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Generation of continuous-variable entanglement in a three-level system

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A single three-level atom interacting with a two-mode cavity is studied. The generation of output entanglement and output squeezing is investigated. It shows that this system can serve as an output-entanglement source. By tuning the pump field or the detuning, strong entanglement can be generated.

1. Introduction. Continuous-variable (C-V) entanglement is known to be a very important resource in quantum computation and quantum communication [1]. The main motivation of introduction continuous variables theory in quantum information originates from a more practical observation: with continuous quadrature amplitudes of the quantized electromagnetic field, the quantum communication protocols is efficiently achieved in quantum optics. The generation of C-V entanglement has attracted much interest after the first realization of unconditional quantum teleportation [2]. Therefore, how to generate high-intensity entangled light has become an energetic research field in quantum optics.

The classical scheme of producing C-V entanglement is Nondegenerate parametric down conversion (NPDC) in a crystal [3, 4] and injecting single-mode-squeezed light to a beam splitter [5 – 7]. Besides, the preparation of the high-intensity entanglement based on atomic coherence has been investigated extensively [8 – 17]. For example, Xiong *et al.* [8] propose a class of generating entanglement amplifiers based on two-mode correlated spontaneous emission lasers (CEL) involving a three-level atom interacting with two modes of the cavity is induced by pumping atom from the lower level. Then, Tan *et al.* [9] extend the work of Ref. [8] and investigate the generation and evolution of entangled light in the Wigner representation. In order to enhance the intensity of C-V entanglement, Zhou *et al.* [10] study the generation of a macroscopic entangled state in a single three-level atom in cavity-QED system even under the presence of cavity losses. Recently, Kiffner *et al.* [11] consider a scheme of generating two-mode entanglement in macroscopic light just using a single four-level atom. Very Recently, in order to estimate the entanglement in a quantum beat laser, Qamar *et al.* [12] consider a quantum beat laser as a source of entangled radiation.

All above researches are centred on generating C-V entanglement in the cavity. In this letter, output entanglement and output squeezing are investigated in a single three-level atom interacting with a two-mode cavity. It shows that this single atom cavity-QED system can generate output entanglement. By tuning the intensity of pump field with Rabi frequency Ω and the detuning δ, strong output squeezing and output entanglement can be produced.

2. The model and calculation. Here, we consider a single three-level atom in a cascade configuration trapped in a two-mode field cavity. The configuration of this system is depicted in Fig.1. The two atomic transi-

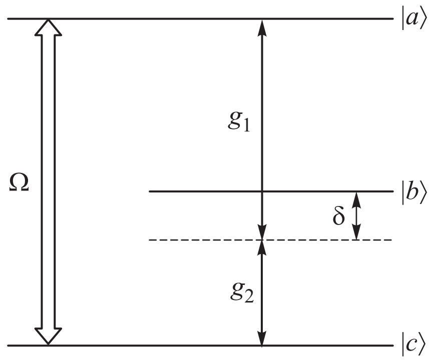

Fig.1. An outline of a single three-level atom and exciton mode in a microcavity driven by an external classical field

tions $|a\rangle \leftrightarrow |b\rangle$ and $|b\rangle \leftrightarrow |c\rangle$ interact with the two-mode cavity with detunings $\mp\delta$. The dipole forbidden atomic transition between $|a\rangle$ and $|c\rangle$ are resonantly driven by a classical field with Rabi frequency Ω. The two-mode cavity interacts with atomic transitions $|a\rangle \leftrightarrow |b\rangle$ and $|b\rangle \leftrightarrow |c\rangle$ with the detunings $\mp\delta$, and the classical field with the Rabi frequency Ω drives the dipole forbidden atomic transition between $|a\rangle$ and $|c\rangle$ resonantly. In rotating wave approximation and in the interaction picture, the Hamiltonian of this system is given by

$$\hat{H}_I = g_1(\hat{a}\hat{\sigma}_{bc} + \hat{a}^\dagger\hat{\sigma}_{cb}) + g_2(\hat{b}\hat{\sigma}_{ab} + \hat{b}^\dagger\hat{\sigma}_{ba}) + \\ + \Omega(\hat{\sigma}_{ac} + \hat{\sigma}_{ca}) - \delta(\hat{\sigma}_{aa} + \hat{\sigma}_{cc}), \quad (1)$$

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where g_1 and g_2 are the atom-field coupling constants. $\hat{a}(\hat{a}^\dagger)$ and $\hat{b}(\hat{b}^\dagger)$ are the creation (annihilation) operators of the two cavity modes, and $\hat{\sigma}_{ij} = |i\rangle\langle j|$ $(i,j = a,b,c)$ are the atomic transition operators.

