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Contrasting Different Scenarios for the Quantum Critical Point

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Competing scenarios for quantum critical points (QCPs) of strongly interacting Fermi systems signaled by a divergent density of states at zero temperature are contrasted. The conventional scenario, which enlists critical fluctuations of a collective mode and attributes the divergence to a coincident vanishing of the quasiparticle strength z, is shown to be incompatible with identities arising from conservation laws prevailing in the fermionic medium. An alternative scenario, in which the topology of the Fermi surface is altered at the QCP, is found to explain the non-Fermi-liquid thermodynamic behavior observed experimentally in Yb-based compounds close to the QCP. It is suggested that combination of the topological scenario with the theory of quantum phase transitions will provide a proper foundation for analysis of the extended QCP region.

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Introduction. Fundamental understanding of the behavior of Fermi systems in the vicinity of quantum phase transitions persists as one of the most challenging objectives of condensed-matter research. As it involves second-order transitions occurring at a critical density ρ_c, the problem is even more difficult than in the classical regime, since the description of quantum fluctuations entails a new critical index, the dynamical critical exponent [1, 2]. In several heavy-fermion metals–notably Yb-based compounds [3]–critical temperatures $T_N(H)$ can be driven to zero by extremely weak magnetic fields H, creating a quantum critical point (QCP). It is commonly believed that low-temperature fluctuation contributions to the free energy, specific heat $C(T)$, and other thermodynamic quantities must then follow power laws in T, the Sommerfeld ratio $C(T)/T$ being divergent at $T \to 0$.

From a pedestrian standpoint, such non-Fermi-liquid (NFL) behavior must extend some distance from the QCP, implying separation at $T = 0$ of a domain of magnetic ordering from a Fermi-liquid (FL) regime–as is indeed the case in the heavy-fermion metal YbAgGe [4]. However, this example is unique; otherwise, the two phases seem to abut each other at the quantum critical point (QCP). Since the standard FL formalism is applicable on the FL side of the QCP, $C(T)/T$ is proportional to the effective mass M^* in this region and it follows that M^* must diverge at the QCP.

Conventional arguments that quasiparticles in Fermi liquids "get heavy and die" [5] at the QCP commonly employ the textbook formula

$$\frac{M}{M^*} = z\left[1 + \frac{1}{v_F^0}\left(\frac{\partial\Sigma(p,\varepsilon)}{\partial p}\right)_0\right], \qquad (1)$$

where $v_F^0 = p_F/M$ and the derivative is evaluated at $p = p_F$ and $\varepsilon = 0$, single-particle (sp) energies being referred to the chemical potential μ. The factor $z = [1 - (\partial\Sigma(p,\varepsilon)/\partial\varepsilon)_0]^{-1}$ is the quasiparticle weight of the sp state at the Fermi surface. The conventional belief, traced back to Ref. [6], holds that the divergence of M^* at the QCP is caused by the vanishing of the z factor in Eq. (1), stemming from the divergence of the derivative $(\partial\Sigma(p,\varepsilon;\rho_c)/\partial\varepsilon)_0$ at implicated second-order phase transition points.

However, this scenario is problematic. As will be seen, the z-factor does not vanish at the points of second-order phase transitions. It will be argued that the divergence of the density of states $N(T)$ at the QCP is in fact associated with a rearrangement of *single-particle degrees of freedom* [7], rather than with critical fluctuations. Even so, the divergence of $N(T)$ at the QCP does give rise to some second-order phase transitions, occurring at $T = T_N$ in the vicinity of the QCP. Accordingly, the full pattern of the temperature interval from 0 to $T \geq T_N$ in the QCP region is determined by an intricate interplay between the two mechanisms. The present analysis is limited to the disordered side of the QCP where the impact of sp degrees of freedom is decisive, while the role of critical fluctuations is suppressed.

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Other regions of the phase diagram will be analyzed elsewhere.

