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Critical disorder effects in Josephson-coupled quasi-one-dimensional superconductors

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Effects of non-magnetic randomness on the critical temperature T_c and diamagnetism are studied in a class of quasi-one dimensional superconductors. The energy of Josephson-coupling between wires is considered to be random, which is typical for dirty organic superconductors. We show that this randomness destroys phase coherence between the wires and T_c vanishes discontinuously when the randomness reaches a critical value. The parallel and transverse components of the penetration depth are found to diverge at different critical temperatures $T_c^{(1)}$ and T_c, which correspond to pair-breaking and phase-coherence breaking. The interplay between disorder and quantum phase fluctuations results in quantum critical behavior at $T = 0$, manifesting itself as a superconducting-normal metal phase transition of first-order at a critical disorder strength.

Quasi-one-dimensional (quasi-1D) organic conductors, including the charge-transfer (Bechgaard) salts of $(TMTSF)_2X$ (where TMTSF stands for tetramethyltetraselenofulvalinium and X = PF_6, ClO_4, NO_3 being a strong electron acceptor or anion) [1] and A-15 compounds [2], attract enhanced interest since the discovery of superconductivity in $(TMTSF)_2PF_6$. Low temperature properties of the organic superconductors are very sensitive to disorder. Alloying anions, X-ray irradiation and cooling rate controlled anion reorientation introduce non-magnetic randomness into the system while leaving the backbone structure and the unit cell of the organic superconductors to a large extent unchanged. There is a common agreement that disorder, introduced by means of these experimental methods, must be characterized as non-magnetic, and yet it was shown [3 – 5] to suppress the superconducting (SC) phase.

Effect of disorder on the SC phase has a long-standing history. According to the Anderson's theorem [6], the SC critical temperature T_c for s-wave pairing is insensitive to the scattering rate on non-magnetic impurities. Magnetic impurities break time-reversal symmetry of the s-pairing, suppress at the same time the SC phase [7]. Strong disorder of non-magnetic impurities may however destroy d-wave pairing [8]. Interplay between superconductivity and Anderson localization in a strongly disordered superconductor was shown [8 – 17] to result in spatial inhomogeneity of the order parameter. High purity of the organic superconductor backbone even in the dirty limit seems to exclude a spatial inhomogeneity of the order parameter modulus along SC wires, offering an opportunity for another mechanism of disorder-driven superconductor-normal metal phase transition. Effects of order parameter phase fluctuations on T_c have also been studied in low-dimensional superconductors [18 – 21]. It is well known that there is no SC phase transition in 1D and two-dimensional (2D) systems [22], since strong fluctuations of the order parameter phase destroy off-diagonal long-range order (ODLRO) in a single SC wire or film. Strong phase fluctuations in clean quasi-1D superconductors have been shown [18, 19] to suppress T_c below a mean-field transition temperature. Classifying the superconductors with small stiffness as bad metals, Emery and Kivelson have evaluated [20] a critical temperature of phase ordering by formally dividing a clean bulk superconductor into small regions with well defined phase, and have shown strong suppression of SC phase by phase fluctuations. Nevertheless, effects of disorder on phase fluctuations are neglected in all of these papers.

In contrast to these previous activities we study in this Letter a suppression of superconductivity as a result of the destruction of the order parameter phase coherence by disorder. We consider weakly linked quasi-one-dimensional superconductors with random Josephson-couplings between pure one-dimensional (1D) SC wires. Singlet pairing is considered to occur inside the wires. Therefore, we assume that non-magnetic randomness does not affect the order parameter amplitude. We demonstrate in this Letter that (i) non-magnetic randomness in the Josephson-coupling destroys correlation of the phases between different chains in quasi-1D super-

conductors even in the classical phase fluctuation regime, (ii) randomness yields quantum critical behavior. A SC-normal metal phase transition occurs at $T = 0$ with increasing the strength of disorder, and that (iii) a suppression of the SC phase occurs discontinuously in both classical and quantum phase fluctuations as a first-order phase transition when the disorder-strength reaches a critical value. We derive that parallel and perpendicular components of the penetration depth, $\lambda_\parallel$ and $\lambda_\perp$ diverge at different critical temperatures $T_c^{(1)}$ and T_c, which correspond to pair-breaking in the wires and to phase coherence breaking between the SC wires, respectively.

Classical fluctuations of the phase. The free energy functional of a quasi-1D superconductor weakly linked with Josephson coupling energy $E_{\mathbf{j},\mathbf{j}+\mathbf{g}}$ between nearest-neighbor chains can be written in the presence of the magnetic field $\mathbf{B}$ as

$$F_{st}=N_s^{(1)}(T)\xi_\parallel\sum_{\mathbf{j}}\int dz\Big\{\frac{\hbar^2}{8m_\parallel\xi_\parallel^2}\Big(\frac{\partial\varphi_{\mathbf{j}}}{\partial z}-\frac{2e\xi_\parallel}{\hbar c}A_z\Big)^2+$$
$$\sum_{\mathbf{g}=\pm1}E_{\mathbf{j},\mathbf{j}+\mathbf{g}}[1-\cos\big(\varphi_{\mathbf{j}}-\varphi_{\mathbf{j}+\mathbf{g}}+\frac{2e\xi_\parallel}{\hbar c}\int_{\mathbf{j}}^{\mathbf{j}+\mathbf{g}}\mathbf{A}_\perp d\mathbf{r}_\perp\big)]+$$
$$+\xi_\parallel a_\perp^2\frac{(\mathbf{B}(\mathbf{r})-\mathbf{B}_{ext})^2}{8\pi}\Big\}, \qquad (1)$$