The time evolution of the Heisenberg equations for the atomic operators $\hat{\sigma}_{bc}$ and $\hat{\sigma}_{ba}$ are written as

$$i\frac{d\hat{\sigma}_{ba}}{dt} = -g_1\hat{a}^\dagger\hat{\sigma}_{ca} + g_2\hat{b}(\hat{\sigma}_{bb} - \hat{\sigma}_{aa}) + \Omega\hat{\sigma}_{bc} - \delta\hat{\sigma}_{ba},$$
$$i\frac{d\hat{\sigma}_{bc}}{dt} = -g_1\hat{a}^\dagger(\hat{\sigma}_{cc} - \hat{\sigma}_{bb}) - g_2\hat{b}\sigma_{ac} + \Omega\hat{\sigma}_{ba} - \delta\hat{\sigma}_{bc}, \tag{2}$$

under the large detuning condition, Eq. (2) can be solved adiabatically by taking $d\hat{\sigma}_{ba}/dt = d\hat{\sigma}_{bc}/dt = 0$. The adiabatic solutions for $\hat{\sigma}_{ba}$ and $\hat{\sigma}_{bc}$ can then be substituted into the Hamiltonian Eq. (1), and can be obtained

$$\begin{aligned}\hat{H}_1 &= \Omega(\hat{\sigma}_{ac} + \hat{\sigma}_{ca}) - \delta(\hat{\sigma}_{aa} + \hat{\sigma}_{cc}) + \\ &+ \frac{1}{\Omega^2 - \delta^2}\{\delta g_1^2(2a^\dagger a + 1)(\hat{\sigma}_{cc} - \hat{\sigma}_{bb}) + \\ &+\delta g_2^2(2\tilde{b}^\dagger\tilde{b} + 1)(\hat{\sigma}_{aa} - \hat{\sigma}_{bb}) + \Omega[(g_1^2a^\dagger a + g_2^2b^\dagger b)\hat{\sigma}_{ac} + \\ &+ (g_1^2aa^\dagger + g_2^2bb^\dagger)\hat{\sigma}_{ca}] + 2g_1g_2\delta(ab\hat{\sigma}_{ac} + a^\dagger b^\dagger\hat{\sigma}_{ca}) + \\ &+ g_1g_2\Omega(ab + a^\dagger b^\dagger)(\hat{\sigma}_{aa} + \hat{\sigma}_{cc} - 2\hat{\sigma}_{bb})\}.\end{aligned} \tag{3}$$

If the atom is initially prepared in the level $|b\rangle$, it will remain confined to this level due to the large detuning approximation. The approximate effective Hamiltonian for this case reduces to

$$\hat{H}_{\text{eff}} = \eta_1a^\dagger a + \eta_2b^\dagger b + \frac{1}{2}(\eta_1 + \eta_2) + \xi(ab + a^\dagger b^\dagger), \tag{4}$$

where

$$\xi = \frac{2g_1g_2\Omega}{\delta^2 - \Omega^2}, \qquad \eta_1 = \frac{2g_1^2\delta}{\delta^2 - \Omega^2}, \qquad \eta_2 = \frac{2g_2^2\delta}{\delta^2 - \Omega^2}.$$

In Eq. (4) the constant term $\frac{1}{2}(\eta_1 + \eta_2)$ does not affect the dynamics of this system. This Hamiltonian can be rewritten as

$$\hat{H}_{\text{eff}} = (\eta_1 + \eta_2)\hat{K}_0 + \xi(\hat{K}_- + \hat{K}_+) + \frac{1}{2}(\eta_1 - \eta_2)\hat{N}_0, \tag{5}$$

where

$$\hat{K}_0 = \frac{1}{2}(a^\dagger a + b^\dagger b + 1), \quad \hat{K}_- = ab,$$
$$\hat{K}_+ = a^\dagger b^\dagger, \ \hat{N}_0 = a^\dagger a - b^\dagger b.$$