Fault lines of the conventional scenario for the divergence of M^*. We begin by exposing inconsistencies of the standard derivation leading to divergence of $(\partial\Sigma(p,\varepsilon)/\partial\varepsilon)_0$ in the vicinity of second-order phase transitions. This derivation is based on a fundamental relation of many-body theory,

$$\frac{\partial\Sigma_{\alpha\delta}(p,\varepsilon)}{\partial\varepsilon}=-\frac{1}{2}\int U_{\alpha\delta\gamma\beta}(\mathbf{p},\varepsilon,\mathbf{l},\varepsilon_1)\frac{\partial G_{\beta\gamma}(l,\varepsilon_1)}{\partial\varepsilon_1}\frac{d\mathbf{l}d\varepsilon_1}{(2\pi)^4 i}, \tag{2}$$

where U is the scattering amplitude, irreducible in the longitudinal particle-hole channel.

The treatment in question retains only the pole part G^q of the sp Green function $G = zG^q + G^r$ and a singular part of the diagram block U that is supposedly responsible for the divergence of $(\partial\Sigma(p,\varepsilon)/\partial\varepsilon)_0$. In the case of critical spin fluctuations addressed in Ref. [6], there is no the direct spin-fluctuation contribution to $(\partial\Sigma(p,\varepsilon)/\partial\varepsilon)_0$, while the exchange term has the form $U_{\alpha\delta\gamma\beta}(\mathbf{p},\varepsilon,\mathbf{l},\varepsilon_1) = g^2\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\gamma\delta}\chi(q,q_0)$ involving the spin susceptibility $\chi(q,q_0)$, where $q = |\mathbf{p}-\mathbf{l}|$, $q_0 = \varepsilon - \varepsilon_1$, and g is a dimensionless effective coupling constant.

On the FL side of the QCP, $\Sigma_{\alpha\delta}(p,\varepsilon) = \Sigma(p,\varepsilon)\delta_{\alpha\delta}$ and $\mathrm{Im}\, G^q(p,\varepsilon) = -2\pi\mathrm{sgn}[\epsilon(p)]\delta(\varepsilon-\epsilon(p))$, where $\epsilon(p)$ is an appropriate sp spectrum. In accord with the FL perspective and conventions, we now write $U_{\alpha\delta\gamma\beta} \equiv \equiv U_o\delta_{\alpha\delta}\delta_{\gamma\beta} + U_s\boldsymbol{\sigma}_{\alpha\delta}\boldsymbol{\sigma}_{\gamma\beta}$, and the single component U_o enters Eq. (2), yielding

$$\frac{\partial\Sigma(p,\varepsilon)}{\partial\varepsilon}=-\int U_o(\mathbf{p},\varepsilon,\mathbf{l},\varepsilon_1)\frac{\partial G(l,\varepsilon_1)}{\partial\varepsilon_1}\frac{d\mathbf{l}d\varepsilon_1}{(2\pi)^4 i}. \tag{3}$$

Applying the identity $\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\gamma\delta} = \frac{3}{2}\delta_{\alpha\delta}\delta_{\gamma\beta} - \frac{1}{2}\boldsymbol{\sigma}_{\alpha\delta}\boldsymbol{\sigma}_{\gamma\beta}$, we have $U_o(q,q_0) = (3/2)g^2\chi(q,q_0)$ and Eq. (2) reduces to [8, 6]

$$\left(\frac{\partial\Sigma(p,\varepsilon)}{\partial\varepsilon}\right)_0 \sim -g^2 z\left(\frac{dp}{d\epsilon(p)}\right)_0\int\chi(q,q_0=0)\frac{qdq}{\pi^2}. \tag{4}$$

