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Generalized Hidden Local Symmetry Model confronts the decay $\tau^- \to \pi^+\pi^-\pi^-\nu_\tau$

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Generalized Hidden Local Symmetry (GHLS) model as the chiral model of pseudoscalar, vector, and axial vector mesons, is confronted the ALEPH data on the decay $\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$. It is shown that the spectrum of this decay in GHLS falls short of the experimental data. The modifications of GHLS based on inclusion of heavier axial vector mesons are studied. It is shown that the scheme with two additional axial vector isovector mesons with masses $m_{a_1'} = 1.59$ GeV and $m_{a_1''} = 1.88$ GeV gives a good description of the ALEPH data.

There is popular chiral model of pseudoscalar, vector, and axial vector mesons and their interactions based on nonlinear realization of chiral symmetry, the so called Generalized Hidden Local Symmetry (GHLS) model [1–3]. One of its virtue is that the sector of electroweak interactions is introduced in such a way that the low energy relations in the sector of strong interactions are not violated upon inclusion of photons and electroweak gauge bosons. Some interesting two- and three-particle decays as, for example, $\rho^0 \to \pi^+\pi^-$ and $\omega \to \pi^+\pi^-\pi^0$, were analyzed in the framework of GHLS [1].

Some time ago GHLS with particular choice of free parameters

$$(a, b, c, d, \alpha_5) = (2, 2, 2, 0, 1) \tag{1}$$

(see Refs. [3–5, 7] and (2) for more detail) was applied to the evaluation of the four-pion process $\rho \to 4\pi$ [4–7] and to the comparison with existing data on the reaction $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ [8, 9]. It was shown that while the results of calculations do not contradict the data [8] at energies near m_ρ, at higher energies near 1 GeV the cross section of above reaction measured in independent experiments [8, 9], by the factor of about 30 exceeds the values evaluated in GHLS [6, 7]. The contributions of higher resonances ρ', ρ'' were included to reconcile the data with calculations [6, 7].

Since axial vector meson $a_1(1260)$ appears only in the intermediate states of the reaction $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$, it would be desirable to study the processes where it manifests directly as in the decay $\tau^- \to \pi^+\pi^-\pi^-\nu_\tau$ which was studied by ALEPH Collaboration [10]. The aim of the present paper is to evaluate the $\pi^+\pi^-\pi^-$ spectrum in the decay of τ^- lepton in the framework of GHLS and compare the results with the ALEPH data. Notice that analogous work in the framework of different chiral model was undertaken, in particular, in Refs. [11, 12].

The basis of the derivation is the lagrangian of the generalized hidden local symmetry model [1, 3] (GHLS). In the sector of strong interactions and in the gauge $\xi_M = 1$, $\xi_L^\dagger = \xi_R = \xi$, it looks as

$$\begin{aligned}
\mathcal{L}^{(\mathrm{GHLS})} = {} & a_0 f_\pi^{(0)2} \mathrm{Tr}\left(\frac{\partial_\mu\xi^\dagger\xi + \partial_\mu\xi\xi^\dagger}{2i} - gV_\mu\right)^2 + \\
& + b_0 f_\pi^{(0)2} \mathrm{Tr}\left(\frac{\partial_\mu\xi^\dagger\xi - \partial_\mu\xi\xi^\dagger}{2i} + gA_\mu\right)^2 + \\
& + c_0 f_\pi^{(0)2} g^2 \mathrm{Tr} A_\mu^2 + \\
& + d_0 f_\pi^{(0)2} \mathrm{Tr}\left(\frac{\partial_\mu\xi^\dagger\xi - \partial_\mu\xi\xi^\dagger}{2i}\right)^2 - \\
& - \frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}^{(V)2} + F_{\mu\nu}^{(A)2}\right) - \\
& - i\alpha_4 g \mathrm{Tr}[A_\mu, A_\nu] F_{\mu\nu}^{(V)} + \\
& + 2i\alpha_5 g \mathrm{Tr}\left(\left[\frac{\partial_\mu\xi^\dagger\xi - \partial\xi\xi^\dagger}{2ig}, A_\nu\right] + \right. \\
& \left. + [A_\mu, A_\nu]\right) F_{\mu\nu}^{(V)},
\end{aligned} \tag{2}$$

