Pis'ma v ZhETF, vol. 93, iss. 9, pp. 547 – 551

Photo-induced nuclear excitation by electron transition

A. Ya. Dzyublik

Institute for Nuclear Research, Kiev, Ukraine

Submitted 25 January 2011
Resubmitted 18 March 2011

We described the nuclear excitation at electron transition (NEET), induced by x-rays, on the basis of strict collision theory. All stages of the process are considered, including formation of the hole in the electronic K-shell, its decay accompanied by excitation of the nucleus, filling of the M-vacancy and subsequent deexcitation of the nucleus. The cross sections for the NEET and photoabsorption of x-rays near K-edge are calculated. The results agree with the data of Kishimoto et al.

1. Introduction. Vacancies in the inner atomic shells are filled by electrons from upper levels mainly with emission of x-ray photons or Auger electrons. But there is one more, although weak, decay channel of vacancies – nuclear excitation by electron transition (NEET) [1]. It arises in those cases when atomic and nuclear transitions have near energies and the same multipolarities. This effect has been already observed for the nuclei ^{197}Au [2], ^{189}Os [3–6] and ^{237}Np [7]. Most precise measurements were performed by Kishimoto et al. [2], who irradiated the golden foil, containig ^{197}Au, by synchrotron radiation with a very narrow bandwidth $\Gamma_s = 3.5\,$eV. Incident x-ray photons ionized the K-orbit of the Au atom, further the electron transition between the M- and K-levels led to excitation of the isotope ^{197}Au due to exchange by virtual photons between the electron and the nucleus. The number of excited nuclei was counted by detecting the emitted L-conversion electrons. Varying the energy of x-ray photons they found that the NEET edge was shifted higher by $40 \pm 2\,$eV than the K-photoabsorption edge and was more steep.

For definiteness, we suppose that the NEET process is ensured by the electronic transitions between the K- and M-orbits, as it was the case in the experiment [2].

The decay probability $P_{\rm NEET}$ of the K-hole through the NEET channel has been calculated in [6, 8, 9], giving

$$P_{\rm NEET} = \left(1 + \frac{\Gamma_M}{\Gamma_K}\right) \frac{E_{\rm int}^2}{\delta^2 + (\Gamma_K + \Gamma_M)^2/4}, \qquad (1)$$

where Γ_K and Γ_M are the widths of the K- and M-holes, $\delta = E_0^n - E_0^a$ is the mismatch of the nuclear E_0^n and atomic E_0^a transition energies. For explicit form of the coupling parameter $E_{\rm int}^2$ see [8].

The initial stage of the NEET process, i.e., the K-hole formation by the incident x-ray photon, has been taken into consideration by Tkalya [10] in the framework of the quantum electrodynamics. For description of the resonances the real energies in the propagators were replaced *ad hoc* by their complex values, which are associated with infinite values of the wave functions at the initial moment $t \to -\infty$. Another unpleasant procedure was a "spreading" of the δ functions, corresponding to the energy conservation law.

In addition, recently new formula for the NEET probability has been proposed [11], which drastically differ from (1). All this served as a motivation for our approach to this task, based on the strict collision theory [12]. Here we consider all stages of the photo-induced NEET, including the formation of the K-hole, filling of the M-vacancy and subsequent decay of the nucleus. Both NEET and x-ray photoaborption are studied on the same basis.

2. Transition matrix. The unperturbed Hamiltonian of the system (nucleus + atomic electrons + quantized electromagnetic field) is

$$\hat{H}_0 = \hat{H}_n + \hat{H}_a + \hat{H}_{rad}, \qquad (2)$$

where $\hat{H}_n$, $\hat{H}_a$ and $\hat{H}_{rad}$ define respectively the Hamiltonians of the nucleus, atomic electrons and quantized electromagnetic field. The latter in the Coulomb gauge is given by

$$\hat{H}_{rad} = \sum_{\mathbf{k}} \sum_{p=\pm 1} \hat{a}^+_{\mathbf{k}p} \hat{a}_{\mathbf{k}p}, \qquad (3)$$

where $\hat{a}^+_{\mathbf{k}p}$ and $\hat{a}_{\mathbf{k}p}$ are the creation and annihilation operators of the photon with the wave vector $\mathbf{k}$ and circular polarization $\boldsymbol{\epsilon}_p$.

