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# Search for long-range forces by means of ultracold neutrons (1)

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A method of using a gravitational spectrometer to search for long-range forces between neutrons and atoms is proposed. The constraints on the strength of long range forces within the range of $10^{-10}$–$10^{-4}$ cm can be obtained from the experiments on measurements of the total cross section of interaction of ultracold neutrons with atoms of noble gases (He, Ne, Ar, $^{86}$Kr) and the data on the coherent neutron scattering length of the nucleus. The first result of such type analysis is presented.

Search for deviations of gravitational interaction from the $1/r^2$ law (inverse-square law) in the range of small distances is extremely important in order to verify both the theories assuming existence of additional dimensions [1, 2] and supersymmetric theories in which existence of new very light particles is supposed. The exchange of these particles leads to additional interactions between nucleons [3–5]. The review of theoretical and experimental works on search for deviations from the inverse-square law is presented in works [6, 7]. In the given work we will discuss forces which can appear at distances $10^{-10}$–$10^{-4}$ cm. From the point of view of search for deviations of gravitational interaction from the inverse-square law these forces should be named the short-range forces. But in nuclear interactions a characteristic scale of distances is of the order of $10^{-13}$ cm, thereby for nuclear physics the interaction at distances of $10^{-10}$–$10^{-4}$ cm are long-range forces. We have stopped on the definition of long-range forces because it is a question of interaction of a neutron with a nucleus. Such a term is specified in the title of our article.

There are different methods of search for long-range forces in the interaction of elementary particles [6–8]. Within the range of $10^{-11}$–$10^{-9}$ cm researches are carried out by means of neutrons at the energy of the order of electron volt [9, 10]. For distances $10^{-4}$–$10^{-2}$ cm the laboratory experiments investigating gravitational interactions of bodies are used [11–17]. Within the range of $10^{-10}$–$10^{-4}$ cm there are rather effective methods using thermal and cold neutrons [9, 18]. In the given article there will be discussed a question of possibility to use ultracold neutrons for the range $10^{-10}$–$10^{-4}$ cm.

The scattering amplitude of a neutron by atoms can be expressed in the following way

$$f(q) = f_{\text{nucl}} + f_{n-e}(q) + f_{\text{long\_range}}(q), \qquad (1)$$

where $f_{\text{nucl}}$ is a nuclear scattering amplitude, which is usually expressed through scattering length $b$, $f_{\text{nucl}} = -b$, $f_{n-e}(q)$ is the amplitude of neutron-electron scattering, which arises due to scattering of a neutron by charges distributed inside the nucleus and the electron shell of atoms. Further we will not consider contribution from the $n-e$ interaction, because this effect occurs mainly for fast neutrons [19]. The following term in the equation (1) does relate to a hypothetical long-range interaction (in comparison to the nuclear one) of a neutron with a nucleus. $f_{\text{long\_range}}(q)$ is a spin independent amplitude of interaction, which can arise as a result of exchange by a scalar or vector boson. In case of a scalar type of interaction the potential of interaction is written as an attractive potential, for a vector boson exchange the potential of interaction is written as a repulsive one

$$\varphi(r) = \frac{\pm g_{\pm}^2 Mhce^{-r/\lambda}}{4\pi r}, \qquad (2)$$

where $M$ is mass of atom of gas in units of nucleon mass $m_n$. In formula (2) the lower sign corresponds to a scalar type of interaction, the upper does to a vector type of interaction.

Correspondently, the amplitude within the Born approximation can be presented in the form

$$f_{\text{long\_range}} = -\frac{m}{2\pi h^2}\int \varphi(r)e^{-i\mathbf{qr}}dV = \frac{m2m}{h^2}\frac{g_{\pm}^2 Mhc}{4\pi}\frac{\lambda^2}{(\lambda q)^2+1}, \qquad (3)$$

where $m$ is reduced mass $m = m_n m_A/(m_n + m_A)$, mass of an atom of gas $m_A = m_n M$, $q = |\mathbf{k}' - \mathbf{k}|$, is a momentum transferred to a neutron, $\mathbf{k}$ and $\mathbf{k}'$ are wave vectors of the particle at rest before and after collision. The momentum $q$ is connected with the neutron recoil energy $\varepsilon$ by a simple relation: $q = \sqrt{2\varepsilon m_n}/h$.

An experimental search for additional terms in the scattering amplitude can be based on the fact that a

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long-range interaction gives contribution to the scattering amplitude at small transferred momentum $q$ or at small scattering angles. The scattering amplitude at $\theta = 0$ or $q = 0$ can be measured at a high accuracy in neutron-optical experiments with an interferometer [18]. This result should be compared with $f_{\rm nucl} = -b$ to find out presence of additional members in equation (1). For example, comparison of measurements with interferometers and experiments with the Bragg diffractometer allows strong enough restrictions [9] to be obtained.

Lately the method of studying quantum states of a neutron in the Earth gravitational field near the matter surface [20] has been actively discussed. However, there is lack of real statistics in these researches therefore we are going to propose in this article more statistical method.

A direct method of research would be the method of a small angle scattering, as existence of long-range forces results in appearing of scattering at small angles. In this method there are obvious problems connected with existence of a small angle scattering resulting from scattering on the texture of the matter and multiple scattering. Besides, initial divergence of a beam does not permit to distinguish scattering at very small angles from the beam divergence.