where $\varphi_{\mathbf{j}}(z)$ denotes the order parameter phase, $\mathbf{A} = \{\mathbf{A}_\perp, A_z\}$ is the vector-potential, and $N_s^{(1)}(T) = N_s^{(1)}(0)\tau(T)$ is the linear density of SC electrons with $\tau(T) = (T_c^{(1)} - T)/T_c^{(1)}$ and $N_s^{(1)}(0) \equiv N_N^{(1)} \simeq p_F/\hbar$ at $T \le T_c^{(1)}$. Dimensionless coordinates $\mathbf{r} = \{\mathbf{j}, z\}$ are introduced on the scale of longitudinal $\xi_\parallel = \hbar^2 N_s^{(1)}(0)/4m_\parallel T_c^{(1)}$ and transverse $\xi_\perp \sim a_\perp$ components of the coherence length. We assume the Josephson energy $E_{\mathbf{j},\mathbf{j}+\mathbf{g}}$ to be a random parameter with Gaussian distribution given by

$$P\{E_{\mathbf{j},\mathbf{j}+\mathbf{g}}\} = \frac{1}{\sqrt{2\pi W^2}}\exp\Big\{-\frac{(E_{\mathbf{j},\mathbf{j}+\mathbf{g}}-E_{\mathbf{g}})^2}{2W^2}\Big\}. \qquad (2)$$

Employing the replica trick one can integrate out the Gaussian disorder to obtain the average value of the free energy $\mathcal{F} = -T\langle\ln Z\rangle$ as

$$\mathcal{F} = -T\int\prod_{\mathbf{j},\mathbf{g}}\frac{N_s^{(1)}\xi_\parallel}{\sqrt{2\pi}}d\zeta_{\mathbf{j},\mathbf{g}}e^{-\frac{N_s^{(1)2}\xi_\parallel^2}{2}\zeta_{\mathbf{j},\mathbf{g}}^2}\times$$
$$\times\ln\int\prod\mathcal{D}\varphi_{\mathbf{j}}e^{-F/T}, \qquad (3)$$

with

$$F = N_s^{(1)}\xi_\parallel\sum_{\mathbf{j}}\int dz\Big\{\frac{\hbar^2}{8m_\parallel\xi_\parallel^2}\Big(\frac{\partial\varphi_{\mathbf{j}}}{\partial z}\Big)^2+$$
$$+\sum_{\mathbf{g}}(E_{\mathbf{g}}-N_s^{(1)}\xi_\parallel W\zeta_{\mathbf{j},\mathbf{g}})[1-\cos(\varphi_{\mathbf{j}}-\varphi_{\mathbf{j}+\mathbf{g}})]\Big\}, \qquad (4)$$

where $\zeta_{\mathbf{j},\mathbf{g}}$ is a Hubbard-Stratonovich auxiliary field. The average value of a given functional $C(\{\varphi_{\mathbf{j}}\})$, e.g. $\cos\varphi_{\mathbf{j}}$ or $\cos(\varphi_{\mathbf{j}} - \varphi_{\mathbf{j}+\mathbf{g}})$, can be obtained according to the relation

$$\langle\langle C(\{\varphi_{\mathbf{j}}\})\rangle\rangle = -T\frac{\delta}{\delta\eta_{\mathbf{j}}}\langle\ln Z\rangle|_{\eta_{\mathbf{j}}=0}$$

by adding the source term $\sum_{\mathbf{j}}\int dz\eta_{\mathbf{j}}C(\{\varphi_{\mathbf{j}}\})$ to the free energy functional, which yields for the correlator

$$\langle\langle C(\{\varphi_{\mathbf{j}}\})\rangle\rangle = \int\prod_{\mathbf{j},\mathbf{g}}\frac{N_s^{(1)}\xi_\parallel}{\sqrt{2\pi}}d\zeta_{\mathbf{j},\mathbf{g}}e^{-\frac{N_s^{(1)2}\xi_\parallel^2}{2}\zeta_{\mathbf{j},\mathbf{g}}^2}\times$$
$$\times\frac{\int\mathcal{D}\varphi C(\{\varphi_{\mathbf{j}}\})e^{-F/T}}{\int\mathcal{D}\varphi e^{-F/T}}, \qquad (5)$$

where the double bracket $\langle\langle\ldots\rangle\rangle$ means averaging over thermodynamic fluctuations and over randomness. In order to estimate an asymptotic behavior of the correlator, e.g. $\langle\langle\cos(\varphi_{\mathbf{j}} - \varphi_{\mathbf{j}+\mathbf{g}})\rangle\rangle$ we write the integrand of Eq.(5) as $\exp\{-N_s^{(1)2}\xi_\parallel^2 f(\zeta_{\mathbf{j},\mathbf{g}})\}$, and apply the stationary-phase approximation to determine an extremal value of the auxiliary field $\bar\zeta_{\mathbf{j},\mathbf{g}}$ minimizing the function $f(\zeta_{\mathbf{j},\mathbf{g}})$. The saddle point value of $\zeta_{\mathbf{j},\mathbf{g}}$ is obtained to be