These operators obey the $SU(1,1)$ commutation relations $[\hat{K}_-, \hat{K}_+] = 2\hat{K}_0$, $[\hat{K}_0, \hat{K}_\pm] = \pm\hat{K}_\pm$, and $[\hat{N}_0, \hat{K}_0] = [\hat{N}_0, \hat{K}_\pm] = 0$. Thus, we can use the $SU(1,1)$ Lie-algebra to expand the unitary evolution operator $\hat{U} = e^{-i\hat{H}_{\text{eff}}t}$ as

$$\hat{U} = e^{(A_+\hat{K}_+)}e^{(\ln A_0\hat{K}_0)}e^{-\frac{it}{2}(\eta_1 - \eta_2)\hat{N}_0}e^{(A_-\hat{K}_-)} \tag{6}$$

where $A_0 = a_0^2$, $A_+ = A_- = (-i\xi t/\phi)a_0\sinh\phi$, with

$$a_0 = \frac{1}{\cosh\phi + it\frac{(\eta_1 + \eta_2)}{2\phi}\sinh\phi},$$
$$\phi^2 = [-(\frac{\eta_1 + \eta_2}{2})^2 + \xi^2]t^2. \tag{7}$$

When the two-mode field is initially prepared in a coherent state $|\alpha, \beta\rangle$. The time evolution of the field state can be obtained

$$|\Psi_f(t)\rangle \Rightarrow \exp(A_+\hat{a}^\dagger\hat{b}^\dagger)|\alpha, \beta\rangle. \tag{8}$$

The $SU(1,1)$ Lie-algebra yields

$$e^{A_+\hat{a}^\dagger\hat{b}^\dagger} = e^{(\vartheta^*\hat{a}\hat{b} - \vartheta\hat{a}^\dagger\hat{b}^\dagger)}e^{A_+^*\hat{a}\hat{b}}e^{g(\hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b} + 1)}. \tag{9}$$

Let $\vartheta = re^{i\varepsilon}$, $g = \ln\cosh r$, where r and ε are determined by the relation

$$A_+ = -e^{i\varepsilon}\tanh r. \tag{10}$$

The squeezed parameters r and ε are

$$r = \tanh^{-1}|A_+|, \ \cos\varepsilon = -\frac{\text{Re}\,A_+)}{|A_+|}, \ \sin\varepsilon = -\frac{\text{Im}(A_+)}{|A_+|}.$$

The state of the system can then be written as

$$|\Psi_f(t)\rangle = e^{(\vartheta^*\hat{a}\hat{b} - \vartheta\hat{a}^\dagger\hat{b}^\dagger)}|\alpha, \beta\rangle = S(\vartheta)D(\alpha)D(\beta)|0,0\rangle, \tag{11}$$

it is obviously a two-mode coherent-squeezed state [18, 19].

3. Output Entanglement. In this section, it shows that this system can serve as an entangled source of the output cavity fields. The numerical results of the C-V entanglement is presented. Employed the methods of input-output theory [20, 21], the entanglement between the two modes outside the cavity can be evaluated. Assume that the two-mode cavity is driven by two external laser fields with strengths μ_1 and μ_2, the Langevin equations of motion for the two-mode cavity fields can be expressed as

$$\dot{a} = -i(\eta_1a + \varepsilon b^\dagger + \mu_1^*) - \frac{\kappa_1}{2}a - \sqrt{\kappa_1}a_{in},$$
$$\dot{b} = -i(\eta_2b + \varepsilon a^\dagger + \mu_2^*) - \frac{\kappa_2}{2}b - \sqrt{\kappa_2}b_{in}, \tag{12}$$

where, a_{in} and b_{in} are the annihilation operators of input fields. $\kappa_{1(2)}$ is the cavity decay rate of mode $a(b)$.