The total Hamiltonian $\hat{H} = \hat{H}_0 + \hat{V}$ contains the perturbation operator

$$\hat{V} = \hat{V}_r^n + \hat{V}_r^a + \hat{V}_c', \qquad (4)$$

where $\hat{V}_r^{(n)}$ and $\hat{V}_r^{(a)}$ are the interaction operators of the nucleus and the atomic electrons with the quantized electromagnetic field, $\hat{V}_c'$ is the residual Coulomb interaction of the nucleus with surrounding electrons. For definite electron with the radius-vector $\mathbf{r}$ it has the form

$$\hat{V}_c' = -e^2 \sum_{i=1}^{Z} \frac{1}{|\mathbf{r}-\mathbf{r}_i|} + \frac{Ze^2}{r}, \tag{5}$$

where the summation is over all protons of the nucleus with the coordinates $\mathbf{r}_i$.

The interactions $\hat{V}_r^{n(a)}$ are defined by well-known expression (see, e.g.,[13]):

$$\hat{V}_r^{n(a)} = -\frac{1}{c}\int d\mathbf{r}\hat{\mathbf{j}}^{n(a)}(\mathbf{r})\hat{\mathbf{A}}(\mathbf{r}), \tag{6}$$

where $\hat{\mathbf{j}}^{n(a)}(\mathbf{r})$ is the flux density operator of the nucleus (atomic electrons), $\hat{\mathbf{A}}(\mathbf{r})$ is the vector potential operator of the quantized electromagnetic field.

The eigenfunctions and eigenvalues of the unperturbed Hamiltonian $\hat{H}_0$ obey the equation

$$\hat{H}_0\chi_b = E_b\chi_b. \tag{7}$$

Let at the initial moment $t \to -\infty$ the system be described by the wave function

$$\chi_a = |I_iM_i\rangle\Phi_0|1_{\mathbf{ke}}\rangle, \tag{8}$$

where the wave function $|I_iM_i\rangle$ describes the initial state of the nucleus with spin I_i and its projection M_i on the quantization axis, Φ_0 the initial state of the atom, $|1_{\mathbf{ke}}\rangle$ the quantized field, containing one x-ray photon with the wave vector $\mathbf{k}$ and polarization $\mathbf{e}$. The corresponding initial energy of the system will be

$$E_a = W_i^n + W_i^a + E, \tag{9}$$

where W_i^n and W_i^a are the initial energies of the nucleus and electrons respectively, E is the energy of incident photon. The magnitude of W_i^a is determined by the binding energies of all atomic electrons B_l, i.e., $W_i^a = -\sum_l B_l$.

At first stage the K-electron, absorbing the x-ray photon, flies away with the wave vector $\boldsymbol{\kappa}$ and kinetic energy ε. In other words, there arises a hole in the K orbit. Such first intermediate state of the system is described by the wave function

$$|c_1\rangle = |I_iM_i\rangle\Phi_{j_im_i}(K)|\boldsymbol{\kappa}\rangle|0\rangle, \tag{10}$$

where $|0\rangle$ stands for the wave function of the vacuum of the electromagnetic field, $\Phi_{j_im_i}(K)$ and $|\boldsymbol{\kappa}\rangle$ are the functions of the atom with K-hole and the ejected K-electron respectively. The corresponding eigenvalue of the unperturbed Hamiltonian $\hat{H}_0$ will be

$$E_1 = W_i^n + W_i^a + B_K + \varepsilon. \tag{11}$$

After that the M-electron, passing down into the K-orbit, transfers its energy to the nucleus (NEET). The whole system undergoes transition into the second intermediate state, described already by

$$|c_2\rangle = |I_eM_e\rangle\Phi_{j_fm_f}(M)|\boldsymbol{\kappa}\rangle|0\rangle, \tag{12}$$

having the energy

$$E_2 = W_e^n + W_i^a + B_M + \varepsilon, \tag{13}$$

where $|I_eM_e\rangle$ and W_e^n are the wave function and energy of the excited nucleus. The transition energies are $E_0^n = W_e^n - W_i^n$ and $E_0^a = B_K - B_M$.