g is the coupling constant to be related to $g_{\rho\pi\pi}$. See (10) below. The notations, assuming the restriction to the sector of the non-strange mesons, are

$$\begin{aligned}
F_{\mu\nu}^{(V)} &= \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu] - ig[A_\mu, A_\nu], \\
F_{\mu\nu}^{(A)} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[V_\mu, A_\nu] - ig[A_\mu, V_\nu],
\end{aligned} \tag{3}$$

$$V_\mu = \left(\frac{\boldsymbol{\tau}}{2} \cdot \boldsymbol{\rho}_\mu\right),$$

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$A_\mu = \left(\frac{\boldsymbol{\tau}}{2}\cdot \mathbf{A}_\mu\right)$, $\xi = \exp i\frac{\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{2f_\pi^{(0)}}$, where $\boldsymbol{\rho}_\mu$, $\boldsymbol{\pi}$ are the vector meson ρ and pseudoscalar pion fields, respectively, $\boldsymbol{\tau}$ is the isospin Pauli matrices. The axial vector field $\mathbf{A}_\mu$ is not literally the field corresponding to the physical $a_1(1260)$ meson. The fact is that the lagrangian (2) contains the $A-\pi$ mixing term. The latter can be removed upon choosing

$$A_\mu = a_\mu - \frac{b_0c_0}{g(b_0+c_0)}A_{(\xi)\mu}, \tag{4}$$

where a_μ is the $a_1(1260)$ meson field, and

$$A_{(\xi)\mu} = \frac{\partial_\mu\xi^\dagger\xi - \partial_\mu\xi\xi^\dagger}{2i}. \tag{5}$$

Free parameters (a_0, b_0, c_0, d_0), and $f_\pi^{(0)}$ of the GHLS lagrangian with index 0 are bare parameters before renormalization (see below); $[,]$ stands for commutator. Hereafter the boldface characters, cross $(\times)$, and dot $(\cdot)$ stand for vectors, vector product, and scalar product, respectively, in the isotopic space. The terms with free parameters $\alpha_{4,5}$ are necessary for cancelation of momentum dependence in $\rho\pi\pi$ vertex. To provide such cancelation, one should set $\alpha_4 = 1 + 2\alpha_5c_0/b_0$ [1, 3]. Removing the $A-\pi$ mixing (4) results in non-canonical coefficient in the kinetic term of the pion field. To make it canonical, one should fulfil the following renormalization:

$$f_\pi^{(0)} = Z^{-1/2}f_\pi,\ \boldsymbol{\pi}\to Z^{-1/2}\boldsymbol{\pi}, (a_0,b_0,c_0,d_0) = = Z(a,b,c,d), \tag{6}$$

where

$$\left(d_0 + \frac{b_0c_0}{b_0+c_0}\right)Z^{-1} = 1.$$

See [1, 3–5] for more detail.

The amplitude of the decay $\tau^-\to\pi^-\pi^-\pi^+\nu_\tau$ incorporates the transition $W^-\to\pi^-\pi^-\pi^+$. In GHLS, the latter is given by the diagram shown in Fig.1. Necessary terms are obtained from the total GHLS lagrangian which includes electroweak sector [1], and look like

$$\Delta\mathcal{L}_{\rm EW} = \frac{1}{2}g_2V_{ud}\mathbf{W}_{\mu\perp}\Big(-f_\pi\partial_\mu\boldsymbol{\pi}_\perp + + \frac{1}{3f_\pi}[\boldsymbol{\pi}\times[\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}]]_\perp + + bgf_\pi^2\mathbf{a}_{\mu\perp} + agf_\pi[\boldsymbol{\pi}\times\boldsymbol{\rho}_{\mu\perp}]\Big), \tag{7}$$

where $\mathbf{a}_\mu$ stands for the field of a_1 meson, $\mathbf{V}_\perp = (V_1, V_2)$ denotes transverse component of the isotopic vector, $\mathbf{W}_{\mu\perp}$ is the W-boson field while g_2 and V_{ud} are the SU(2) electroweak coupling constant and corresponding element of the Kobayashi-Maskawa matrix, respectively. The amplitudes of transitions $a_1\to3\pi$ and $\pi\to3\pi$ were