A key assumption made in Ref. [6], and generally adopted in subsequent treatments, is the Ornstein-Zernike (OZ) form

$$\chi(q,q_0=0)\equiv\chi_{\mathrm{OZ}}(q)=\frac{4\pi}{\xi^{-2}+q^2} \tag{5}$$

for the static correlation function $\chi(q)$, with the correlation length ξ diverging at the critical point. Inserting this ansatz into Eq. (4) together with $v_0 = (d\epsilon(p)/dp)_0 = = v_F^0 M/M^*$, and assuming that the momentum dependence of $\Sigma(p,\varepsilon)$ is not altered dramatically, one arrives at

$$\left(\frac{\partial\Sigma(p,\varepsilon)}{\partial\varepsilon}\right)_0 \sim -\frac{g^2}{v_F^0}\ln(p_F\xi), \tag{6}$$

which implies that $(\partial\Sigma(p,\varepsilon;\rho)/\partial\varepsilon)_0$ diverges at $\xi\to\infty$. However, the applicability of the OZ approximation to evaluation of $(\partial\Sigma(p,\varepsilon)/\partial\varepsilon)_0$ in the QCP domain has never been proved.

This deficiency exhorts us to check the compatibility of the OZ approximation in homogeneous matter with a set of identities involving the derivative $(\partial\Sigma(p,\varepsilon)/\partial\varepsilon)_0$, all having the same structure as Eq. (2). In so doing, we observe that Eq. (2), which follows from particle-number conservation with the aid of the scalar gauge transformation $\Psi(t)\to\Psi(t)e^{iVt}$, is but one instance of a class of similar identities [9]. Any conservation law existing in the medium generates a corresponding identity involving $(\partial\Sigma(p,\varepsilon)/\partial\varepsilon)_0$. For example, momentum conservation in homogeneous, isotropic matter, associated with the vector gauge transformation $\Psi(t)\to\Psi(t)e^{i\mathbf{pA}t}$, results in the well-known Pitaevskii relation [10]

$$\frac{\partial\Sigma(p,\varepsilon)}{\partial\varepsilon}=-\int U_o(\mathbf{p},\varepsilon,\mathbf{l},\varepsilon_1)\frac{\partial G(l,\varepsilon_1)}{\partial\varepsilon_1}\frac{(\mathbf{pl})}{p^2}\frac{d\mathbf{l}d\varepsilon_1}{(2\pi)^4 i}. \tag{7}$$

Analogously, in the model of Ref. [6] the spin operator σ_3 commutes with the Hamiltonian, and the gauge transformation $\Psi(t)\to\Psi(t)e^{i\sigma_3 Vt}$ leads to the relation

$$\frac{\partial\Sigma(p,\varepsilon)}{\partial\varepsilon}=-\int U_s(\mathbf{p},\varepsilon,\mathbf{l},\varepsilon_1)\frac{\partial G(l,\varepsilon_1)}{\partial\varepsilon_1}\frac{d\mathbf{l}d\varepsilon_1}{(2\pi)^4 i}. \tag{8}$$

Even more conservation laws exist in nuclear and dense quark matter, each providing an identity like Eq. (2).

The standard manipulations applied to relation (2), leading to the result (6) via ansatz (5), can now be repeated for any such conservation identity. Irrespective of which identity is chosen, a divergent result is obtained for $(\partial\Sigma(p,\varepsilon;\rho_c)/\partial\varepsilon)_0$. Importantly, however, the signs of the divergent components of this derivative *do* depend on the choice made. For example, in the case of critical spin fluctuations, adoption of the anzatz (5) results in divergence of the derivative $(\partial\Sigma(p,\varepsilon;\rho_c)/\partial\varepsilon)_0$ whether Eq. (3) or Eq. (8) is adopted, but *different* signs are delivered, since in the OZ approximation the blocks U_o and U_s have *opposite* signs. Thus, Eq. (3) provides an "acceptable" negative sign, whereas Eq. (8) gives a "wild" *positive* sign (and a meaningless limit for z). In the case of short-wave-length fluctuations with critical wave number q_c, a similar wild result is obtained from Eq. (7) because the prefactor of the divergent part of

$(\partial\Sigma(p,\varepsilon)/\partial\varepsilon)_0$ differs from that derived [11] from Eq. (3) by a factor $\cos\theta_c = 1 - q_c^2/2p_F^2$. Since the nonsingular components of the block U are incapable of compensating the divergent OZ contributions to $(\partial\Sigma(p,\varepsilon;\rho_c)/\partial\varepsilon)_0$, we must conclude that the result (6) is fallacious, and that more sophisticated methods must be applied to clarify the situation in the critical-point region.