In principle, two decay branches of the state $|c_2\rangle$ are possible: de-excitation of the nucleus and decay of the M-hole. But the decay of the hole is by many orders faster than that of the nucleus. In particular, the width of the first excited level of ^{197}Au is $\Gamma_n = 2.38\cdot 10^{-7}$ eV, while the K-hole width $\Gamma_K = 52$ eV and the M-hole width $\Gamma_M = 14.3$ eV [2]. Thus, at first there are electron transitions, which lead to filling of the M-hole. For brevity, we shall talk about single electron transition from any upper level with binding energy B', followed by emission of the photon with the energy $\hbar\omega'$.

As a result, our system occurs in another state $|d\rangle$ with the energy

$$E_d = W_e^n + W_i^a + B' + \varepsilon + \hbar\omega'. \tag{14}$$

The nucleus next decays into a final state $|I_fM_f\rangle$, emitting γ quantum with the energy $\hbar\omega_\gamma$. Of course, the internal conversion is also possible. The energy of the whole system in the final state $|b\rangle$ then becomes

$$E_b = W_f^n + W_i^a + B' + \varepsilon + \hbar\omega' + \hbar\omega_\gamma. \tag{15}$$

Such multi-step scattering process is determined by the resonant part of the transition operator

$$\hat{T} = \hat{V}_r^n\hat{G}^+(E_a)\hat{V}_r^a \tag{16}$$

with the Green's operator

$$\hat{G}^+(E_a) = (E_a + i\eta - \hat{H})^{-1}, \quad \eta\to +0. \tag{17}$$

It is useful to introduce the operators

$$\begin{gathered}\hat{A} = E_a + i\eta - \hat{H},\\ \hat{B} = E_a + i\eta - \hat{H}_0 - \hat{V}_d,\\ \hat{V}' = \hat{V} - \hat{V}_d,\end{gathered} \tag{18}$$

where $\hat{V}_d = \hat{P}_d\hat{V}$ and $\hat{P}_d$ are the projection operators on the vectors $|d\rangle$. Using the operator identity (see also [12])

$$\hat{A}^{-1} - \hat{B}^{-1} = \hat{B}^{-1}(\hat{B}-\hat{A})\hat{A}^{-1}, \tag{19}$$

we transform the transition operator (16) to

$$\hat{T} = \hat{V}_r^n \frac{1}{E_a + i\eta - \hat{H}_0 - \hat{V}_d} \hat{T}', \tag{20}$$

where the reduced transition operator $\hat{T}'$ is given by

$$\hat{T}' = \hat{V}' + \hat{V}'\hat{G}^+(E_a)\hat{V}'. \tag{21}$$

Then the matrix elements for (20) will be

$$T_{ba} = \sum_{d,c_2,c_1} \langle b|\hat{V}_r^n|d\rangle \frac{1}{E_a - E_d + i\Gamma_n/2} \times \\ \times \langle d|\hat{V}_r^a|c_2\rangle G_{21}^+(E_a)\langle c_1|\hat{V}_r^a|a\rangle, \tag{22}$$

where Γ_n is the nuclear width. The Green's matrix $G_{c'c}^+(E_a)$ is governed by algebraic equations [14]

$$(E_a - E_{c'})G_{c'c}^+(E_a) = \delta_{c'c} + \sum_{c''} R_{c'c''}^+(E_a)G_{c''c}^+(E_a), \tag{23}$$

where the R matrix is represented by the expansion

$$R_{cc'}^+(E_a) = V_{c,c'} + \sum_{b\neq c,c'} \frac{V_{cb}V_{bc'}}{E_a + i\eta - E_b} + \ldots \tag{24}$$

The diagonal elements of the R matrix are $R_{11} \approx \approx -i\Gamma_1/2$ and $R_{22} \approx -i\Gamma_2/2$, where $\Gamma_1 = \Gamma_K - \Delta\Gamma$ and $\Gamma_2 = \Gamma_M + \Gamma_n + \Delta\Gamma$, while the broadening $\Delta\Gamma$ describes the inverse to NEET process, namely, the bound internal conversion (BIC):

$$\Delta\Gamma = \frac{E_{\rm int}^2}{\delta^2 + (\Gamma_K - \Gamma_M)^2/4}(\Gamma_K - \Gamma_M), \tag{25}$$

where the coupling parameter

$$E_{\rm int}^2 = \sum_{m_f,M_e} |R_{21}|^2. \tag{26}$$

Omitting here Γ_M we get the BIC width in stripped ions [15]. The difference $\Gamma_K - \Gamma_M$ is due to filling of the M-vacancy by outer electrons, which decreases the BIC rate.