In this article one suggests reconsidering an approach to the method of a small angle scattering and switching to registration of small recoil energy instead of small angles of scattering. A new method suggests using gas of ultracold neutrons (UCN) as a target that is collided with the flux of atoms being in the same trap. Criterion of a signal of scattering caused by long-range forces is transfer of ultimately small recoil energy $\sim 10^{-7}$ eV, which can be registered with the help of a trap of ultracold neutrons.

For thermal neutrons the recoil energy to a neutron $\sim 10^{-7}$ eV corresponds to a scattering angle $2 \cdot 10^{-3}$ radian, which is within divergence of an incident neutron beam. For cold neutrons this scattering angle is twice higher, but it does not yet exceed divergence of a neutron beam.

The method of a trap of ultracold neutrons filled with the investigated gas (He, Ar and other heavy one-atomic noble gases.) allows recoil energy about $10^{-7}$ eV to be registered. On the other hand the total cross section of UCN with gas will not include areas with recoil energy smaller than $10^{-7}$ eV. Therefore it is possible to compare the scattering amplitude from interferometer measurements $f(0)$ with the amplitude obtained by the UCN method.

One of possible schemes of an experiment is presented in Fig. 1. It allows to use the available equipment of PNPI in ILL. UCN fill the trap at an opened valve (*2*) and closed valves (*6*) and (*7*). The absorber (*3*) is placed in the bottom position at a distance "$h$" from the trap bottom. When equilibrium density in the trap (*1*) is achieved, the valve (*2*) is closed. The UCNs are stored in the trap for the predetermined time $t_{\rm hold}$ to form the spectrum with maximal UCN energy $mgh$. Then the absorber (*3*) is pulled up to the upper position near the top of the trap.

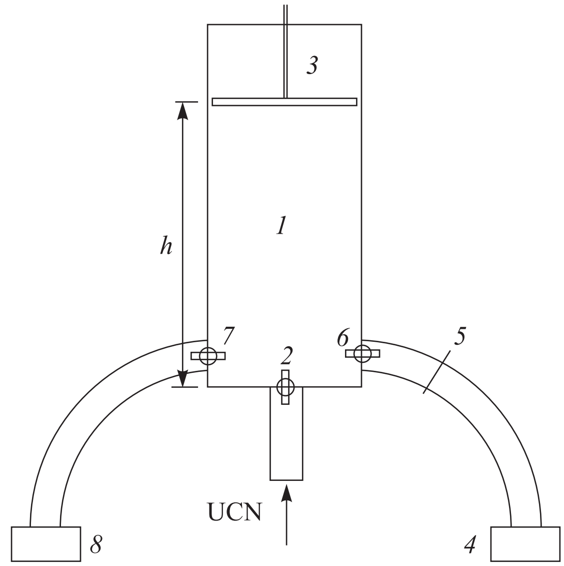

Fig. 1. The experiment setup: *1* – trap of UCN, *2* – the input valve, *3* – the absorber-former of spectrum of UCN, *4* – the detector of lower barrier neutrons, *5* – a foil with the boundary velocity above the one of UCN after spectrum formation, *6* – the valve for the background measurement with the detector of upper barrier neutrons, *7* – the valve for output of UCN after storage, *8* – the detector for counting of UCN after storage

Ultracold neutrons interact with walls of the trap and with the investigated gas, which fills the trap. The trap and the investigated gas are maintained at the room temperature. The temperature of UCN gas is $10^{-3}$ K. In coherent reflection from the matter the energy of UCN is conserved. Non-elastic scattering occurs when a neutron is scattered by atoms of gas and when UCN penetrate into substance in reflecting from the wall. In both cases there is energy transfer by the order of $kT$. The energy transfer of the order of $10^{-7}$ eV is of low probability for the above mentioned processes. Nevertheless at reflection from the substance there is a quasi-elastic scattering which was revealed experimentally [21]. Due to long-range interaction with atoms of gas the quasi-elastic scattering with the recoil energy of the order $10^{-7}$ eV would be also possible. Processes with the energy transfer of the order of $kT$ and $\sim 10^{-7}$ eV are easily distinguished in the present installation, as UCN having obtained energy $\sim kT$ leave an experimental trap. Such neutrons are not detected. Neutrons which have obtained a small recoil energy can still be stored in the trap, if their energy near the bottom is less than critical energy of the trap. Critical energy of the foil is equal to $mgh$. Therefore neutrons which have obtained a small recoil energy can overcome a potential barrier of the foil (*5*) and finally can be registered by a detector (*4*). For registration of these neutrons the valve (*6*) is opened just after lifting the absorber. The closed valve (*6*) is used for measuring the background with the detector (*4*). To distinguish processes of quasi-elastic scattering on the surface of the trap from scattering with the investigated gas measurements are to be made both with the investigated gas and without it.

The detector (*8*) is used to measure the number of UCN in the trap after different storage time. In this case the valve (*7*) is opened rather than the valve (*6*). Measuring UCN storage time at different gas pressure, we can determine the total cross section of UCN interaction with atoms. As UCN are sensitive to a small energy transfer, this cross section will include interaction due to long-range forces. We can compare the obtained result with nuclear scattering cross section. Additionally we can make measurements (either with gas or without it) when an absorber is not lifted as well as with a lifted absorber. In the first case cross section will include the effect of very small energy transfer because such neutrons can reach absorber. This effect will not be taken into account in case with a lifted absorber. Thus one can measure the effect of very small energy transfer and compare with the expected value due to nuclear scattering. Probably in the latter case it can be a statistical problem because of measuring a small difference of two big values. Therefore the method with foil and a detector (*4*) will be a preferable one.