We call attention here to the situation for classical second-order phase transitions, where the OZ correlation function (5) is altered by scattering of the fluctuations themselves [12]. As a result, the actual correlation function $\chi(r,\rho_c)$ decays more rapidly at large distance r than $\chi_{OZ}(r,\rho_c) \propto 1/r$. In momentum space, $\chi(q\to 0,\rho_c)$ behaves [12] as $1/q^{2-\eta}$, with $\eta > 0$, compared with $\chi_{OZ}(q) \propto 1/q^2$. If a similar alteration of $\chi(q\to 0,\rho_c)$ occurs at $T\to 0$, then the integration leading to (6) is saturated at $\rho\to\rho_c$, ensuring that $z(\rho_c)\neq 0$.

Topological scenario for the QCP. With the condition $z(\rho_c) = 0$ ruled out, the effective mass in Eq. (1) can only diverge at a density ρ_∞ where the factor in square brackets, or equivalently the group velocity, changes sign. Such a QCP can be examined based on the FL equation [10]

$$\mathbf{v}(\mathbf{p}) = \frac{\partial\epsilon(\mathbf{p})}{\partial\mathbf{p}} = \frac{\partial\epsilon^0_{\mathbf{p}}}{\partial\mathbf{p}} + \int f(\mathbf{p},\mathbf{p}_1)\frac{\partial n(\mathbf{p}_1)}{\partial\mathbf{p}_1}\,d\tau_1, \qquad (9)$$

where $\epsilon^0_{\mathbf{p}}$ is the bare sp spectrum and $d\tau$ is the volume element in 3D or 2D momentum space. The $T = 0$ group velocity, being a continuous function of the interaction function $f(\mathbf{p},\mathbf{p}_1)$, changes its sign on the Fermi surface at the critical density ρ_∞. In 3D homogeneous matter the critical condition is

$$1 = f_1(p_\infty,p_\infty)p_\infty M/3\pi^2, \qquad (10)$$

where f_1 is the first harmonic of f and $p_\infty = (3\pi^2\rho_\infty)^{1/3}$. In this scenario, the QCP is associated with a rearrangement of *single-particle degrees of freedom*; no collective parameter is involved, and the symmetry of the ground state is not broken. Such *topological* phase transitions, induced by the interactions between quasiparticles, have been discussed for over two decades [7].

For a homogeneous medium there are in general two ways to realize a divergent density of states $N(0,\rho_\infty)$. Both options are associated with bifurcation points p_b of the equation $\epsilon(p,\rho_\infty) = 0$. As a condition for the divergence of the effective mass M^*, Eq. (10) refers to the case [13] $p_b = p_F$ in which the sp spectrum $\epsilon(p)$ has an inflection point. In the second option, where $p_b \neq p_F$, M^* remains finite, while $N(0,\rho_\infty)$ diverges due to vanishing of the group velocity at the bifurcation point.

Thus far we have dealt only with homogeneous systems. A comparable analysis of NFL behavior of heavy-fermion metals must include the effects of anisotropy, which are of special importance in the QCP region. An early study of topological phase transitions in anisotropic electron systems in solids, induced by electron-electron interactions, was carried out in Ref. [14].

Here it will be instructive to address the 2D electron liquid in a quadratic lattice, assuming the QCP electron Fermi line to be approximately a circle of radius p_∞, with the origin shifted to $(\pi/a,\pi/a)$. Since the group velocity $v_n(\mathbf{p}) = \partial\epsilon(\mathbf{p})/\partial p_n$ now has a well pronounced angular dependence, the topological *anisotropic* QCP is to be specified by the vanishing of $v_n(p,\phi;T=0,\rho_\infty)$ at the single point $p = p_\infty, \phi = 0$. On the disordered side of the QCP, where $\partial v_n(p,\phi)/\partial\phi > 0$, one has

