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On kinetic theory of energy losses in randomly heterogeneous medium

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We derive equation describing distribution of energy losses of the particle propagating in fractal medium with quenched and dynamic heterogeneities. We show that in the case of the medium with fractal dimension $2 < D < 3$ the losses Δ are characterized by the sublinear anomalous dependence $\Delta \sim x^{\alpha}$ with power-law dependence on the distance x from the surface and exponent $\alpha = D - 2$.

This letter is devoted to studying statistical properties of the collisional energy losses suffered by a high energy particle passing through a randomly inhomogeneous disordered medium. Exploration of the properties of complex media with the help of test particles propagating through it is one of the most important scientific instruments used in physics allowing, in particular, to study a response of the medium to particle beams or radiation coming through the medium under study. A range of possible applications is quite broad, from the physics of high energy collisions to polymer physics, to name the few. In all cases the main quantity under study is the distribution of energy losses of the test particle studied as a function of the distance covered by the test particle in the medium.

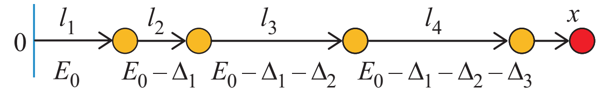
The original context of the problem was related to examination of ionization losses of high energy particles in ordinary homogeneous amorphous matter (energy straggling), see, e.g., [1]. The microscopic picture underlying the energy losses in this case was, evidently, that of a series of inelastic collisions of the projectile with atomic electrons resulting in excitation/ionization of the corresponding atoms. In between the scattering events the projectile trajectory is ballistic and its energy does not change. In the continuum limit the problem can be reformulated, for a high energy projectile and energy losses small compared to the energy of the incident particle, in terms of a one-dimensional kinetic equation suggested by Landau [2], in which the role of time is played by the distance along the straight line trajectory covered by the projectile in the medium.

The focus of the present paper is on the statistical properties of energy straggling in matter characterized by strong random inhomogeneities with the special emphasis on the case of fractal medium. Our consideration of the microscopic energy loss model in a strongly inhomogeneous random medium with power-like correlations is realized at the level analogous to the Continuous Random Walk (CTRW)[3] that was, in particular, actively explored to describe thermal diffusion of particles in a fractal medium [4–7]. Physically, the main difference between the stochastic process describing the energy loss and the CTRW is in straight line trajectory of high energy particle, while CTRW describes haotic trajectories of particle induced by random Brownian forces. From mathematical point of view the distinction of these processes is that the energy loss problem corresponds to a sum of random positive quantities – energy losses at scattering events, whereas random particle displacements in CTRW can have any sign.

Let us formulate the basic microscopic model of energy loss studied in this paper and consider a high energy particle incident on the medium containing randomly placed scattering centers. In the high energy approximation the trajectory of the projectile is a straight line and the particle is assumed to interact with all the scattering centers that happen to lie at the projectile's trajectory. More precisely, let us consider the particle with high energy E_0 entering the medium at the point $x = 0$. Our main goal is to compute the distribution $f(\Delta, x)$ of its energy loss Δ at some depth x from the surface.

The energy loss Δ at the point x in a given event is fully characterized by the set of energy losses at each of scattering event $\{\Delta_i\}$, $i = 1, \ldots, n$ so that $\Delta = \Delta_1 + \Delta_2 + \ldots + \Delta_n$. The corresponding configuration of the scattering centers is, in turn, fully specified by the set of distances $l_1, l_2, \ldots, l_n, l_{n+1}$, where l_1 is the distance between the surface and the first scattering center, $\{l_i\}$ are the distances between the i'th and $(i+1)$'th scattering centers and, finally, l_{n+1} is the distance between the last scattering center and the observation point x $(x = l_1 + l_2 + \ldots + l_{n+1})$, see figure.

In the assumption of independent energy losses at different scattering centers with the distribution of energy losses $w(\varepsilon)$ the probability distribution $f(\Delta, x)$ of the cumulative energy loss Δ at some point x is fully described by the probability densities $\psi(l)$ of having a



Particle propagating in the medium looses the energy $\Delta = \Delta_1 + \Delta_2 + \ldots + \Delta_n$ in n scattering events taking place at points separated by intervals $\{l_i\}$ along the trajectory of the projectile

spatial distance l between two scattering centers, see figure. For large number of scattering events the energy losses do not depend on the probability density $\varphi(l_1)$ of the first event, and in follows we will simply assume that $\varphi(l_1) = \psi(l_1)$. The problem of random loss is similar to that of the usual random walk, with the distance x and local energy loss Δ replacing the time and displacement correspondingly. From this analogy the energy loss distribution function $f(\Delta,x)$ is described by CTRW-like equation:

$$f(\Delta,x) = \delta(\Delta)\,\Psi(x) + \\ + \int_0^x dx'\psi(x-x') \int_0^\Delta d\varepsilon w(\varepsilon)\, f(\Delta-\varepsilon,x'), \qquad (1)$$

where $\Psi(x) = \int_x^\infty dy\psi(y)$ is the probability of having no scattering events in the interval $[0,x]$. Eq. (1) is conveniently solved by using the double Laplace transform

$$\tilde{f}(p,q) \equiv \int_0^\infty d\Delta e^{-p\Delta} \int_0^\infty dx e^{-qx} f(\Delta,x).$$

Introducing the finction $\tilde{g}(q) \equiv \tilde{\psi}(q)/[1-\tilde{\psi}(q)]$ the corresponding equation for $\tilde{f}(p,q)$ can be written in the form

$$\tilde{f}(p,q) = 1/q + \tilde{g}(q)\,[\tilde{w}(p) - 1]\,\tilde{f}(p,q), \qquad (2)$$

which, in turn, corresponds to the following version of the original kinetic equation (1):

$$f(\Delta,x) = \delta(\Delta) + \int_0^x dx' g(x-x') \times \\ \times \int_0^\infty d\varepsilon w(\varepsilon)\,[f(\Delta-\varepsilon,x') - f(\Delta,x')]. \qquad (3)$$

The function $g(r)$ is found by inverse Laplace transform of the function $\tilde{g}(q)$ and it has the meaning of the average density of scatterings along the direction of particle propagation at the distance r from the last scattering.