$$\bar\zeta_{\mathbf{j},\mathbf{g}} = \frac{W}{T}\int dz\Big(\langle\cos(\varphi_{\mathbf{j}}(z)-\varphi_{\mathbf{j}+\mathbf{g}}(z))\rangle-$$
$$-\frac{\langle\cos(\varphi_{\mathbf{j}}(z)-\varphi_{\mathbf{j}+\mathbf{g}}(z))\cos(\varphi_{\mathbf{j}}(0)-\varphi_{\mathbf{j}+\mathbf{g}}(0))\rangle}{\langle\cos(\varphi_{\mathbf{j}}(0)-\varphi_{\mathbf{j}+\mathbf{g}}(0))\rangle}\Big).$$

The constant $N_s^{(1)}\xi_\parallel$ on the exponent can be estimated to be equal to $N_s^{(1)}\xi_\parallel \simeq \epsilon_F/T_c^{(1)} \sim 10^3$ for the organic superconductors with ϵ_F being the Fermi energy, which ensures a sharply peaked saddle point of the integrand. The thermodynamic averages in the expression of $\bar\zeta_{\mathbf{j},\mathbf{g}}$ are taken with the free energy functional, given by Eq.(4) at the saddle point $\zeta_{\mathbf{j},\mathbf{g}} = \bar\zeta_{\mathbf{j},\mathbf{g}}$. So, a contribution of the non-magnetic randomness to the effective free-energy functional is proportional to the variance of the phase correlator, which gives an idea on the form of the disorder-dependent term in the effective functional.

The critical temperature for the quasi-1D superconductors can now be found from Eq.(5),written for $\cos\varphi_{\mathbf{j}}$ by using the self-consistent mean-field method

[18], which consists in replacing the phase correlations of the cosine term by

$$\sum_{\mathbf{g}} E_{\mathbf{g}}[1-\cos(\varphi_{\mathbf{j}}(z)-\varphi_{\mathbf{j}+\mathbf{g}}(z))] \to$$

$$\to E_{\perp}[1-\langle\langle\cos(\varphi)\rangle\rangle_{eff}\cos(\varphi(z))],$$

where $E_{\perp}=\sum_{\mathbf{g}} E_{\mathbf{g}}$. For a clean system $\langle\langle\cos(\varphi)\rangle\rangle_{eff}$ was chosen [18] to be equal to $\langle\cos(\varphi)\rangle$. For the disordered superconductor we choose

$$\langle\langle\cos\varphi\rangle\rangle_{eff}=\langle\langle\cos\varphi\rangle\rangle-N_s^{(1)}\xi_{\parallel}\frac{\langle\langle\cos\varphi\rangle^2\rangle-\langle\langle\cos\varphi\rangle\rangle^2}{\langle\langle\cos\varphi\rangle\rangle}.$$

Such a form of $\langle\langle\cos\varphi\rangle\rangle_{eff}$ is similar to the expression of the saddle point value for the averaged order parameter. Taking advantage of the smallness of $(E_{\perp}-N_s^{(1)}\xi_{\parallel}W\zeta)\langle\langle\cos(\varphi)\rangle\rangle_{eff}$ near T_c, we expand both the numerator and the denominator of the integrand of Eq.(5), written for $\langle\langle\cos(\varphi)\rangle\rangle_{eff}$, in this parameter. The thermodynamic averages become pure one-dimensional after this expansion, which can be taken easily, yielding a power series of ζ for the integrand. Therefore, the integration over ζ is immediately performed. Since all higher order in $\langle\langle\cos(\varphi)\rangle\rangle_{eff}$ terms of the expansion vanish at $T=T_c$, we get the equation for T_c

$$1=\frac{E_{\perp}N_s^{(1)}\xi_{\parallel}}{T_c}\left(1-\frac{W^2\xi_{\parallel}N_s^{(1)}\eta^2}{T_cE_{\perp}}\right)\times$$

$$\times\int\langle\cos(\varphi(0))\cos(\varphi(z))\rangle dz, \qquad (6)$$

where η is the coordination number. The phase correlator in Eq.(6) is calculated in the clean limit of the 1D free energy functional, obtained from Eq.(1) by setting $E_{\mathbf{j}\mathbf{j}+\mathbf{g}}=0$, which returns (see Ref.[22])

$$\langle\cos(\varphi(0))\cos(\varphi(z))\rangle=\exp\{-|z|/r_c\}, \qquad (7)$$

where $r_c=\hbar^2N_s^{(1)}(T)/2m_{\parallel}\xi_{\parallel}T$. Using dimensionless T_c-shift $t=\sqrt{\eta\epsilon_F E_{\perp}}\left(\frac{1}{T_c}-\frac{1}{T_c^{(1)}}\right)$ and disorder parameter

$$q=\frac{W^2}{E_{\perp}}\sqrt{\frac{2m_{\parallel}\xi_{\parallel}^2\eta}{\hbar^2E_{\perp}}}=\frac{W^2}{2E_{\perp}T_c^{(1)}}\sqrt{\frac{\eta\epsilon_F}{E_{\perp}}},$$