$$v_n(p,\phi;T=0,\rho) = v_n(p,\phi=0) + a_\phi\phi^2 + a_\rho(\rho-\rho_\infty), \qquad (11)$$

where $v_n(p,\phi=0) = a_p(p-p_\infty)^2$ as in the inflection-point case treated in Ref. [13]. With the relation $n(\epsilon) = [1+\exp(\epsilon/T)]^{-1}$, the QCP density of states

$$N(T,\rho) \propto \frac{1}{T}\int n(\epsilon)(1-n(\epsilon))\frac{d\epsilon\,d\phi}{v_n(p(\epsilon),\phi;T,\rho)} \qquad (12)$$

determines the corresponding specific heat $C(T) = TdS/dT \sim TN(T)$ and thermal expansion coefficient $\beta(T) \sim -\partial S(T,\rho)/\partial P \sim -T\partial N(T,\rho)/\partial\rho$. To evaluate $N(T,\rho_\infty)$, we employ the relation

$$\epsilon(p,\phi) = a_p(p-p_\infty)^3/3 + a_\phi(p-p_\infty)\phi^2, \qquad (13)$$

which follows from Eq. (11) at $\rho = \rho_\infty$. Solution of this cubic algebraic equation yields $p - p_\infty$ as a function of ϵ. Inserting this function into Eq. (11) for the group velocity $v_n(\epsilon,\phi)$, the integral in Eq. (12) is readily evaluated. Upon introducing dimensionless variables $w = (a_p/3\epsilon)^{1/3}(p-p_\infty)$ and $u = \sqrt{a_\phi/a_p}(a_p/3\epsilon)^{1/3}\phi$, one obtains

$$\int\frac{d\phi}{v_n(p(\epsilon),\phi;T)} = \frac{a}{a_\phi^{1/2}v_n^{1/2}(p(\epsilon),\phi{=}0;T)}\,, \qquad (14)$$

where $a \simeq 2$ is a numerical factor given by the integral

$$a = \int_0^\infty \frac{du}{w^2(u)+u^2}, \qquad (15)$$

in which $w(u)$ is a real solution of the cubic equation $w^3 + 3u^2w = 1$. (We note that when Eq. (11) is extended to finite T, the group velocity $v_n(p,\phi=0,T)$

acquires an additional term $\propto T^{2/3}$ [13], but its inclusion reduces to an insignificant renormalization of the numerical factor a). The results

$$C(T,\rho_\infty)\propto \int n(\epsilon)(1-n(\epsilon))\frac{d\epsilon}{v_n^{1/2}(p(\epsilon),\phi=0;T)} \propto T^{2/3} \quad (16)$$

and

$$\beta(T,\rho_\infty)\propto \int n(\epsilon)(1-n(\epsilon))\frac{v_n'(\rho_\infty)d\epsilon}{v_n^{3/2}(p(\epsilon),\phi=0;T)} \propto O(1), \quad (17)$$

then follow, where $v_n'(\rho)\equiv \partial v_n(\rho)/\partial\rho = a_\rho$ by Eq. (11). The results (16) and (17) may be combined to determine the behavior of the Grüneisen ratio,

$$\Gamma(T,\rho_\infty)=\beta(T,\rho_\infty)/C(T,\rho_\infty)\propto T^{-2/3}, \quad (18)$$

which is at variance with the FL result $\Gamma \propto O(1)$.

Imposition of an external magnetic field greatly enlarges the scope of challenging NFL behavior, as reflected in the *magnetic* Grüneisen ratio $\Gamma_{\rm mag}(T,H) = -(\partial S(T,H)/\partial H)/C(T,H)$. We now analyze this key quantity within the topological scenario, again following the path established in Refs. [13, 15]. The original Fermi line is split into two, with consequent modification of field-free relations such as (9) and (12) through the appearance of half the sum of quasiparticle occupancies $n_\pm(\epsilon)=\{1+\exp[(\epsilon\pm\mu_e H)/T]\}^{-1}$. As a consequence, v_n^{-1} is replaced by half the sum of quantities $(\partial\epsilon(p,\phi)/\partial p_n)^{-1}$ evaluated at $\epsilon(p,\phi)\pm\mu_e H$. Analytic integration over ϕ still goes through and yields half the sum of square roots of these quantities. In the limit $T\gg\mu_e H$, terms linear in $r=\mu_e H/T$ cancel each other, such that the net result is proportional to r^2, leading to