In general, the function $g(r)$ depends on characteristics of the medium, and it can be related to the so-called structure function of the medium

$$G(\mathbf{r}) = \left\langle \sum\nolimits_{n\neq 0} \delta(\mathbf{x}_i - \mathbf{x}_{i+n} - \mathbf{r}) \right\rangle, \qquad (4)$$

where $\mathbf{x}_i$ are coordinates of the i-th scattering center, by

$$g(r) = a^2 G(r), \qquad (5)$$

a^2 is the scattering area of the particle. In many important cases of the scattering of particles in a complex heterogeneous medium a microstructure of the medium remains unknown, but its structure function $G(r)$ can be directly measured experimentally. In these cases the kinetic equation (3) can be used to predict the spectrum of energy losses in such a medium. The knowledge of the spectrum is extremely important, for example, in the problem of radiation damage of the medium which is determined not only by total adsorbed energy but also by the shape of the distribution of energy losses.

Eq. (3) may be considered as a generalization of the Landau equation for ionization losses in amorphous media [1], which can be written in integral form:

$$f(\Delta,x) = \delta(\Delta) + \\ + \frac{1}{a}\int_0^x dx' \int_0^\infty d\varepsilon w(\varepsilon)\,[f(\Delta-\varepsilon,x') - f(\Delta,x')]. \quad (6)$$

This equation is obtained from Eq. (3) in the case of the constant linear density of scattering centers $g(x-x') = 1/a$.

Analytical solution of generalized kinetic equation (3) can be found in the case of scattering medium with fractal dimension D, when the Laplace transform of the function $g(r)$ has the form:

$$\tilde{g}(q) = (aq)^{-\alpha}, \quad \alpha = D - 2. \qquad (7)$$

In the infrared limit $x \gg a$ one can approximate the Laplace transform of the function $w(\varepsilon)$ by $\tilde{w}(p) \simeq 1 - p\bar{\varepsilon}$, where $\bar{\varepsilon} = \int \varepsilon w(\varepsilon) d\varepsilon$ is the average energy loss in a scattering event. Calculating the inverse Laplace transform of Eq. (2), we get

$$f(\Delta,x) = \frac{1}{\bar{\varepsilon}}\left(\frac{a}{x}\right)^\alpha W_\alpha\left[\frac{\Delta}{\bar{\varepsilon}}\left(\frac{a}{x}\right)^\alpha\right], \qquad (8)$$

where W_α is the Wright function[8]

$$W_\alpha(z) = \sum\nolimits_{l=0}^\infty \frac{(-z)^l}{l!\Gamma(1-\alpha-\alpha l)}. \qquad (9)$$

Using the distribution function (8) one can compute the average energy loss at some depth x:

$$\langle\Delta(x)\rangle = [\bar{\varepsilon}/\Gamma(1+\alpha)]\,(x/a)^\alpha, \qquad 0<\alpha<1. \quad (10)$$

We conclude that in the case of the fractal medium the average energy loss is characterized by fractional sublinear dependence on the distance. The equations (8), (10) constitute the main result of the paper.

Studied in this paper problem of random energy loss can be considered as "dual" to the problem of thermal random walk motion of low energy particles. The thermal diffusion of particles in a fractal medium is known to be anomalously slow, see, e.g., [9]. The reason of the subdiffusional motion in fractal porous medium is that a particle is trapped in dead end pores and bottlenecks, so that diffusion is slowed down and becomes anomalous. The physics of anomalous random energy loss in the fractal medium is different and is related with the presence of long-range correlations in positions of scattering centers. Approach developed in this work can also be applied to describe particle propagation in the system with dynamic heterogeneities formed at the critical point of phase transition.

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1. A. I. Akhiezer and N. F. Shul'ga, *High-energy electrodynamics in matter*, Kharkov Inst. of Phys. and Techn., Ukraine Gordon and Breach pub., 1996.
2. L. D. Landau, Journ. of Phys. **8**, 201 (1944).
3. E. W. Montroll and G. H. Weiss, J. Math. Phys. **6**, 167 (1965).
4. J. Klafter, A. Blumen, and G. Zumofen, J. of Stat. Phys. **36**, 561 (1984).
5. H. E. Stanley, J. of Stat. Phys. **36**, 843 (1984).
6. A. Blumen, J. Klafter, B. S. White, and G. Zumofen, Phys. Rev. Lett. **53**, 1301 (1985).
7. H. Isliker and L. Vlahos, Phys. Rev. E **67**, 026413 (2003).
8. R. Gorenflo, Y. Luchko, and F. Mainardi, Fractional Calculus and Applied Analysis **2**, 383 (1999); arXiv:math-ph/0701069.
9. Z.-T. Wang, Applied Math. and Mech. **21**, 1145 (2000).