Eqs.(6), (7) yield

$$1=t^2(1-qt). \qquad (8)$$

Expanding the physical solution of this cubic equation in the weak disorder regime (small q) the T_c-shift obeys

$$\frac{1}{T_c}=\frac{1}{T_c^{(1)}}+\frac{1}{\sqrt{\eta\epsilon_F E_{\perp}}}+\frac{1}{T_c^{(1)}}\left(\frac{W}{2E_{\perp}}\right)^2, \qquad (9)$$

showing that T_c decreases with increasing randomness like W^2. For a pure system Eq.(9) gives the dependence $T_c\sim E_{\perp}^{1/2}$, in agreement with Efetov and Larkin in Ref. [18]. This expression shows that even a small interchain-coupling sets up an ODLRO in the system, and consequently, the critical temperature increases with $E_{\perp}$. On the other hand, disorder reduces T_c due to "melting" of the order parameter phase coherence between neighboring chains. The exact solution of Eq.(8) gives three roots, among which the physical one is confined to the finite q-range as shown in Fig.1 by the bold line.

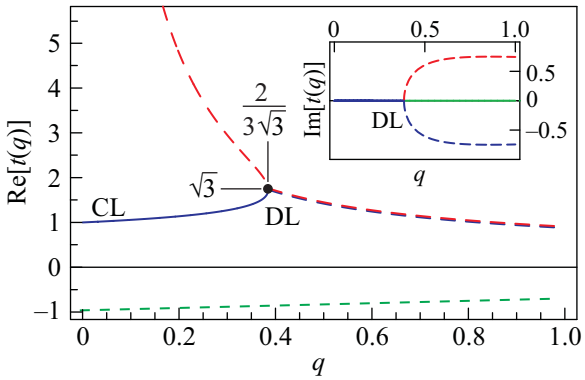

Fig.1. The physical solution $t(q)$, giving the $T_c(W)$-dependence, within the full range from clean limit (CL: $q=0$) to the dirty limit (DL: $q_c=2/3\sqrt{3}$)) is highlighted as the bold (blue) curve. Formal solutions of the cubic Eq.(8) are shown for completeness. $T_c(q)$ vanishes abruptly at $q=q_c$

According to this solution, the critical temperature decreases monotonically with increasing q in the interval of $0\le q\le q_c=2/3\sqrt{3}$. The SC phase hence suppressed beyond the critical disorder-value

$$W_c^2=\frac{4E_{\perp}T_c^{(1)}}{3}\sqrt{\frac{E_{\perp}}{3\eta\epsilon_F}},$$

being transformed into a normal metallic phase for $W^2>W_c^2$. The critical temperature drops to zero at $W^2=W_c^2$ with a jump of size

$$\Delta T_c=T_c^*=\left(\sqrt{\frac{3}{\eta\epsilon_F E_{\perp}}}+\frac{1}{T_c^{(1)}}\right)^{-1}.$$

Thus the SC-normal metal phase transition appears as a 1st-order transition.

In order to find the behavior of t near the disorder limit (DL in Fig.1) $\{t^*,q^*\}=\{\sqrt{3},2/3\sqrt{3}\}$, we expand $\delta t=t^*-t$ in terms of $\delta q=q^*-q$, which gives

$\delta t = 3^{3/4}\sqrt{\delta q}$. In other words, $\delta T_c = T_c - T_c^*$ behaves as

$$\delta T_c = \frac{3^{3/4} T_c T_c^*}{E_\perp (4\eta\epsilon_F E_\perp (T_c^{(1)})^2)^{1/4}} (W_c^2 - W^2)^{1/2}$$

in the vicinity of the breakdown point

$$\{T_c^*, W_c^2\} = \left\{ \left(\sqrt{\frac{3}{\eta\epsilon_F E_\perp}} + \frac{1}{T_c^{(1)}} \right)^{-1}, \frac{4E_\perp T_c^{(1)}}{3} \sqrt{\frac{E_\perp}{3\eta\epsilon_F}} \right\}.$$

Quantum phase fluctuations regime. We shall improve the calculation of the phase-correlators by taking into account the transverse rigidity of the system, which provides a more realistic determination of the transition temperature in the quantum fluctuation regime. We start from the Lagrangian, for simplicity at $\mathbf{B} = 0$

$$\mathcal{L} = \frac{K\xi_\parallel(0)}{8} \sum_{\mathbf{j}} \int dz [\hbar \dot{\varphi}_{\mathbf{j}}(z)]^2 - F_{eff}\{\varphi\}, \qquad (10)$$

where $\dot{\varphi}$ denotes the time derivative of the phase. The dynamical term in $\mathcal{L}$ can be interpreted as the electrostatic energy of charged wires [19, 21]

$$E_{el} = \frac{1}{2} \sum_{\mathbf{i},\mathbf{j}} \int dz \int dz' C_{\mathbf{i},\mathbf{j}}(z - z') V_{\mathbf{i}}(z) V_{\mathbf{j}}(z')$$

produced according to the first Josephson equation $\dot{\varphi} = (2e/\hbar)V$; $C_{\mathbf{i},\mathbf{j}}(z - z')$ are the specific coefficients of electrostatic induction. Rewriting E_{el} in terms of time-derivative of phases, the Fourier transform $K(\mathbf{q}_\perp, q_z)$ of the new coefficients

