for interface-roughness scattering versus condensate density N_B for $m_B = 0.6m_e$, $e_B = 2e$, and $\varepsilon_L = 10$

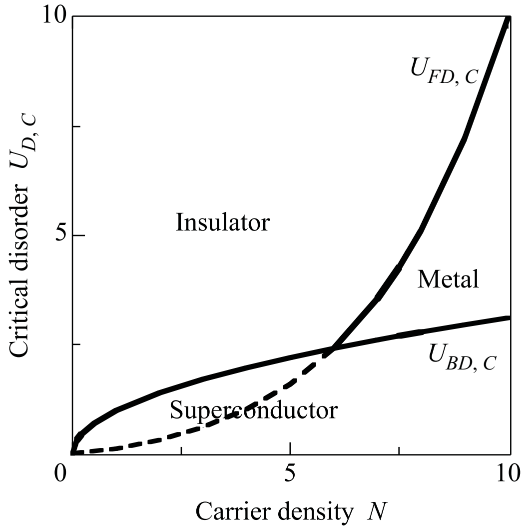

Fig.3. Critical disorder potential $U_{D,C}$ versus carrier density $N = N_B + N_F$ within a two fluid model for fermions (F) and bosons (B)

occurs at $U_{FD,C}$. An extended metallic phase exists between the superconductor and the insulator, see the right-hand side of the schematic Fig.3. In this metallic phase the Bose condensate is localized and the (metallic) electrons determine the transport properties. It is possible that such a metallic phase has already been seen in experiment [18 – 20].

For $U_{BD,C} < U_{FD,C}$ we expect that

$$T_C = T_{C0}(1 - \sigma_C/\sigma_m)^\nu \qquad (9)$$

is a good approximation, because localisation effects for electrons can be neglected in this case. We stress that the data of $Mo_{0.79}Ge_{0.21}$ films [21], also described by the theory [15] for BCS superconductors, can nicely be described by $T_C = T_{C0}(1 - \sigma_C/\sigma_m)^2$ with $T_{C0} = 7.3\,$K and $\rho_C = 1/\sigma_C = 2300\Omega$. Note, that for Eq.(9) no assumptions have been made concerning the influence of disorder on the microscopic parameters of the attractive force creating the superconducting particles.

In the present paper we used an one-subband model with infinite confinement. While an one-subband model is well adapted for a Bose condensate, because all bosons are condensed to the lowest energy state, it might be questionable for fermions. Moreover, only for infinite confinement the $\sigma \propto L^6$ law for IRS is expected. For finite confinement the $\sigma \propto L^6$ power law is replaced by a weaker dependence $\sigma \propto L^\alpha$ where α depends on the confinement strength and the carrier density [22]. Such a weaker dependence has already been seen in semiconductor QW's [4] and might explain the width dependence of the normal state resistance in thin metallic Bi-films showing a SIT at low temperature, see Fig.2 in [2]. We suggest that measurements of the conductivity versus film width could give direct information about the confinement strength and the microscopic parameters Δ and Λ.

Finally, it should also be mentioned that the morphology of these thin metallic films [1, 2, 13] is far from being well understood. Doped semiconductors might be an alternative approach for the study of the SIT. In addition we admit that in most materials the SIT happens for a BCS superconductor and our results for a dirty charged Bose condensate might only be of qualitative relevance. On the other hand one knows that Bose condensation (strong coupling) and BCS-superconductors (weak coupling) are related [23]. We stress that Eq.(4) is valid not only for a charged Bose condensate but also for a neutral Bose condensate and is independent of the dimension of the system. We assumed that disorder does not influence the order parameter: the disorder considered is not pair-breaking. This is an approximation. But our calculation clearly shows than even in this case there exists a maximum amount of disorder, described by $A = 1$, and superconductivity is only expected if $A < 1$. For larger disorder the system is insulating.

We claim that our theory [7, 8] describes an upper limit of disorder where superconductivity with $R = 0$ can be found. The novel ideas of the present paper are: (i) IRS is the important source of disorder in thin uniform films and (ii) IRS also applies to bosons. IRS can only be discussed in films of finite width, here considered for the first time.

In conclusion, we have proposed and investigated interface-roughness scattering for superconducting particles in thin metallic films. A Bose condensate in a quantum well of finite width (thickness) with Coulomb interaction and disorder has been used as model. We showed that interface-roughness scattering leads to a superconductor-insulator transition at a critical width or a critical boson density, in agreement with experiment on thin films. We deduced that disorder leads to a strong reduction of the critical temperature and we discussed this reduction qualitatively and quantitatively. A superconductor-metal transition is predicted using a phenomenological two-fluid model. The new metallic phase consist of a localized Bose condensate with conducting electrons.

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