Another possible scheme of measurements is to use only the detector (*4*) but change position of foil (put in, take out) to make measurements with the same detector. Such a scheme was used in our measurements of lower energy upscattering of UCN in process of storage in the trap [21].

Summing up, we can conclude that two experimental methods have been discussed: the method of total cross-section measurements and the method of above-barrier neutron measurements. Potential sensitivity of both methods will be discussed later.

Let us carry out calculation of the differential cross section depending on the recoil energy transferred to the ultracold neutron. To simplify the problem we will assume that before collision UCN was at rest. The atoms of gas are scattered at UCN. Neutrons obtain the recoil energy. We will consider the amplitude of an additional long range interaction of a neutron with the atom consisting of $M$ nucleons.

The differential cross section of scattering of a neutron with an atom should take into account the amplitude of nuclear scattering and that of scattering (3) due to an additional contribution from the potential (2):

$$d\sigma = |f_{\rm nucl} + f_{\rm long\_range}|^2 d\Omega =$$
$$= \left(b^2_{\rm free\_nucl} \pm \frac{g^2_\pm M}{\pi}\frac{b_{\rm free\_nucl} m_n c^2}{\hbar c}\frac{\lambda^2}{(\lambda q)^2+1} + \right.$$
$$\left. + f^2_{\rm long\_range}\right) d\Omega, \tag{4}$$

where the element of solid angle $d\Omega$ is connected with the energy of an incident atom – $E_A$ through the following formula:

$$d\Omega = \frac{\pi(M+1)^2}{M}\frac{d\varepsilon}{E_A}. \tag{5}$$

For the differential cross section the following expression has been derived

$$d\sigma = |f_{\rm nucl} + f_{\rm long\_range}|^2 = d\Omega =$$
$$= \pi\left(\frac{(M+1)^2}{M} b^2_{\rm free\_nucl} \pm \frac{g^2_\pm M(M+1)}{\pi}\frac{b_{\rm free\_nucl} m_n c^2}{\hbar c} \times \right.$$
$$\left. \times \frac{\lambda^2}{2m_n\varepsilon\lambda^2/\hbar^2+1} + \frac{(M+1)^2}{M} f^2_{\rm long\_range}\right)\frac{d\varepsilon}{E_A}. \tag{6}$$

Now we will integrate expression (6) over the recoil energy from $\varepsilon_1$ to $\varepsilon_2$.

$$\sigma(\varepsilon_2, \varepsilon_1, E_A) = \pi\left[\frac{(M+1)^2}{M} b^2_{\rm free\_nucl}(\varepsilon_2 - \varepsilon_1) \pm \right.$$
$$\pm g^2_\pm M \frac{\hbar c b_{\rm free\_nucl}(M+1)}{2\pi}\ln\left(\frac{2m_n\lambda^2\varepsilon_2/\hbar^2+1}{2m_n\lambda^2\varepsilon_1/\hbar^2+1}\right) +$$
$$+ (g^2_\pm M)^2\left(\frac{m_n c}{2\pi\hbar}\right)^2 \times$$
$$\left. \times \frac{M\lambda^4(\varepsilon_2-\varepsilon_1)}{(2m_n\lambda^2\varepsilon_2/\hbar^2+1)(2m_n\lambda^2\varepsilon_1/\hbar^2+1)}\right]\frac{1}{E_A}. \tag{7}$$

The integral UCN scattering cross section when UCN will be escaped from the trap with critical energy ($E^{\rm trap}_{UCN}$) is:

$$\sigma^{\rm escape}(\varepsilon_{\max}, E^{\rm trap}_{UCN}, E_A) =$$
$$= \pi\left[4b^2_{\rm free\_nucl}\left(1 - \frac{E^{\rm trap}_{UCN}(M+1)^2}{4E_A M}\right) \pm \right.$$
$$\pm g^2_\pm M \frac{\hbar c b_{\rm free\_nucl}(M+1)}{2\pi E_A} \times$$

$$\times\ln\left(\frac{8m_n\lambda^2E_AM/\hbar^2(M+1)^2+1}{2m_n\lambda^2E_{UCN}^{\text{trap}}/\hbar^2+1}\right)+$$
$$+(g_\pm^2M)^2\left(\frac{m_ncM}{\pi\hbar(M+1)}\right)^2\times$$
$$\times\frac{\lambda^4}{(8m_nM\lambda^2E_A/(M+1)\hbar^2+1)(2m_n\lambda^2E_{UCN}^{\text{trap}}/\hbar^2+1)}\Bigg]. \quad (8)$$

The lower energy UCN scattering cross section when UCN will be still storage in the trap is:

$$\sigma^{\text{low}}(E_{UCN}^{\text{trap}},E_A)=$$
$$=\pi\left[\frac{(M+1)^2}{M}b_{\text{free\_nucl}}^2\frac{E_{UCN}^{\text{trap}}}{E_A}\pm\right.$$
$$\pm g_\pm^2M\frac{\hbar cb_{\text{free\_nucl}}(M+1)}{2\pi E_A}\ln(2m_n\lambda^2E_{UCN}^{\text{trap}}/\hbar^2+1)+$$
$$\left.+(g_\pm^2M)^2\left(\frac{m_nc}{2\pi\hbar}\right)^2\frac{M\lambda^4E_{UCN}^{\text{trap}}/E_A}{(2m_n\lambda^2E_{UCN}^{\text{trap}}/\hbar^2+1)}\right]. \quad (9)$$

It should be mentioned that we have assumed for simplification that initial UCN energy is equal to zero. Such a simplification does not matter but makes the calculation much easier.