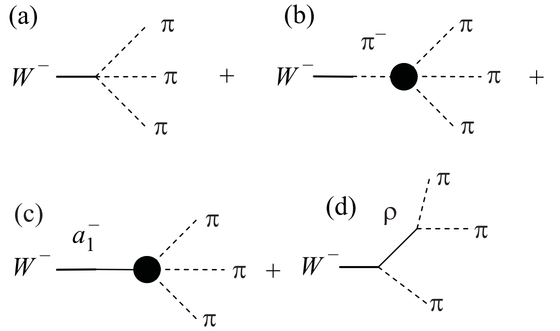

Fig.1. Diagrams describing the transition $W^-\to\pi^-\pi^-\pi^+$. Shaded circles depict the transition including both the point-like and ρ-exchange contributions. Permutations of pion momenta are understood

given in Ref. [4, 5] and Ref. [13], respectively. Here we do not fix GHLS parameters to their "canonical" values (1) [1] and allow them to be free. Then the amplitude of the decay $a_1^-(q)\to\pi^+(q_1)\pi^-(q_2)\pi^-(q_3)$ should be rewritten as follows: $M_{a_13\pi}\equiv M[a_1^-(q)\to\pi^+(q_1)\pi^-(q_2)\pi^-(q_3)]$,

$$iM_{a_13\pi} = \frac{agr}{2f_\pi}\epsilon_\mu\left(A_1q_{1\mu} + A_2q_{2\mu} + A_3q_{3\mu}\right), \tag{8}$$

where ϵ_μ is the polarization four-vector of a_1 meson, and

$$A_1 = (1+\hat P_{23})\left\{\frac{\beta[(q_3, q_1-q_2) - qq_3 + m_\pi^2] - qq_3}{D_\rho(q_1+q_2)} + \right. \left. + \frac{4r^2(\beta-1)q_2q_3 + q^2 - qq_1}{2m_\rho^2}\right\},$$
$$A_2 = \frac{\beta[(q_3, q_1-q_2) + qq_3 - m_\pi^2] + qq_3}{D_\rho(q_1+q_2)} + + \frac{(q_2, q_1-q_3)}{D_\rho(q_1+q_3)} - \frac{2r^2(\beta-1)q_1q_3 + qq_1}{m_\rho^2}. \tag{9}$$

Hereafter $\hat P_{ij}$ interchanges pion momenta q_i and q_j, (q_i, q_j) stands for the scalar product, and $A_3 = \hat P_{23}A_2$. Note that

$$g_{\rho\pi\pi} = \frac{ag}{2},\ m_\rho^2 = ag^2f_\pi^2,\ m_{a_1}^2 = (b+c)g^2f_\pi^2, \tag{10}$$

$f_\pi = 92.4$ MeV is the pion decay constant. Parameters r and β are the combinations of the GHLS parameters:

$$r = \frac{b}{b+c},\ \beta = \frac{\alpha_5}{r}. \tag{11}$$

The amplitude of the decay $W^-(q)\to\pi^+(q_1)\pi^-(q_2)\pi^-(q_3)$ corresponding to the diagrams Fig.1 is

$$iM = \frac{g_2V_{ud}}{2f_\pi}\epsilon_\mu^{(W)}J_\mu, \tag{12}$$

where $\epsilon_\mu^{(W)}$ is the polarization four-vector of W^- boson and the axial decay current J_μ looks like

$$J_\mu = -q_{1\mu} + \frac{q_\mu}{D_\pi(q)}\left[m_\pi^2 - qq_1 + \frac{am_\rho^2}{2}\times\right.$$
$$\left.\times\,(1+\hat{P}_{23})\frac{(q_2, q_1-q_3)}{D_\rho(q_1+q_3)}\right] - \frac{ar^2 m_{a_1}^2}{2D_{a_1}(q)}\times$$
$$\times\left\{A_1 q_{1\mu} + A_2 q_{2\mu} + A_3 q_{3\mu} - \frac{2q_\mu}{m_{a_1}^2}\times\right.$$
$$\times\,(1+\hat{P}_{23})\left[(m_\pi^2 + q_1q_2)(q_3, q_1-q_2)\times\right.$$
$$\left.\left.\times\left(\frac{\beta}{D_\rho(q_1+q_2)} - \frac{r^2(\beta-1)}{m_\rho^2}\right)\right]\right\} +$$
$$+\,\frac{am_\rho^2}{2}(1+\hat{P}_{23})\frac{(q_1-q_3)_\mu}{D_\rho(q_1+q_3)}. \qquad (13)$$