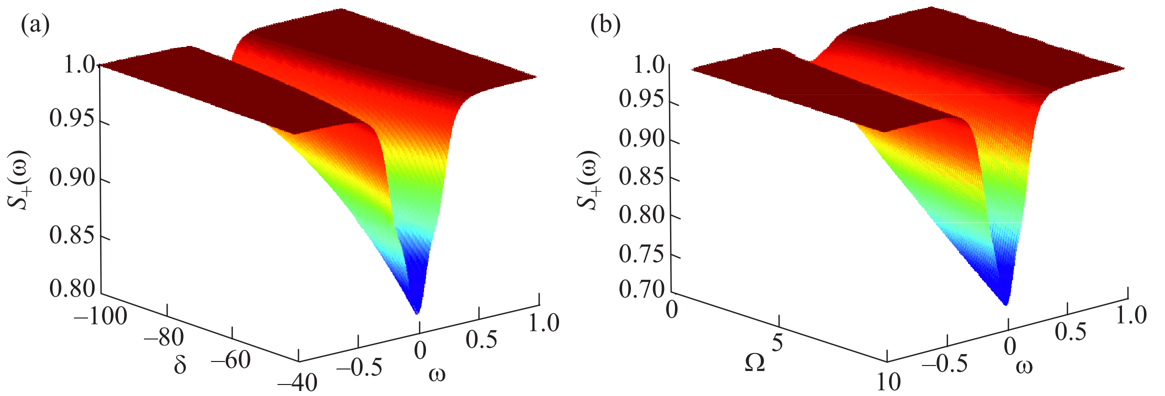

Fig.2. The output squeezing spectrum $S_+(\omega)$ as a function of parameters ω, δ and Ω. The parameters are $g_2 = 2g_1$, $\kappa_1 = \kappa_2$=0.2g_1, (a) $\Omega = 5g_1$; (b) $\delta = -50g_1$

With the transformations $a = a' + \alpha_0$ and $b = b' + \beta_0$, Eq. (12) can be rewritten as

$$\dot{a}' = -i(\eta_1 a' + \varepsilon b'^\dagger) - \frac{\kappa_1}{2}a' - \sqrt{\kappa_1}a_{in},$$
$$\dot{b}' = -i(\eta_2 b' + \varepsilon a'^\dagger) - \frac{\kappa_2}{2}b' - \sqrt{\kappa_2}b_{in}, \tag{13}$$

where

$$\alpha_0 = \frac{-2i\mu_1^*(\kappa_2 + 2i\eta_2) - 4\mu_2^*\varepsilon}{(\kappa_1 + 2i\eta_1)(\kappa_2 + 2i\eta_2) + 4\varepsilon^2},$$
$$\beta_0 = \frac{-2i\mu_2^*(\kappa_1 + 2i\eta_1) - 4\mu_1^*\varepsilon}{(\kappa_1 + 2i\eta_1)(\kappa_2 + 2i\eta_2) + 4\varepsilon^2}. \tag{14}$$

Using the input-output relationship $a_{jout} = a_{jin} + \sqrt{\kappa_j}a_j$, one can perform fourier transformation to Eq. (13)

$$a_{\rm out}(\omega) = \sqrt{\kappa_1}\alpha_0\delta(\omega) - \frac{(\alpha_1^*\alpha_2 + \varepsilon^2)a_{in}(\omega) - i\varepsilon\sqrt{\kappa_1\kappa_2}b_{in}^\dagger(-\omega)}{\alpha_1\alpha_2 - \varepsilon^2},$$
$$b_{\rm out}(\omega) = \sqrt{\kappa_2}\beta_0\delta(\omega) - \frac{(\beta_1^*\beta_2 + \varepsilon^2)b_{in}(\omega) - i\varepsilon\sqrt{\kappa_1\kappa_2}a_{in}^\dagger(-\omega)}{\beta_1\beta_2 - \varepsilon^2}, \tag{15}$$

where $\alpha_1 = \kappa_1/2 + i(\eta_1 - \omega)$, $\alpha_2 = \kappa_2/2 - i(\eta_2 + \omega)$, $\beta_1 = \kappa_2/2 + i(\eta_2 - \omega)$, and $\beta_2 = \frac{\kappa_1}{2} - i(\eta_1 + \omega)$.

The squeezing spectrum can be defined as [22]

$$\langle I_\pm(\omega)\rangle\langle I_\pm(\omega')\rangle + \langle I_\pm(\omega')\rangle\langle I_\pm(\omega)\rangle = 2S\pm(\omega)\delta(\omega + \omega'), \tag{16}$$

where $I_\pm(\omega)$ can be defined as

$$I_+(\omega) = \frac{1}{\sqrt{2}}[a(\omega) + a^\dagger(\omega) - b(\omega) - b^\dagger(\omega)],$$

$$I_-(\omega) = \frac{1}{\sqrt{2}i}[a(\omega) - a^\dagger(\omega) + b(\omega) - b^\dagger(\omega)].$$

According to this define, one find that $S_+(\omega)$ is equal to $S_-(\omega)$ for uncorrelated vacuum input noise.