Formulae (7)–(9) are written for the fixed kinetic energy of an atom. For further calculations we should integrate over the flux of incident atoms. As it has been noted above, the installation setup enables to measure the total sections of interaction of UCN with gas using the detector *8*, and the differential cross sections of very small energy transfer using the detector of the above-barrier neutrons *4*. In the following paragraph the first experimental opportunities are considered in detail.

The probability of UCN storage in a trap is the sum of probability of UCN losses:

$$(\tau_{\text{stor}}^{\text{total}})^{-1}=\tau_n^{-1}+(\tau_{\text{stor}}^{\text{gas}})^{-1}+(\tau_{\text{stor}}^{\text{walls}})^{-1}, \quad (10)$$

where $\tau_n^{-1}$ is the probability of neutron decay, $(\tau_{\text{stor}}^{\text{gas}})^{-1}$ is the probability of UCN losses due to interaction with atoms of gas and $(\tau_{\text{stor}}^{\text{walls}})^{-1}$ is the probability of UCN losses due to interaction with trap walls.

The probability of UCN losses due to neutron interaction with atoms of gas can be measured as the difference of UCN storage probability in a trap with some gas density and with zero gas density:

$$(\tau_{\text{stor}}^{\text{gas}})^{-1}(n_A)=(\tau_{\text{stor}}^{\text{total}})^{-1}(n_A)-(\tau_{\text{stor}}^{\text{total}})^{-1}(n_A=0). \quad (11)$$

Now let us calculate the magnitude of the value $(\tau_{\text{stor}}^{\text{gas}})^{-1}(n_A)$ taking into account an additional contribution from the long-range interaction. The probability of UCN losses due to collision with atoms of gas can be written as follows:

$$(\tau_{\text{stor}}^{\text{gas}})^{-1}(n_A)=\int\limits_{E_{\min}}^{\infty}d\Phi(E_A)\int\limits_{\varepsilon_{\min}}^{E_A\frac{4M}{(M+1)^2}}d\sigma(\varepsilon)+$$
$$+n_AV_{2200}\sigma_{\text{capt}}^0=n_A\bar V_A\sigma_A^{\text{total}}, \quad (12)$$

where $d\Phi(E_a)/dE_A$ is a flux of atoms incident on an ultracold neutron, $d\sigma/d\varepsilon(\varepsilon)$ is a differential cross section depending on the recoil energy, according to formula (6), $\sigma_{\text{capt}}^0$ is capture cross section reduced to neutron velocity $V_{2200}=2.2\cdot10^5\,\text{cm}\cdot\text{s}^{-1}$, $\sigma_A^{\text{total}}$ is the total cross section, which consists of the scattering cross section ($\sigma_{\text{scat}}$) and capture cross section ($\sigma_{\text{capt}}$): $\sigma_A^{\text{total}}=$ $=\sigma_{\text{scat}}+\sigma_{\text{capt}}^0V_{2200}/\bar V_A$ (the scattering cross section ($\sigma_{\text{scat}}$) takes into account a nuclear and long-range interaction), $E_{\min}$ is minimum energy of atoms, after colliding with them a neutron is able to escape the trap $E_{\min}=E_{UCN}^{\text{trap}}(M+1)^2/4M$, $\varepsilon_{\min}$ is minimum recoil energy when UCN escape from the trap. (For simplification UCN initial energy is equal to zero, then $\varepsilon_{\min}=E_{UCN}^{\text{trap}}$), $E_A4M/(M+1)^2$ is maximum neutron recoil energy. The flux of atoms is

$$\frac{d\Phi(E_A)}{dE_A}=\frac{n_A\bar V_A}{(kT)^2}E_A\exp\left\{-\frac{E_A}{kT}\right\}, \quad (13)$$

where $n_A$ is the number of atoms in $\text{cm}^{-3}$ at temperature $T$, $\bar V_A$ is the average velocity of atoms of mass $m_nM$ at temperature $T$, $\bar V_A=4(kT/2\pi m_nM)^{1/2}$. The gas density

$$n_A[\text{cm}^{-3}]=2.687\cdot10^{16}\times P_A[\text{mbar}]\times293/T[\text{K}], \quad (14)$$

where $P_A$ is the experimentally measured gas pressure.

We can rewrite formula (12) in the following form:

$$(\tau_{\text{stor}}^{\text{gas}}n_A\bar V_A)^{-1}-\sigma_{\text{capt}}^0\bar V_{th}/\bar V_A=\frac{\pi(M+1)^2}{M}b_{\text{free\_nucl}}^2\times$$
$$\times\int\limits_{E_{UCN}^{\text{trap}}\frac{(M+1)^2}{4M}}^{\infty}dE_A\int\limits_{E_{UCN}^{\text{trap}}}^{E_A\frac{4M}{(M+1)^2}}\frac{E_A}{(kT)^2}e^{-E_A/kT}\times$$
$$\times\left[1\pm\frac{g_\pm^2M^2}{\pi(M+1)}\frac{m_nc^2}{b_{\text{free\_nucl}}\hbar c}\frac{\lambda^2}{2m_n\varepsilon\lambda^2/\hbar^2+1}+(g_\pm^2M)^2\times\right.$$
$$\left.\times\frac{M^2}{(M+1)^2}\left(\frac{m_nc}{2\pi\hbar_{\text{free\_nucl}}}\right)^2\frac{\lambda^4}{2m_n\lambda^2\varepsilon/\hbar^2+1)^2}\right]\frac{d\varepsilon}{E_A}. \quad (15)$$