$$\Gamma_{\rm mag}(T\gg\mu_e H)\propto T^{-2}. \quad (19)$$

More specifically, upon integration over ϕ in the field-perturbed formulas for $C(T,H)$ and $S(T,H)$, the ensuing expressions involve half the sum $(\epsilon+\mu_e H)^{-1/3} + (\epsilon-\mu_e H)^{-1/3}$, multiplied by a factor depending only on $n(\epsilon)$. Integrating over the dimensionless variable $y=\epsilon/T$, the ratio $S(T,H)/C(T)$ is determined as a function of r^2 only, yielding the result (19) at $r\ll 1$.

In the opposite limit $r\gg 1$, the density of states $N(T=0,H)$ diverges at a critical magnetic field H_∞, where the function $v_n(p,\phi,T=0,H_\infty)$ vanishes on *one of the two Fermi lines* $p^\pm(\phi)$ specified by

$$\epsilon(p^\pm,\phi)\pm\mu_e H=0. \quad (20)$$

The field-induced splitting that alters the relevant Fermi-surface group velocity can be compensated – for example, by doping–thereby providing the means for driving the system toward the QCP.

At $H>H_\infty$, the key quantity $v_n(p,\phi=0;T=0,H)\equiv v_n(H)$ becomes positive and FL behavior is recovered, as in the isotropic case [15, 13]. To evaluate the critical index specifying the divergence of the density of states $N(T=0,H\to H_\infty)\propto v_n^{-1/2}(H)$, we calculate the spectrum from Eq. (20) (as in Ref. [13]) and insert the result into Eq. (11), obtaining $v_n(H)\propto(H-H_\infty)^{2/3}$ and $N(T=0,H)\propto(H-H_\infty)^{-1/3}$. Thus

$$C(T\to 0,H)=S(T\to 0,H)\propto T(H-H_\infty)^{-1/3} \quad (21)$$

and $\beta(T\to 0,H)\propto T(H-H_\infty)^{-1}$, so that $\Gamma(T\to 0)\propto (H-H_\infty)^{-2/3}$. Importantly, we arrive at

$$\Gamma_{\rm mag}(T\to 0,H)=\frac{1}{3}(H-H_\infty)^{-1}. \quad (22)$$

Such a divergence was first predicted within scaling theory [16], in which the peak of $\Gamma_{\rm mag}(T=0,H)$ is located at H_c, the end point of the line $T_N(H)$ where $T_N(H_c)=0$. In the topological scenario, H_∞ does not coincide with H_c.

It is worth noting that all the results obtained above for the case of a 2D electron liquid on a quadratic lattice are readily transcribed for a 3D anistropic system. In particular, if we assume that the pre-QCP electron Fermi surface in the latter system is an ellipsoid $p_\infty(\theta)$, the group velocity at the FL side of the QCP is given by an expression

$$v_n(p,\theta;T{=}0,\rho)=b_p(p_n-p_\infty(\theta))^2+b_\theta\theta^2+b_\rho(\rho-\rho_\infty) \quad (23)$$

completely analogous to Eq. (11). In the corresponding formula for the density of states $N(T,\rho)$, the integrations now go over $dp_n d\sigma$, where $d\sigma=2\pi p^2(\theta)\cos\theta[1+(dp/d\theta)^2/p^2(\theta)]^{1/2}d\theta$. At low T the main contribution to $N(T,\rho)$ comes from the region of small $\theta\propto T$, where the ratio $(dp/d\theta)^2/p^2(\theta)$ is negligibly small, and one arrives at the expression

$$N(T,\rho)\propto\frac{1}{T}\int n(\epsilon)(1-n(\epsilon))\frac{d\epsilon\, d\theta}{v_n(p(\epsilon),\theta;T,\rho)} \quad (24)$$

for the QCP density of states, which has of exactly the same form as Eq. (12).