Karamyan and Carroll [11] assumed, that by means of $\Gamma_{\rm BIC}$ one can eliminate large discrepancy of their estimates with experimental value of the NEET probability $P_{\rm NEET} = 4.5\cdot 10^{-8}$ for ^{197}Au [2]. However, $\Delta\Gamma \sim P_{\rm NEET}(\Gamma_K - \Gamma_M) \ll \Gamma_K$, i.e., the role of $\Gamma_{\rm BIC}$ as well as of Γ_n is negligible.

Therefore the solution of Eqs.(23) takes the form

$$G_{11}^+(E_a) \approx \frac{1}{\Delta E - \varepsilon + i\Gamma_K/2}, \tag{27}$$

$$G_{21}^+(E_a) \approx \frac{R_{21}}{(\Delta E - \varepsilon + i\Gamma_K/2)\,(\Delta E - \delta - \varepsilon + i\Gamma_M/2)}. \tag{28}$$

Here $\Delta E = E - B_K$ specifies the energy excess of the x-ray photon over the K-absorption threshold B_K.

3. NEET cross section. The cross section for the transition from $|a\rangle$ to $|b\rangle$ is given by

$$\sigma_{a\to b} = \frac{2\pi}{\hbar c}\,|T_{ba}|^2\,\delta(E_a - E_b), \tag{29}$$

where c is the light velocity.

After substitution of (28) into (29) we have to average yet $\sigma_{a\to b}$ over the initial states and sum over all possible final ones $|b\rangle$. Such summation includes integration over the states of emitted γ quantum with frequency ω_γ, x-ray photon ω' and outgoing electron. The volume element in the integral for this electron $d\boldsymbol{\kappa}/(2\pi)^3$ we rewrite as $d\Omega_{\vec{\kappa}}\rho_e(\varepsilon)d\varepsilon$ with the solid angle $d\Omega_{\vec{\kappa}}$ and the density of electronic states in the continuous spectrum $\rho_e(\varepsilon) = m\kappa/(2\pi)^3\hbar^2$, where m is the electron mass. Performing first integration over ω' with the aid of the δ function, we find that

$$E_a - E_d = \hbar(\omega_\gamma - \omega_\gamma^{(0)}), \tag{30}$$

where $\hbar\omega_\gamma^{(0)} = W_e^n - W_f^n$ is the transition energy to the final state of the nucleus.

By integrating then over ω_γ we find the reaction cross section be equal to product of the NEET cross section $\sigma_{\rm NEET}(E)$ and the branching ratio of the partial radiative nuclear width $\Gamma_n^\gamma(e\to f)$ for the transition from $|I_eM_e\rangle$ to $|I_fM_f\rangle$ and the total width of the excited state Γ_n:

$$\sigma_r(E) = \frac{\Gamma_n^\gamma(e\to f)}{\Gamma_n}\sigma_{\rm NEET}(E), \tag{31}$$

where the NEET cross section

$$\sigma_{\rm NEET}(E) = E_{\rm int}^2\left(\frac{\Gamma_M}{2\pi}\right) \times \tag{32}$$

$$\times \int_0^\infty \frac{\sigma_{\rm ion}(\varepsilon)d\varepsilon}{[(\varepsilon-\Delta E)^2+(\Gamma_K/2)^2][(\varepsilon-\Delta E+\delta)^2+(\Gamma_M/2)^2]},$$

with the ionization cross section

$$\sigma_{\rm ion}(\varepsilon) = \frac{2\pi}{\hbar c}\sum_{m_i}\int d\Omega_{\vec{\kappa}}|\langle c_1|\hat{V}_r^a|a\rangle|^2\rho_e(\varepsilon), \tag{33}$$

depending on the energy ε of the ejected electron.