After integration we can extract contribution due to long-range forces in the following form:

$$\Delta_1^{\text{excape}}=$$

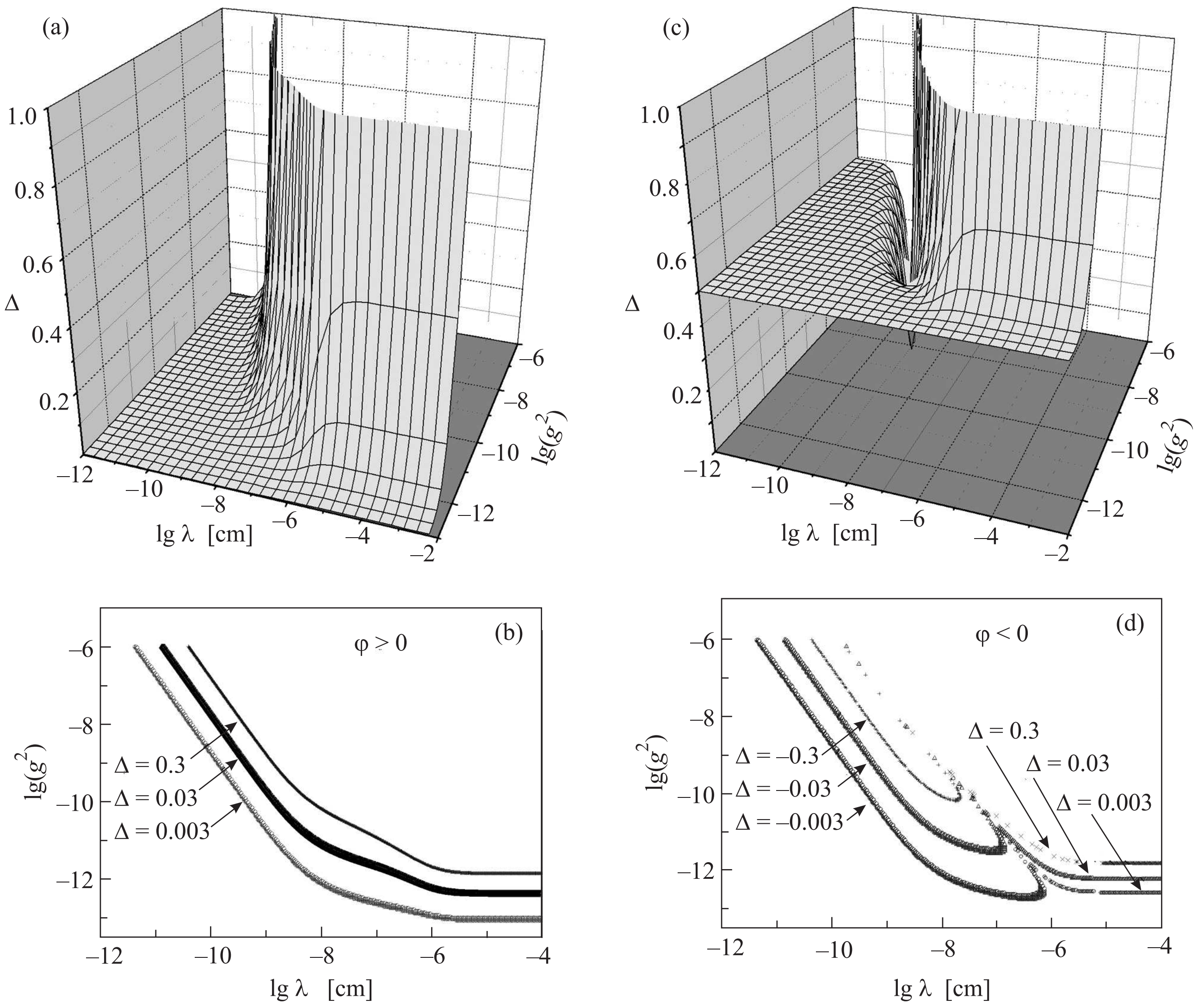


Fig. 2. Dependence of an expected experimental effect $\Delta$ on parameters $g^2$ and $\lambda$ for helium: (a) the case of a repulsive potential, i.e. an exchange by a vector boson; (b) the relation between parameters $g^2$ and $\lambda$ for a repulsive potential if the effect $\Delta$ is equal to 0.3, 0.03 and 0.003; (c) the case of an attractive potential, i.e. an exchange by a scalar boson; (d) the relation between parameters $g^2$ and $\lambda$ for an attractive potential if the effect $\Delta$ is equal to $\pm 0.3$, $\pm 0.03$ and $\pm 0.003$

$$= \left[ \frac{(\tau^{\text{gas}}_{\text{stor}} n_A \bar{V}_A)^{-1} - \sigma^0_{\text{capt}} V_{2200}/\bar{V}_A}{4\pi b^2_{\text{free\_nucl}}} e^{\frac{E^{\text{trap}}_{UCN}}{kT}\frac{(M+1)^2}{4M}} - 1 \right] =$$