$$K_{\mathbf{i},\mathbf{j}}(z - z') = \frac{1}{4e^2} C_{\mathbf{i},\mathbf{j}}(z - z'),$$

has the physical meaning of a compressibility. In Eq.(10) we neglect a spatial dispersion of the compressibility and take $K(\mathbf{q}_\perp, q_z) = K = \text{const}$. $F_{eff}\{\varphi\}$ is the functional F in Eq.(4), written at the saddle point $\tilde{\zeta}_{\mathbf{j},\mathbf{g}}$ of the averaged free energy $\mathcal{F}$. The saddle point of $\mathcal{F}$ is found to be

$$\tilde{\zeta}_{\mathbf{j},\mathbf{g}} = \frac{W}{\mathcal{F}_{eff}} \int dz \langle\langle [1 - \cos(\varphi_{\mathbf{j}}(z) - \varphi_{\mathbf{j}+\mathbf{g}}(z))] \rangle\rangle,$$

where

$$\mathcal{F}_{eff} = -T \ln \int \prod_{\mathbf{j},\mathbf{g}} \mathcal{D}\varphi_{\mathbf{j}} e^{-F_{eff}/T}.$$

So, $\mathcal{F}_{eff}$ has to be calculated self-consistently. Note that the model would be calculated more rigorously by replacing the 1D wire with discrete analogue of Josephson-coupled cells, and considering a strongly anisotropic 3D Josephson network. After averaging over disorder, one can introduce the 'order parameters' as $\zeta_a = \langle\langle e^{i\varphi_{\mathbf{j}}^a} \rangle\rangle$ and $q_{a,b}^{\pm} = \langle\langle e^{i\varphi_{\mathbf{j}}^a} \rangle \langle e^{\pm i\varphi_{\mathbf{j}'}^b} \rangle\rangle$, where ζ_a and $q_{a,b}$ are the order parameters corresponding to the SC and glassy phases with a, b being the replica indices. The model can be mapped to the solvable Sherrington-Kirkpatrick model for the long-ranged phase-phase correlations, nevertheless a solution of the model for the short-ranged (nearest-neighbor) phase-phase correlation case, realized in our model, is hard task.

The Hamiltonian, expressed in terms of the phases $\varphi_{\mathbf{j}}$ and canonical conjugate momenta $\Pi_{\mathbf{j}}$ as

$$\mathcal{H} = \sum_{\mathbf{j}} \hbar \int \Pi_{\mathbf{j}} \dot{\varphi}_{\mathbf{j}} dz - \mathcal{L},$$

becomes

$$\mathcal{H} = \sum_{\mathbf{j}} \int dz \left\{ 2\frac{\Pi_{\mathbf{j}}^2(z)}{K\xi_\parallel(0)} + \frac{\hbar^2 N_s^{(1)}(T)}{8m_\parallel \xi_\parallel} \left[\left(\frac{\partial \varphi_{\mathbf{j}}}{\partial z} \right)^2 + \right. \right.$$
$$\left. \left. + \sum_{\mathbf{g}} \delta_{cl}^2 [1 - \cos(\varphi_{\mathbf{j}}(z) - \varphi_{\mathbf{j}+\mathbf{g}}(z))] \right] \right\}, \qquad (11)$$

where

$$\Pi_{\mathbf{j}} = \frac{1}{\hbar} \frac{\delta \mathcal{L}}{\delta \dot{\varphi}_{\mathbf{j}}} = \frac{1}{4} \hbar K \xi_\parallel(0) \dot{\varphi}_{\mathbf{j}},$$

and δ_{cl} as given by

$$\delta_{cl}^2 = \delta_0^2 \left[1 - \frac{W^2 N_s^{(1)} \xi_\parallel}{E_\perp \mathcal{F}_{eff}} \langle\langle [1 - \cos(\varphi_{\mathbf{j}}(z) - \varphi_{\mathbf{j}+\mathbf{g}}(z))] \rangle\rangle \right] \qquad (12)$$

represents either the dimensionless anisotropy-parameter or the transverse rigidity of the random system, while δ_0 in Eq.(12) being the transverse rigidity of the pure system

$$\delta_0 = \left(\frac{E_\perp}{\hbar^2/8m_\parallel \xi_\parallel^2} \right)^{1/2} = \frac{(\epsilon_F E_\perp)^{1/2}}{T_c^{(1)}}. \qquad (13)$$

δ_0 is a natural small parameter of the quasi-1D superconductors, which ensures small interchain-coupling energies in comparison with the intrachain Cooper-pair energy. Indeed, expressing $E_\perp$ through the interchain tunneling integral $J_\perp$ [18] as

$$E_\perp \simeq J_\perp^2/\epsilon_F \text{ yields } \delta_0 = J_\perp/T_c^{(1)}.$$

The quantum description is realized by expressing $\varphi_{\mathbf{q}}$ and $\Pi_{\mathbf{q}}$ as a linear superposition of Bose operators $b_{\mathbf{q}}$ as

$$\varphi_{\mathbf{q}} = \left(\frac{\pi\alpha\bar{\omega}}{\omega(\mathbf{q})} \right)^{1/2} (b_{\mathbf{q}} + b_{-\mathbf{q}}^\dagger)$$

and

$$\Pi_{\mathbf{q}} = i\Big(\frac{\omega(\mathbf{q})}{4\pi\alpha\bar{\omega}}\Big)^{1/2}(b_{-\mathbf{q}} - b^{\dagger}_{\mathbf{q}}).$$