In the above expressions, D_ρ, D_π, and D_{a_1} are the inverse propagators of π, ρ, and a_1 mesons, respectively. Their expressions are given in Ref. [4]. The terms corresponding to the diagrams (a), (b), (c), and (d) in Fig.1 are easily identified by these propagators. Note that the divergence of the axial decay current is

$$q_\mu J_\mu = \frac{m_\pi^2}{D_\pi(q)}\left[q^2 - qq_1 + \frac{a}{2}m_\rho^2(1+\hat{P}_{23})\times\right.$$
$$\left.\times\,\frac{(q_2, q_1-q_3)}{D_\rho(q_1+q_3)}\right] - ar^2\frac{m_{a_1}^2 - q^2}{D_{a_1}(q)}\times$$
$$\times\,(1+\hat{P}_{23})(m_\pi^2 + q_1q_2)(q_3, q_1-q_2)\times$$
$$\times\left[\frac{\beta}{D_\rho(q_1+q_2)} - \frac{r^2(\beta-1)}{m_\rho^2}\right]. \qquad (14)$$

One can see that it vanishes at the a_1 mass shell in the limit of vanishing pion mass.

The spectrum of the three pion state in the decay $\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$ normalized to its branching fraction is [14]

$$\frac{dB}{ds} = \frac{(G_F V_{ud})^2(m_\tau^2 - s)^2}{2\pi(2m_\tau)^3\Gamma_\tau}\times$$
$$\times\left[(m_\tau^2 + 2s)\rho_t(s) + m_\tau^2\rho_l(s)\right], \qquad (15)$$

$s = q^2$. The transverse and longitudinal spectral functions are, respectively,

$$\rho_t(s) = \frac{1}{3\pi s f_\pi^2}\int d\Phi_{3\pi}\left(\frac{|qJ|^2}{s} - |J|^2\right),$$
$$\rho_l(s) = \frac{1}{\pi s^2 f_\pi^2}\int d\Phi_{3\pi}|qJ|^2, \qquad (16)$$

where $d\Phi_{3\pi}$ is the element of Lorentz-invariant phase volume of the system $\pi^-\pi^-\pi^+$. The numerical integration shows that due to Eq. (14) ρ_l is by at least three orders of magnitude smaller than ρ_t in all allowed kinematical range $9m_\pi^2 < s < m_\tau^2$ and by this reason it is neglected in what follows.

The "canonical" choice (1) of free GHLS parameters [1] with $m_{a_1} = 1.23$ GeV results in the spectrum shown with the dot-dashed line in Fig.2. Upon variation of free parameters with the single a_1 resonance contribution results in the curve drawn with the dashed line. It corresponds to $m_{a_1} \approx 1.54$ GeV, $a \approx 1.75$, $r \approx 1.05$, $\beta \approx 0.84$ with $\chi^2 = 690/112 d.o.f.$ To improve the fit heavier resonances a_1', a_1'' were included in a way analogous to $a_1(1260)$. Note that there are indications on such resonances both theoretical [15, 16] and experimental [17–19]. The total set of the fitted parameters is first taken to be