$$= \pm \frac{g^2_{\pm} M(M+1)}{8\pi b_{\text{free\_nucl}}} \left(\frac{\hbar c}{kT}\right) e^{z(\lambda)} E_1(z(\lambda)) +$$

$$+ \frac{g^4_{\pm}}{4} \frac{M^2(M+1)^2}{(8\pi b_{\text{free\_nucl}})^2} \left(\frac{\hbar c}{kT}\right)^2 \frac{e^{z(\lambda)} E_2(z(\lambda))}{z}, \qquad (16)$$

where $E_1(z)$ and $E_2(z)$ are exponential integrals. The value

$$z(\lambda) \equiv \frac{(M+1)^2}{4M} \left( \frac{\hbar^2}{2m_n kT\lambda^2} + \frac{E^{\text{trap}}_{UCN}}{kT} \right)$$

is a function of a few variables: $M$, $\lambda$ and $T$.

The expression in the squared brackets (on the left side of formula (16)) is an expected experimental effect due to the long-range interaction. It is defined as $\Delta^{\text{escape}}$ because it is an effect with escape of UCN from the trap.

Let us calculate the value of an expected experimental effect $\Delta^{\text{escape}}$ depending on values $g^2$ and $\lambda$. The case of repulsive potential (a vector boson) and the case of attractive potential (a scalar boson) significantly differ in the form of the effect. In the case of a vector boson the effect is positive for any values $g^2$ and $\lambda$. For a scalar boson the effect can change a sign depending on values $g^2$ and $\lambda$. In Fig. 2a, c the form of a possible effect for both cases is shown. Fig. 2b, d shows the correlation between $g^2$ and $\lambda$, which arises when this surface is crossed by the planes $\Delta^{\rm escape} = \pm 0.3$, $\Delta^{\rm escape} = \pm 0.03$ and $\Delta^{\rm escape} = \pm 0.003$. In case if $\varphi > 0$, the value $\Delta^{\rm escape}$ can be only positive. In case $\varphi < 0$, $\Delta^{\rm escape}$ can have any sign. Therefore at $\Delta^{\rm escape} > 0$ definition of a sign on potential from an experiment is ambiguous. As a rule we will be compelled to analyze both cases: with $\varphi > 0$ and with $\varphi < 0$.

As seen from Fig. 2 the method of comparing the scattering cross section of UCN and nuclear scattering cross section becomes insensitive in the area of $10^{-8}$–$10^{-4}$ cm. This is due to slightly increasing logarithmic dependence in (8) at a sufficiently high energy of incident atoms at room temperature (lowering the temperature of the gas may give some progress). Thus the integral measurement method of UCN essentially measures the scattering cross section in the field of the forces less than $10^{-8}$ cm ($\lambda < 10^{-8}$). Method of measuring low-energy neutrons is sensitive down to $\lambda \approx \lambda\!\!\!^{-}_{UCN}$ i.e. $10^{-6}$ cm ($\lambda\!\!\!^{-2}_{UCN} = \hbar^2/2m_n E^{\rm trap}_{UCN}$). (In the formula (9) under the logarithm is ratio of $\lambda$ to $\lambda\!\!\!^{-}_{UCN}$.) To identify long-range forces with $\lambda > 10^{-6}$ cm, one should compare the scattering cross section of UCN with the value of $4\pi b^2_{\rm free\_in}$, where $b_{\rm free\_in}$ is scattering length is measured at the neutron interferometer: $b_{\rm free\_in} = b_{\rm free\_nucl} + b_{\rm long\_range}$.

The proposed scheme of the experiment for measurement of integrated cross sections by the UCN method purposefully for the problem of long-range forces has not been realized yet. However we can use results of the experiment on measurement of $(P\tau)$ – values, published in Ref. [22]. We can use results of measurements for He and Ar. The simplest situation occurs for $^4$He, which does not possess a neutron capture. The data on argon can be used as well, since the capture section for argon is known rather well.

For helium the cross section determined by experiment [22] through value $(P\tau)$can be expressed from formula $\sigma^{\rm He}_{\rm scat} + \sigma^0_{\rm He\_capt} V_{2200}/\bar V_{\rm He} = (n_{\rm He}\bar V_{\rm He}\tau^{\rm gas}_{\rm stor})^{-1} = (P\tau^{\rm gas}_{\rm stor}\cdot 2.687\cdot 10^{16}\bar V_{\rm He})^{-1}$, where $2.687\cdot 10^{16}$ is the number of helium atoms in 1 cm$^3$ at pressure 1 mbar, $\bar V_{\rm He}$ is an average velocity of helium atoms at the room temperature (293 K) $\bar V_{\rm He} = 1.240\cdot 10^5$ cm·s$^{-1}$; $P\tau = (467\pm 33)$ mbar·s. Then, $\sigma^{\rm He}_{\rm scat} + \sigma^0_{\rm He\,capt} V_{2200}/\bar V_{\rm He} = (0.642\pm 0.045)\cdot 10^{-24}$ cm$^2$, it is measured with accuracy of 7%. The capture cross section of natural He because of admixture of He$^3$ is equal to $0.0075\cdot 10^{-24}$ cm$^2$ for velocity 2200 m/s, correspondingly $\sigma^0_{\rm He\_capt} V_{2200}/\bar V_{\rm He} = 0.0133\cdot 10^{-24}$ cm$^2$. Then scattering cross section $\sigma^{\rm He}_{\rm scat}(\exp P\tau) = (0.629\pm 0.045)\cdot 10^{-24}$ cm$^2$. It is coherent scattering cross section $\sigma^{\rm He}_{\rm coh\_scat}(\exp P\tau)$, incoherent scattering cross section for natural He is equal to zero. The coherent neutron scattering length with helium is measured with accuracy of 1%. It is presented in tables [23] as length of coherent neutron scattering with the bound nucleus since the tables submit data for most nuclei in the bound state with the substance $b_{\rm bound\_nusl,\,He} = 3.26(3)\cdot 10^{-13}$ cm. Since in our case one discusses scattering with a free nucleus, we should recalculate the scattering length for bound He nucleus in regard to the scattering length with a free nucleus of helium using the equation: $b_{\rm free\_nucl,\,He} = b_{\rm bound\_nucl,\,He} M_{\rm He}/(M_{\rm He}+1) = 0.2608(24)\cdot 10^{-12}$ cm. Accordingly, the cross sections with a free nucleus, calculated from the scattering length with a free nucleus will be: $\sigma^{\rm He}_0 = 4\pi b^2_{\rm free\_nucl,\,He} = 0.855(16)\cdot 10^{-24}$ cm$^2$. Then,