In the framework of the self-consistent harmonic approximation (SCHA), we rewrite the expansion of the cosine operator in Eq.(11) in terms of the bosonic particle number operator $\hat{N}_{\mathbf{q}} = b^{\dagger}_{\mathbf{q}} b_{\mathbf{q}}$ [19, 21] as

$$\hat{\mathcal{H}} = \sum_{\mathbf{q}} \hbar\omega(\mathbf{q}, T)\left(b^{\dagger}_{\mathbf{q}} b_{\mathbf{q}} + 1/2\right), \qquad (14)$$

where the eigenfrequency of oscillation $\omega(\mathbf{q}, T)$ is given

$$\omega(\mathbf{q}, T) = \bar{\omega}\left[q_z^2 + \delta_{cl}^2 e^{-S_{\alpha}^{(0)}(\mathbf{g},T)} 2(2 - \cos q_x - \cos q_y)\right]^{1/2}. \qquad (15)$$

We express the amplitude of the frequency $\bar{\omega} = (N_s^{(1)}(T)/m_{\parallel} K\xi_{\parallel}^2)^{1/2}$ as $\bar{\omega} = 2\pi\alpha T_c^{(1)}\tau^{1/2}/\hbar$, where

$$\alpha = 2\left(m_{\parallel}/K\hbar^2 N_s^{(1)}\right)^{1/2}/\pi. \qquad (16)$$

The parameter α, which is an essential parameter of the theory, can assume values between zero and one [18].

The factor $\exp\{-S_{\alpha}(\mathbf{g}, T)\}$ in Eq.(15) is obtained as

$$S_{\alpha}(\mathbf{g}, T) = \frac{2\pi\alpha\bar{\omega}}{N}\sum_{\mathbf{q}} \frac{1 - \cos(\mathbf{q}_{\perp}\mathbf{g})}{\omega(\mathbf{q}, T)}\left(N_{\mathbf{q}} + \frac{1}{2}\right), \qquad (17)$$

where N is the number of unit cells per volume, and $N_{\mathbf{q}} = \{\exp(\hbar\omega(\mathbf{q}, T)/T) - 1\}^{-1}$ is Planck's distribution function for phonons. A physical meaning of $\exp\{-S_{\alpha}(\mathbf{g}, T)\}$ is an average of $\cos(\varphi_{\mathbf{j}}(z) - \varphi_{\mathbf{j}+\mathbf{g}}(z))$ over all one-phonon states, $\exp\{-S_{\alpha}(\mathbf{g}, T)\} = \langle\langle\cos(\varphi_{\mathbf{j}}(z) - \varphi_{\mathbf{j}+\mathbf{g}}(z))\rangle\rangle^{(T)}$, [19]. Application of the SCHA results in a *renormalization* of the parameter δ_{cl}, changing thus the oscillation frequency $\omega(\mathbf{q}, T)$ by means of the phase-phase correlator

$$\delta_{cl}^2 \to \delta_{qu}^2(T) = \delta_{cl}^2 \exp\{-S_{\alpha}(\mathbf{g}, T)\}. \qquad (18)$$

Note that the SCHA is valid under the condition

$$\sum_{\mathbf{q}} |A_{\mathbf{q}}|^2 \langle N_{\mathbf{q}}\rangle = \frac{2\pi\alpha\bar{\omega}}{N}\sum_{\mathbf{q}} \frac{1 - \cos(\mathbf{q}_{\perp}\mathbf{g})}{\omega(\mathbf{q}, T)}\langle N_{\mathbf{q}}\rangle < 1,$$

which means that few phonons are excited in the system.

It is easier to see that $\mathcal{F}_{eff}$ can be calculated as $\mathcal{F}_{eff} = -T\ln Tr\{e^{\hat{\mathcal{H}}/T}\}$ by neglecting the dynamical term ($K = 0$ or $\alpha = 0$) in Eq.(11), which gives $\mathcal{F}_{eff} = T$.

Let us start with the $T = 0$ limit: expressing the phase-phase correlator $e^{-S_{\alpha}(\mathbf{g},0)}$ in terms of

$$S_{\alpha}(\mathbf{g}, 0) = \frac{\pi\alpha\bar{\omega}}{N}\sum_{\mathbf{q}} \frac{1 - \cos(\mathbf{q}_{\perp}\mathbf{g})}{\omega(\mathbf{q}, 0)}$$

gives

$$e^{-S_{\alpha}(\mathbf{g},0)} = (\delta_{qu}(0))^{\alpha} \equiv \delta_{qu}^{\alpha},$$

which implies that even small interchain-coupling stabilizes ODLRO, hence also a finite T phase transition should exist. In order to get an explicit expression for the dependence of δ_{qu} on δ_0 and on disorder, we have to solve the equation $\delta_{qu}^2 = \delta_{cl}^2 e^{-S_{\alpha}(\mathbf{g},0)}$ together with Eq.(12) for δ_{cl}, the latter of which also depends on δ_{qu}. Thus the equation for the reduced transverse rigidity $\delta_{qu}^* = \delta_{qu}/\delta_{qu}^{(0)}$, where $\delta_{qu}^{(0)} = \delta_0^{\frac{2}{2-\alpha}}$ is the renormalized transverse rigidity for the clean system at $T = 0$, assumes the form