For such an electron, moving in the Coulomb field of the nucleus, $\sigma_{\rm ion}(\varepsilon)$ tends to a constant $\sigma_{\rm ion}$, if $\varepsilon \to 0$ [16]. If we replace $\sigma_{\rm ion}(\varepsilon)$ by this constant, integration in (32) is simplified, giving

$$\sigma_{\rm NEET}(E) = P_{\rm NEET} F_{\rm NEET}(E) \sigma_{\rm ion}, \tag{34}$$

where the edge factor $F_{\rm NEET}(E)$ specifies the energy dependence of the NEET near the threshold:

$$F_{\rm NEET}(E) = \frac{1}{(1+\Gamma_M/\Gamma_K)} \times \\ \times \frac{1}{[\delta^2 + (\Gamma_K - \Gamma_M)^2/4]} \sum_{i=1}^{3} f_i(E), \tag{35}$$

with

$$f_1(E) = \frac{\Gamma_M}{\Gamma_K}\left[\delta^2 - \left(\frac{\Gamma_K}{2}\right)^2 + \left(\frac{\Gamma_M}{2}\right)^2\right] \times \\ \times \left[\frac{1}{2} + \frac{1}{\pi}\arctan\left(\frac{2\Delta E}{\Gamma_M}\right)\right], \tag{36}$$
$$f_2(E) = \frac{\delta\Gamma_M}{2\pi}\ln\left[\frac{(\Delta E)^2 + (\Gamma_K/2)^2}{(\Delta E - \delta)^2 + (\Gamma_M/2)^2}\right],$$
$$f_3(E) = \left[\delta^2 + \left(\frac{\Gamma_K}{2}\right)^2 - \left(\frac{\Gamma_M}{2}\right)^2\right] \times \\ \times \left[\frac{1}{2} + \frac{1}{\pi}\arctan\left(\frac{2(\Delta E - \delta)}{\Gamma_M}\right)\right].$$

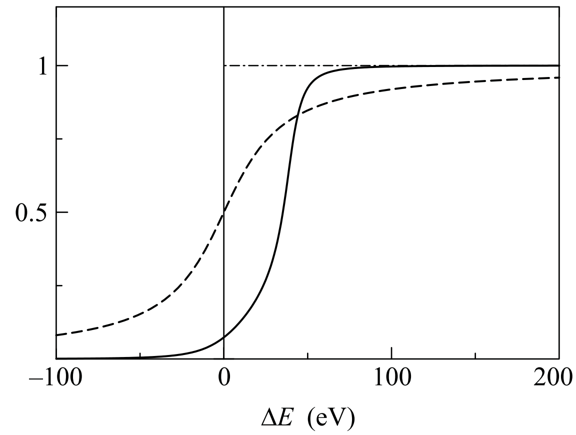
Far from the threshold, when $\Delta E \to \infty$, the factor $F_{\rm NEET}(E) \to 1$ and only then the NEET cross section (34) reduces to the product of the NEET probability $P_{\rm NEET}$ and the ionization cross section $\sigma_{\rm ion}$, which was adopted in [2]). The result $F_{\rm NEET}(\infty) = 1$ can be also obtained by direct contour integration of (32), keeping $\sigma_{\rm ion}$ constant. Note that in (36) the term $f_3(E)$ for ^{197}Au is by order of magnitude larger than $f_1(E)$ and $f_2(E)$ due to inequality $\Gamma_i >> \Gamma_f$ (see also the figure).