To evaluate entanglement of the output fields, the fluctuated criterion proposed by Duan et al. [23] is employed

$$S_+(\omega) + S_-(\omega) < 2. \tag{17}$$

If above condition is satisfied, the spectrum $S_\pm(\omega)$ is not only squeezing but also entangled. Assumed that the input field is in the vacuum, from Eq. (15) one can give the expression of $S_+(\omega)$ as

$$S_+(\omega) = \frac{|\varepsilon^2 + \alpha_1^*\alpha_2|^2 - i\sqrt{\kappa_1\kappa_2}\varepsilon(\alpha_1\alpha_2^* - \alpha_2\alpha_1^*) + \kappa_1\kappa_2\varepsilon^2}{2|\alpha_1\alpha_2 - \varepsilon^2|^2} + (\alpha_j \to \beta_j). \tag{18}$$

From Eq. (15) and Eq. (18), it can be clearly seen that the squeezing spectrum is independent of the parameters α_0 and β_0.

In Fig.2, output squeezing spectrum $S_+(\omega)$ for the two-mode light as a function of parameters ω, δ and Ω is plotted. It shows that it is possible to obtain the squeezing and C-V entanglement for output photons. The maximum squeezing and maximum entanglement can be observed for the parameter $\omega = 0$, while with the absolute value $|\omega|$ increasing, squeezing spectrum reduces to zero corresponding to $S_+(\omega) = 1$. Moreover, the output squeezing is decreasing with the detuning δ enhancing (Fig.2a). The degree of squeezing increases with the parameter Ω which represents the pump fields (Fig.2b). Thus, by increasing the pump field or reducing the detuning, strong output squeezing and output entanglement can be generated.

In order to investigate the influence of the decay rate γ on the output squeezing and output entanglement. In

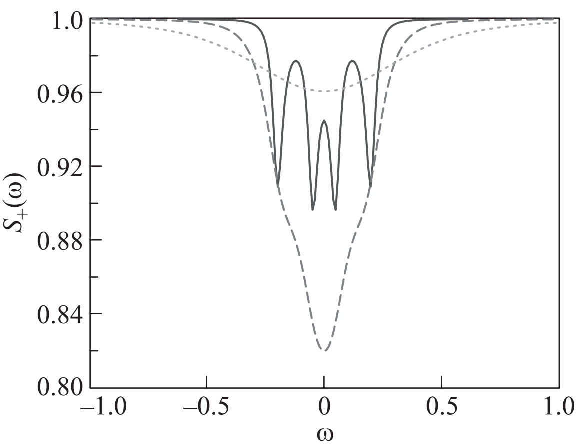

Fig.3. The output squeezing spectrum $S_+(\omega)$ as a function of ω with the different decay rates κ_1 and κ_2. The parameters are $g_2 = 2g_1$, $\Omega = 5g_1$, $\delta = -40g_1$ and (1) $\kappa_1 = \kappa_2$=0.05g_1; (solid line); (2) $\kappa_1 = \kappa_2$=0.2g_1 (dashed line); (3) $\kappa_1 = \kappa_2$=g_1 (dotted line)

Fig.3 we show that the squeezing spectrum changes with different decay rates of the cavity. For $\kappa_1 = \kappa_2 = 0.2g_1$ or $\kappa_1 = \kappa_2 = g_1$, the splits from one valley into four minima are seen from the figure, that is, a minimum values corresponding to maximum squeezing and entanglement can be obtained. Generally only one valley can be seen for $\kappa_1 = \kappa_2$ [24], however for $\kappa_1 = \kappa_2 = 0.05g_1$, four symmetrical minimum values corresponding to the maximum squeezing are observed. This abnormal behavior originates the nonzero and asymmetric parameters η_1 and η_2 ($\eta_1 \neq \eta_2$ it can be seen from Eq. (5)). With the increasing of the decay rate κ, this abnormal splits disappear. Moreover, one can find that an appropriate value κ can ensure the successful squeezing and entanglement. For example, in this system choosing $\kappa_1 = \kappa_2 = 0.2g_1$ is better than $\kappa_1 = \kappa_2 = 0.05g_1$, g_1. From the figure one can also see that the bandwidth of the squeezing becomes wide with the increasing of the decay rates.

4. Conclusion. In summary, using a single three-level atom driven by a pump field, a scheme for the generation of output entanglement and squeezing is proposed. According to the numerical results, the influence of the Rabi frequency Ω, the detuning δ and the decay rate γ on the output squeezing and entanglement is investigated. It shows that, by increasing the strength of pump field or reducing the detuning, strong output squeezing and C-V entanglement can be generated. In addition, one find that choosing an appropriate value κ can ensure the successful squeezing and entanglement.

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