$$\Delta^{\rm escape}_{1,\rm He} = \left[\frac{\sigma^{\rm He}_{\rm scat}(\exp P\tau)e^{\frac{E^{\rm trap}_{UCN}}{kT}\frac{(M+1)^2}{4M}}}{\sigma^{\rm He}_0 = 4\pi b^2_{\rm free\_nucl,\,He}} - 1\right] = = -0.264\pm 0.054.\ (4.9\sigma). \quad (17)$$

At the confidence level of 95% ($2\sigma$) the effect lies in the range $(-0.36; -0.16)$. In Fig. 3 area $(g^2, \lambda)$ for $\Delta^{\rm escape}_1$

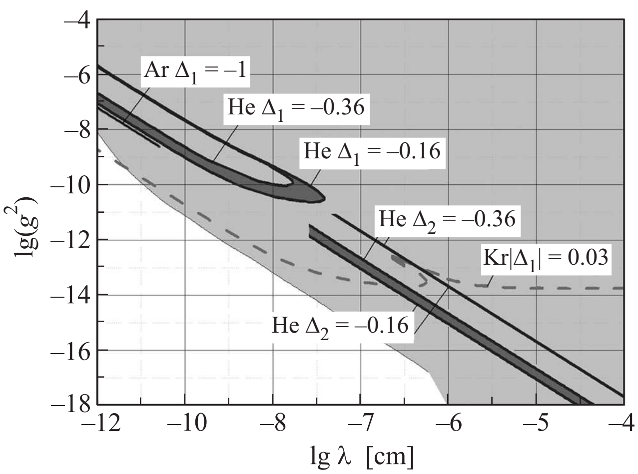


Fig. 3. The shaded area is values $g^2$ and $\lambda$ from the experimental data $(P\tau)$ for He and from present analysis. The straight line near the shaded area is values $g^2$ and $\lambda$ from the experimental data $(P\tau)$ for argon and from present analysis. The painted area is excluded values $g^2$ and $\lambda$ from works [9, 10]. The possible constrains $^{86}$Kr for are shown by dotted lines at the accuracy of measurement of $(P_A\tau^{\rm gas}_{\rm stor})$ and the coherent neutron scattering length about 2%

is shown at the confidence level of 95% for the case of an attractive potential. There is no solution for repulsive potential.

For argon there are the following tabular data: for coherent scattering amplitude [23] $b_{\rm bound\_nucl,\,Ar} = 0.1909(6)\cdot 10^{-12}$ cm, $\sigma^{\rm Ar}_0 = 4\pi b^2_{\rm bound\_nucl,\,Ar}(M_{\rm Ar} + 1))^2 = 0.436(3)\cdot 10^{-24}$ cm$^2$, the capture cross section

[24] $\sigma^0_{\rm Ar\_capt} = 0.675(9)\cdot 10^{-24}\,{\rm cm}^2$ and incoherent scattering cross section $\sigma^{\rm Ar}_{\rm incoh\_scat} = 0.225(5)\cdot 10^{-24}\,{\rm cm}^2$. From experiment [22] $(P\tau)_{\rm Ar} = 260 \pm 6.7\,$mbar·s, then $\sigma^{\rm Ar}_{\rm coh\_scat} + \sigma^{\rm Ar}_{\rm incoh\_scat} + \sigma^0_{\rm Ar\_capt}V_{2200}/\bar V_{\rm Ar} = 3.65(9)\cdot \cdot 10^{-24}\,{\rm cm}^2$, where $\bar V_{\rm Ar} = 3.921\cdot 10^4\,{\rm cm\cdot s^{-1}}$. Then $\sigma^{\rm Ar}_{\rm coh\_scat}(\exp P\tau) = -0.365(10)\cdot 10^{-24}\,{\rm cm}^2$ and

$$\Delta^{\rm escape}_{1,\rm Ar} = \left[\frac{\sigma^{\rm Ar}_{\rm coh\_scat}(\exp P\tau)e^{\frac{E^{\rm trap}_{UCN}}{kT}\frac{(M+1)^2}{4M}}}{\sigma^{\rm Ar}_0 = 4\pi b^2_{\rm free\_nucl,\,Ar}} - 1\right] = = -1.84 \pm 0.24. \quad (18)$$

*This result demands realization of new experiments.* Nevertheless we can present the area $g^2$, $\lambda$ based on the available $(P\tau)$ data for He and Ar. For He the value $\Delta^{\rm escape}_{1,\rm He}$ lies in a range between $-0.37$ and $-0.16$. For argon we can use minimal possible value $\Delta^{\rm escape}_{1,\,\rm Ar} = -1$. This analysis is presented also in Fig. 3.