4. K-absorption. The absorption cross section of x-rays by K electrons according to the optical theorem (see, e.g., [12]) is determined by the imaginary part of the transition matrix element T_{aa}, which is associated with the elastic scattering amplitude of x-ray photons to zero angle:

$$\sigma_a(E) = \frac{2}{\hbar c}{\rm Im}T_{aa}. \tag{37}$$

Since the NEET matrix element $|R_{21}| \ll \Gamma_f$, the absorption of x-rays by K-electrons proceeds mainly without energy transfer to the nucleus. In other words, $|G^+_{21}(E_a)| \ll |G^+_{11}(E_a)|$, so that with good accuracy

$$T_{aa} = \sum_{m_i} \int \frac{d\boldsymbol{\kappa}}{(2\pi)^3}\langle a|\hat V^a_r|c_1\rangle G^+_{11}(E_a)\langle c_1|\hat V^a_r|a\rangle, \tag{38}$$



The NEET edge function $F_{\rm NEET}(E)$ (solid curve) and K-absorption factor $F_{abs}(E)$ (dashed line), calculated for the nucleus ^{197}Au, versus the energy E of x-ray photons, where $\Delta E = E - B_K$

where the explicit form of G^+_{11} is presented in (27). For the K-absorption cross section we have then the following expression:

$$\sigma_a(E) = \frac{\Gamma_i}{2\pi}\int_0^\infty \frac{\sigma_{\rm ion}(\varepsilon)d\varepsilon}{(\varepsilon - \Delta E)^2 + (\Gamma_i/2)^2}. \tag{39}$$

Replacing again $\sigma_{\rm ion}(\varepsilon)$ by the constant $\sigma_{\rm ion}$, one gets

$$\sigma_a(E) = F_{abs}(E)\sigma_{\rm ion}, \tag{40}$$

where the K-absorption edge factor is

$$F_{abs}(E) = \frac{1}{2} + \frac{1}{\pi}\arctan\left[\frac{2\Delta E}{\Gamma_i}\right]. \tag{41}$$

5. Averaging. The cross sections $\sigma_a(E)$ and $\sigma_{\rm NEET}(E)$ should be yet averaged over the energy distribution of incident photons. We shall approximate it by the Lorentz function:

$$w_s(E) = \frac{\Gamma_s/2\pi}{(E - \bar E)^2 + (\Gamma_s/2)^2}. \tag{42}$$

Starting from (39), we easily find that dependence of the averaged cross section $\bar\sigma_a(\bar E)$ on the mean energy of photons $\bar E$ is defined by the factor

$$\mathcal{F}_{abs}(\bar E) = \frac{1}{2} + \frac{1}{\pi}\arctan\left[\frac{2(\Delta\bar E)}{\Gamma_{abs}}\right], \tag{43}$$

where $\Delta\bar E = \bar E - B_K$, and the width $\Gamma_{abs} = \Gamma_s + \Gamma_i$.

In analogy, retaining in (35) only the leading term $f_3(E)$, we see that the averaged NEET cross section $\bar\sigma_{\rm NEET}(\bar E)$ is proportional to the factor

$$\mathcal{F}_{\rm NEET}(\bar E) = \frac{1}{2} + \frac{1}{\pi}\arctan\left[\frac{2(\Delta\bar E - \delta)}{\Gamma_{\rm NEET}}\right], \tag{44}$$

where the width $\Gamma_{\rm NEET} = \Gamma_s + \Gamma_f$.

It useful to describe K-absorption and NEET near their edges by derivatives of (43), (44) over $\bar{E}$:

$$\mathcal{F}'_{abs}(\bar{E}) = \frac{(\Gamma_{abs}/2)^2}{(\Delta\bar{E})^2 + (\Gamma_{abs}/2)^2},$$
$$\mathcal{F}'_{\rm NEET}(\bar{E}) = \frac{(\Gamma_{\rm NEET}/2)^2}{(\Delta\bar{E}-\delta)^2 + (\Gamma_{\rm NEET}/2)^2}. \qquad (45)$$

From here we see that Γ_{abs} and $\Gamma_{\rm NEET}$ can be really interpreted as the edge widths (see also [2]).