$$(\delta_{qu}^*)^{3-2\alpha} = (\delta_{qu}^*)^{1-\alpha} - q_{qu}, \qquad (19)$$

where the quantum parameter of randomness q_{qu} reads

$$q_{qu} = \frac{CW^2}{2E_{\perp}^2}\delta_0^{\frac{2}{2-\alpha}}. \qquad (20)$$

The numerical solution of Eq.(19) is depicted in Fig.2. The reduced $T = 0$ transverse rigidity $\delta_{qu}^*(q_{qu})|_{T=0}$ is

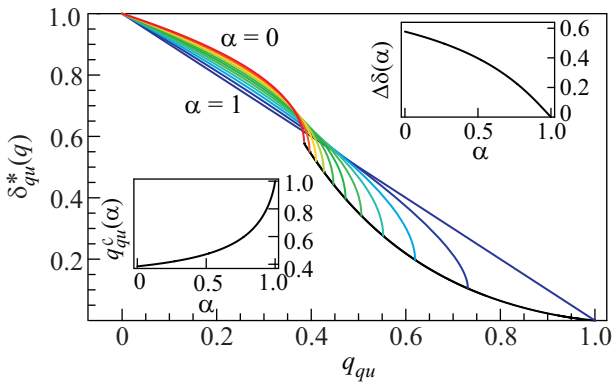

Fig.2. The dependence of the reduced $T = 0$ transverse rigidity $\delta_{qu}^*(q_{qu})$ on the disorder-strength parameter q_{qu} is shown for $0 \le \alpha \le 1$ in steps of $\Delta\alpha = 0.1$. At $q_{qu} = q_{qu}^c$, $\delta_{qu}^*(q_{qu})$ drops to zero for $\alpha < 1$ and vanishes continuously only at $\alpha = 1$. Inserts show the α-variation of the jump (upper right corner) and of its position $q_{qu}^c(\alpha)$ (lower left)

shown to decrease with increasing disorder for (fixed) $\alpha < 1$, and suddenly drops to zero at the critical disorder strength $q_{qu} = q_{qu}^c$. Hence the quantum critical behavior corresponds to a first order phase transition. Fig.2 shows how the breakdown point shifts with increasing α to larger disorder, and the jump vanishes for $\alpha \to 1$. Eq.(19) becomes linear for $\alpha = 1$ and gives, by inferring the $q_{qu}(W)$-relation from Eq.(20),

$$\delta_{qu}(W)|_{T=0,\alpha=1} = \delta_0^2\left[1 - \frac{CW^2\epsilon_F}{2E_{\perp}T_c^{(1)2}}\right]. \qquad (21)$$

Here, the transverse rigidity $\delta_{qu}(W)|_{T=0,\alpha=1}$ decreases linearly with increasing W^2 and vanishes at $W_c^2 = 2E_\perp \left(T_c^{(1)}\right)^2/C\epsilon_F$. The quantum critical behavior in the model is however controlled by two parameters, the strength of randomness $q_{qu}(W)$ and the parameter of quantum dynamics α. For $\alpha < 1$, the superconductor-normal metal phase transition at $T = 0$ is always discontinuous, and only turns into second-order at $\alpha = 1$.

Let us now study the *finite T behavior* of the transverse rigidity. The phase transition in a quasi-1D superconductor occurs at some temperature $T = T_c$ when the transverse rigidity in the ensemble of phases $\{\varphi_{\mathbf{j}}(z)\}$ vanishes. The energy spectrum $\omega(\mathbf{q}_\perp, q_z)$ of the collective excitations is reorganized and the transverse $\mathbf{q}_\perp$-dependent part of $\omega(\mathbf{q}_\perp, q_z)$ vanishes at $T = T_c$, i.e. symmetry breaking occurs in the bosonic excitation at $T = T_c$. Inserting the solution of Eq.(17) for $T < \alpha T_c^{(1)}$ into $\delta_{qu}^2(T) = \delta_{cl}^2 e^{-S_\alpha(\mathbf{g},T)}$ and using Eq.(18), we obtain

$$\delta_{qu}^2(T) = \delta_{qu}^2(0)\left(\frac{T}{\alpha T_{c0}}\right)^\alpha \exp\left\{-C\frac{T}{T_{c0}}\frac{\delta_{qu}(0)}{\delta_{qu}(T)}\right\}, \tag{22}$$

where a new temperature scale is introduced by means of $T_{c0} = \delta_{qu}(0)T_c^{(1)}$, and C is a constant $C \sim 1$. In terms of

$$y = \left(\frac{\alpha T_{c0}}{T}\right)^{\alpha/2}\frac{\delta_{qu}(T)}{\delta_{qu}(0)} \text{ and } \theta = \left(\frac{T}{T_{c0}}\right)^{1-\frac{\alpha}{2}}\frac{C}{2}\alpha^{\alpha/2},$$

Eq.(22) assumes the form $y = \exp\{-\theta/y\}$, which has a non-zero solution only for $\theta \le e^{-1}$. The finite solution of this equation vanishes discontinuously at $\theta = \theta_c = e^{-1}$, giving the following value for T_c

$$T_c = T_{c0}\alpha^{-\frac{\alpha}{2-\alpha}}(2/eC)^{\frac{2}{2-\alpha}}. \tag{23}$$

The magnitude of the jump in $y(\theta_c)$ is e^{-1}, and hence the phase transition is of first-order.