The result of this analysis is incredibly strange, moreover, we can see from Fig. 3 that this results for He and Ar are excluded by results of analysis [9, 10] which was obtained from neutron scattering cross section at the energy 1 ev, from neutron scattering length and from taking into account the contribution from $n-e$ scattering. (In principle the sensitivity of proposed method at the accuracy 2% of measurement of $(P_A, \tau^{\rm gas}_{\rm stor})$ values is comparable with sensitivity of methods discussed in introduction [9]. For example at the accuracy of measurement of $(P_A\tau^{\rm gas}_{\rm stor})$ and the coherent neutron scattering length 2% for $^{86}$Kr possible constrains are shown by dotted lines in Fig. 3. The effect of long-range forces is proportional to nuclear mass therefore the usage of Ne and $^{86}$Kr will be very effective. These atoms are chosen because of small capture cross section.)

Now it is necessary to notice that the above presented analysis assumed that the scattering lengths in tables [23] are nuclear scattering lengths. However, (as has already been pointed out above) if long-range interaction really exist the measurements of scattering lengths by means interferometers ($b_{\rm free\_in}$) should include also the scattering length of long-range interaction ($b_{\rm free\_in} = b_{\rm free\_nucl} + b_{\rm long\_range}$) since the scattering length is measured at zero scattering angle, i.e. $4\pi b^2_{\rm free\_in} = 4\pi(b_{\rm free\_nucl} + b_{\rm long\_range})^2$. Therefore more correct analysis gives the following formula which can be considerably simplified for $\lambda > 10^{-7}$ cm:

$$\Delta^{\rm escape}_2 = \Bigg[\frac{(\tau^{\rm gas}_{\rm stor}n_A\bar V_A)^{-1} - \sigma^0_{\rm capt}V_{2200}/\bar V_A}{4\pi b^2_{\rm free\_in}}\times \times\exp\left\{\frac{E^{\rm trap}_{UCN}}{kT}\frac{(M+1)^2}{4M}\right\} - 1\Bigg] =$$

$$\pm g^2_\pm M\left(\frac{(M+1)}{8\pi b_{\rm free\_in}}\left(\frac{\hbar c}{kT}\right)e^{z(\lambda)}E_1(z(\lambda)) - -\left(\frac{M}{M+1}\right)\frac{\lambda^2}{\pi b_{\rm free\_in}}\frac{m_n c}{\hbar}\right) + +(g^2_\pm M)^2\left(\frac{(M+1)^2}{4(8\pi b_{\rm free\_in})^2}\left(\frac{\hbar c}{kT}\right)^2\frac{e^{z(\lambda)}E_2(z(\lambda))}{z} + +\left(\frac{M}{M+1}\right)^2\left(\frac{m_n c\lambda^2}{2\pi\hbar b_{\rm free\_in}}\right)^2 - -\left(\frac{m_n cM\lambda^2}{16\pi^2\hbar b^2_{\rm free\_in}}\right)\left(\frac{\hbar c}{kT}\right)e^{z(\lambda)}E_1(z(\lambda))\right) \approx \approx \mp g^2_\pm M\left(\frac{M}{M+1}\right)\frac{\lambda^2}{\pi b_{\rm free\_in}}\frac{m_n c}{\hbar} + +(g^2_\pm M)^2\left(\frac{M}{M+1}\right)^2\left(\frac{m_n c\lambda^2}{2\pi\hbar b_{\rm free\_in}}\right)^2. \quad (19)$$

The analysis using the simplified formula (19) for $\Delta^{\rm escape}_2$ and $\lambda > 10^{-7}$ cm is shown also in Fig. 3. The area $(g^2, \lambda)$ for $\Delta^{\rm escape}_2$ is shown for the case of a repulsive potential at the confidence level of 95%. There is no solution for an attractive potential. One can see that the determined area of values $g^2$ and $\lambda$ is again excluded by results of analysis [9, 10]. The similar analysis could not be realized for Ar because correction for capture cross section is too large.

The presented discrepancy for He and Ar can be caused by the different reasons: 1) systematical errors of experiment, 2) effect of collective interaction of UCN with atoms of gas, since UCN wave length is about $5\cdot 10^{-6}$–$10^{-5}$ cm but distance between atoms is about $10^{-6}$ cm at the gas pressure 10 mbar. In this case the detailed measurements of dependence of $P_A\tau^{\rm gas}_{\rm stor}$ value from the gas pressure are necessary.

To clarify the problem new measurements are necessary not only for $P\tau$ data but also for capture cross section at UCN energy. The experimental measurements of the flux of above-barrier neutrons will be particularly necessary. It will be helpful to find the possible systematical errors of experiment as well as the possible effect of collective interaction of UCN with atoms of gas at the pressure about 10 mbar.

The method of measuring the flux of above-barrier neutrons will be considered in the next article.

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