Discussion. The results (16)–(19), (21), and (22) are in agreement with available experimental data [17–20] obtained by the Steglich group in comprehensive studies of the thermodynamic properties of Yb-based heavy-fermion metals. These data also provide a test of modern phenomenological scaling theories of the QCP [16, 21]. The outcome of this test, as aired in Refs. [19, 21], is that no single model based on 2D or 3D fluctuations can describe these data, which require the following set of critical indexes having low probability: dimensionality $d = 1$, correlation-length exponent $\nu = 2/3$, and dynamical exponent $\gamma = 3/2$.

Recent studies of peaks in the specific heat $C(T, H)$ in Yb-based compounds reveal another difficulty confronting the phenomenological theory of second-order phase transitions in the QCP region. According to this theory, at $H = 0$ the difference $T - T_N$ is the single relevant parameter determining the structure of the fluctuation peak of the Sommerfeld ratio $C(T)/T$. However, comparative analysis of corresponding experimental data [20, 17] in $YbRh_2Si_2$ and $YbRh_2(Si_{0.95}Ge_{0.05})_2$ shows clearly that the structure of this peak *is not universal*. As the QCP is approached, the peak gradually shrinks to naught, again bringing into question the applicability of the spin-fluctuation scenario in its vicinity.

Thus, while the spin-fluctuation mechanism remains applicable at *finite* $T \simeq T_N(H)$, it becomes *inadequate* at the QCP itself. Accordingly, the relevant critical indexes of scaling theory must be inferred anew from appropriate experimental data on the *fluctuation peak* located at $T_N(H)$. Furthermore, the existing description of thermodynamic phenomena in the extended QCP region, including the peak at $T_N(H)$, must be revised by integrating the topological scenario with the theory of quantum phase transitions [1].

The posited suppression of critical fluctuations in $YbRh_2(Si_{0.95}Ge_{0.05})_2$, which is situated extremely close to the QCP in the sense that $H_c = 0.027$ T, conflicts with the conclusion of Ref. [22] that the system is on the verge of a ferromagnetic instability. The latter assertion is based on extraction of the Stoner factor from measurements of the Sommerfeld-Wilson (SW) ratio $R_{\rm SW}(T) \propto \chi(T)/C(T)$. However, such an extraction is straightforward only in homogeneous matter, where the magnetic part of the Hamiltonian is specified by the Bohr magneton μ_B. In dealing with electron systems of solids, this strategy is inconclusive unless a reliable replacement $\mu_{\rm eff}$ for μ_B is known. The authors of Ref. [22] have chosen the effective Bohr magneton $\mu_{\rm eff}$ to be 1.4–1.6 μ_B, as determined from data on the magnetic susceptibility itself. Such a choice suffers from double counting. If instead one uses the value $\mu_{\rm eff} = 4.54\mu_B$, appropriate for the atomic state of Yb^{3+}, then the Stoner factor derived from the data remains below 3. Thus the conflict is resolved.

Conclusion. The conventional view of quantum critical phenomena, in which the quasiparticle weight z vanishes at points of related $T = 0$ second-order phase transitions, is incompatible with a set of identities based on gauge transformations associated with prevailing conservation laws. We have traced the failure of the standard scenario to the inapplicability of the Ornstein-Zernike form $\chi^{-1}(q) = q^2 + \xi^{-2}$ for the static correlation function $\chi(q)$ in the limit $\xi \to \infty$. We have discussed an alternative topological scenario and demonstrated that its predictions for the thermodynamics of systems on the disordered side of the QCP are in agreement with available experimental data. Based on these data, we infer that close to the QCP the role of single-particle degrees of freedom is paramount, while the effects of critical fluctuations build up on the ordered side as the system moves away from the QCP.

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