6. Discussion. Our results (34)–(36) coincide with those of Tkalya [10] apart from the missed factor $1/\pi$ at F_1, F_2 in [10]. The behavior of the NEET and K-absorption cross sections for ^{197}Au in the vicinity of the K-absorption threshold is shown in the figure. The calculations are done with the aid of Eqs. (35), (36) and (41), using $\delta = 40$ eV and the parameters Γ_K, Γ_M listed above. Our approach allows to understand why the NEET events for ^{197}Au appear in the vicinity of the K-absorption threshold $\Delta E = 0$ and rapidly grow at another resonant point $\Delta E = \delta$. For this aim we rewrite the energy conservation law $E_b = E_a$ as

$$\varepsilon = \Delta E - \delta - \hbar(\omega' - \omega'_0). \qquad (46)$$

Here we took into account that the energy of γ-quantum, emitted by the nucleus, practically coincides with the transition energy $\hbar\omega_\gamma^{(0)}$. Photoemission of K-electrons is possible even at $\Delta E \approx 0$ owing to large width Γ_M of the M-hole. More definitely, the NEET channel becomes open ($\varepsilon > 0$) when x-ray photons, emitted during the filling of the M-hole, have the energy $\hbar\omega' \sim \hbar\omega'_0 - \delta$. This occurs on the wing of emission line. Therefore the NEET process, being weak at $\Delta E \approx 0$, sharply grows only at $\Delta E \approx \delta$ in correspondence with [2].

Taking the energy $W_0^n = 77.351$ keV of the first excited level for ^{197}Au [17], and comparing it with the energy of atomic transition in the gold $W_0^a = 77.300$ keV [18], we are led to their mismatch $\delta = 51$ eV, whereas the NEET edge was observed at $\delta = 40 \pm 2$ eV [2]. Nevertheless, recently for ^{197}Au it was reported [19] that $W_e^n = 77.339 \pm 0.003$ keV. Adopting this value we have $\delta = 39 \pm 3$ eV in agreement with [2].

The width of the NEET edge $\Gamma_{\rm NEET}$ is much less than that of the K-absorption edge Γ_{abs}, because $\Gamma_K \gg \Gamma_M$. For ^{197}Au by means of Eqs. (45) we found $\Gamma_{\rm NEET} = 17.8$ eV and $\Gamma_{abs} = 55.5$ eV, that well correlates with the experimental data $\Gamma_{\rm NEET} = 14 \pm 9$ eV, $\Gamma_{abs} = 58 \pm 3$ eV [2].

1. M. Morita, Progr. Theor. Phys. **49**, 1574 (1973).
2. S. Kishimoto, Y. Yoda, Y. Kobayashi et. al., Phys. Rev. C **74**, 031301(R) (2006).
3. K. Otozai, R. Arakawa, and T. Saito, Nucl. Phys. A **297**, 97 (1978).
4. A. Shinohara, T. Saito, M. Shoi et al., Nucl. Phys. A **472**, 151 (1987).
5. L. Lakosi, N. C. Tam, and I. Pavliczek, Phys. Rav. C **52**, 1510 (1995).
6. I. Ahmad, R. W. Dunford, H. Esbensen et al., Phys. Rev. C **61**, 051304 (2000).
7. T. Saito, A. Shiohara, and K. Otozai, Phys. Lett. B **92**, 293 (1980).
8. E. V. Tkalya, Nucl. Phys. A **539**, 209 (1992).
9. M. R. Harston, Nucl. Phys. A **690**, 447 (2001).
10. E. V. Tkalya, Phys. Rev. A **75**, 022509 (2007).
11. S. A. Karamian and J. J. Carroll, Laser Phys. **20**, 977 (2010).
12. M. L. Goldberger and K. M .Watson, *Collision Theory*, J. Wiley, New York, 1964.
13. A. S. Davydov, *Theory of Atomic Nucleus*, Fizmatgiz, Moscow, 1958.
14. A. Ya. Dzyublik, Teor. Mat. Fiz. **87**, 86 (1991) [Sov. J. Theor. Math. Phys. **87**, 393 (1991)].
15. F. F. Karpeshin, Part. Nucl. **37**, 523 (2006).
16. L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, Nauka, Moscow, 1974.
17. Z. Chummei, Nucl. Data Sheets **62**, 433 (1991).
18. J. A. Bearden and A. F. Burr, Rev. Mod. Phys. **39**, 125 (1967).
19. V. I. Kirischuk, N. V. Strilchuk, I. M. Vyshnevskyi, and V. A. Zheltonozhsky, Abstracts of 2-nd Int. Conf. NPAE-2008, Kiev, 2008, p. 98.