Meissner effect. The current density is calculated according to

$$\frac{1}{c}\mathbf{J}(z,\mathbf{j}) = -T\frac{\delta}{\delta\mathbf{A}}\langle\ln Z(\mathbf{A})\rangle.$$

For simplicity we present here only the diamagnetic contribution to the i-th ($i = \|, \perp$) component of the current

$$J_i^{dia}(z,\mathbf{j}) = -\frac{c}{4\pi\lambda_i^2}A_i(z,\mathbf{j}), \tag{24}$$

where the longitudinal- $\lambda_\|$ and the transverse $\lambda_\perp$ components of the penetration depth are obtained as

$$\lambda_\|^{-2} = \frac{4\pi e^2 N_s^{(1)}}{c^2 m_\| a_\perp^2};\quad \frac{\lambda_\|^2}{\lambda_\perp^2} = \frac{2m_\| a_\perp^2 E_\perp}{\hbar^2}\langle\langle\cos(\varphi_{\mathbf{j}} - \varphi_{\mathbf{j}+\mathbf{g}})\rangle\rangle. \tag{25}$$

Although $\lambda_\|(T)$ diverges at $T = T_c^{(1)}$ due to pair breaking in the SC wires, $\lambda_\perp(T)$ diverges at the global SC transition temperature $T = T_c$, where the phase coherence between neighboring wires is destroyed. Randomness in the Josephson coupling shifts T_c to lower temperatures and, therefore, the magnetic field parallel to the SC wires penetrates easier into the organic superconductor. On the other hand, the randomness does not break the Cooper pairs, keeping the penetration of a perpendicular magnetic field into the SC wires unchanged.

In this Letter we studied disorder-effects on T_c and on the diamagnetism of Josephson-coupled quasi-1D superconductors. Interplay of disorder with quantum phase fluctuations plays a central role for superconductor-normal metal phase transitions in quasi-1D superconductors. The quantum criticality is controlled by two quantities, namely disorder strength and dynamical parameter of phase fluctuations. The present model's quantum criticality signals the existence of a quantum critical phase between SC- and normal phase. Its nature, whether it is a "mixing" of a glassy and CDW or SDW phases, needs further investigation.

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1. C. Bourbonnais and D. Jérome, arXiv:0904.0617v1 [cond-mat.str.el] 3 Apr. 2009.
2. M. Weger and I. Goldberg, Solid State Phys. **28**, 2 (1973).
3. M.-Y. Choi et al., Phys. Rev. B **25**, 6208 (1982).
4. T. Ishiguro, K. Yamaji, and G. Saito, *Organic Superconductors*, Springer-Verlag, Heidelberg, 1998.
5. N. Joo, P. A.-Senzier, C. Pasquier et al., J. Euro. Phys. B **40**, 43 (2004).
6. P. W. Anderson, J. Phys. Chem. Solids **11**, 26 (1959).
7. A. A. Abrikosov and L. P. Gor'kov, Zh.Eksp.Teor.Fiz. **35**, 1558 (1958) [Sov. Phys. JETP **8**, 1090 (1959)].
8. M. Ma and P. A. Lee, Phys. Rev. B **32**, 5658 (1985).
9. A. Ghosal et al., Phys. Rev. B **65**, 014501 (2001).
10. R. Oppermann, Physica A**167**, 301 (1990).
11. B. Spivak et al., Phys. Rev. B **64**, 132502 (2001).
12. S. Sachdev, P. Werner, and M. Troyer, Phys. Rev. Lett. **92**, 237003 (2004).
13. B. Spivak, P. Oreto, and S. A. Kivelson, Phys. Rev. B **77**, 214523 (2008).

14. A. Maestro et al., Phys. Rev. Lett. **101**, 035701 (2008).
15. V. ,M. Galitski and A. I. Larkin, Phys. Rev. Lett. **87**, 087001 (2001).
16. M. A. Skvortsov and M. V. Feigel'man, Phys. Rev. Lett. **95**, 057002 (2005).
17. A. M. Finkel'stein, Pis'ma v Zh. Eksp. Teor. Fiz. **45**, 37 (1987) [JETP Lett. **45**, 46 (1987)].
18. K. B. Efetov and A. I. Larkin, Zh.Eksp.Teor.Fiz. **66**, 2290 (1974) [Sov. Phys. JETP **39**, 1129 (1975)].
19. Yu. A. Firsov and G. Yu. Yashin, Zh.Eksp.Teor.Fiz. **72**, 1450 (1977) [Sov. Phys. JETP **45**, 761 (1977)].
20. V. J. Emery and S. A. Kivelson, Nature **374**, 434 (1995); Phys. Rev. Lett. **74**, 3253 (1995).
21. E. P. Nakhmedov and Yu. A. Firsov, Physica C **295**, 150 (1998).
22. T. M. Rice, Phys. Rev. A **140**, 1889 (1965).