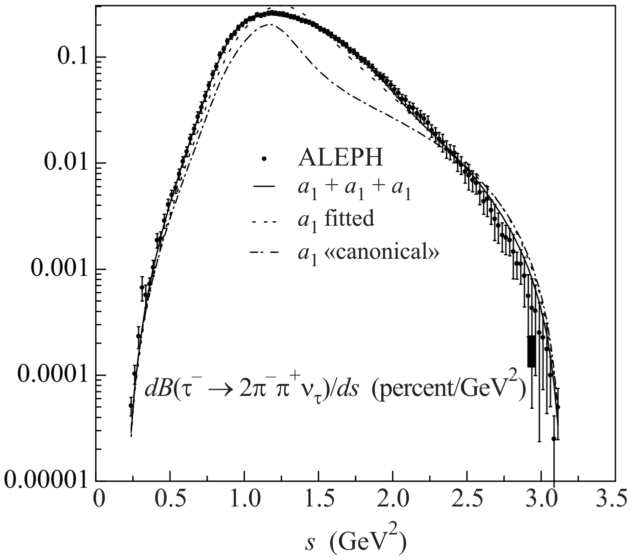

Fig.2. Spectrum of $\pi^-\pi^-\pi^+$ in τ decay normalized to the branching fraction $B_{\tau^-\to\pi^-\pi^-\pi^+\nu_\tau}$. The ALEPH data are from Ref. [10]. See the text for more detail

$$(m_{a_1}, a, r, \beta, m_{a_1'}, a', r', \beta', w', m_{a_1''}, a'', r'', \beta'', w''),$$

where w' parameterizes the coupling $a_1'\rho\pi$ as $g_{\rho\pi\pi}w'r'/f_\pi$. Compare with Eq. (8). Analogously for a_1''. The fit chooses $w' = 1$ and turns out to be insensitive to this parameter leaving $\chi^2 = 122/102 d.o.f.$ The quality of the fit is considerably improved upon fixing $w' = 1$ but adding new parameter ψ'-the phase of the a_1' contribution. Such phase imitates possible mixing among a_1, a_1', a_1'' resonances. The results of such type of the fit are

$$
\begin{gathered}
m_{a_1} = 1.332 \pm 0.015 \text{ GeV},\ a = 1.665 \pm 0.011, \\
r = 0.332 \pm 0.007,\ \beta = 8.5 \pm 0.3, \\
m_{a_1'} = 1.59 \pm 0.01 \text{ GeV},\ a' = 0.99 \pm 0.01, \\
r' = 0.96 \pm 0.01, \beta' = 0.07 \pm 0.02, \\
\psi' = 28^\circ \pm 1^\circ, \\
m_{a_1''} = 1.88 \pm 0.02 \text{ GeV}, a'' = 0.46 \pm 0.01, \\
r'' = 1.45 \pm 0.02, \beta'' = 0.91 \pm 0.05, \\
w'' = 1.14 \pm 0.01,
\end{gathered} \tag{17}
$$

with $\chi^2 = 79/102 d.o.f.$ Corresponding curve is shown in Fig.2 with the solid line. Using (10), (11), and obtaining $g_{\rho\pi\pi}$ from $\Gamma_{\rho\pi\pi}$ [17] one can compare the fitted GHLS parameters with the "canonical" ones (1). To this end one should invoke the condition of cancelation of the point-like $\gamma\pi^+\pi^-$ vertex in GHLS:

$$\frac{a}{2} = d + \frac{bc}{b+c}. \tag{18}$$

Notice that the relation (18) removes also the point-like $W^-\pi^-\pi^0$ vertex in GHLS model. One finds

$$
\begin{aligned}
(a, b, c, d, \alpha_5) = (&1.665 \pm 0.011, 1.5 \pm 0.1, 3.0 \pm 0.1, \\
&-0.16 \pm 0.04, 2.8 \pm 0.1).
\end{aligned} \tag{19}
$$

In principle, the vector isovector resonances ρ', ρ'' could be present only in the final states of the present axial vector isovector channel $\pi^+\pi^-\pi^-$, via the transition $a_1 \to \rho'(\rho'')\pi \to 3\pi$ (analogously for a_1', a_1''). However, their inclusion requires the introduction of new free parameters $g_{a_1\rho'(\rho'')\pi}$ (analogously for a_1', a_1'') in addition to those 14 ones already present. See (18). On the grounds of reasonable adequacy we neglect ρ', ρ'' at the present stage of the study, especially because the coupling constants $g_{\rho'\pi\pi}$ and $g_{\rho''\pi\pi}$ are presumably small as compared to $g_{\rho\pi\pi}$ [20].

To summarize, the simplest variant of GHLS model as applied to the decay $\tau^- \to \pi^+\pi^-\pi^-\nu_\tau$ meets troubles when describing the form of 3π spectrum. One should invoke the contributions of heavier axial vector mesons in order to reconcile calculations with available data.

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