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Two interacting particles in a random potential

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We study the scaling of the localization length of two interacting bosons in a one-dimensional random lattice with the single particle localization length. We consider the short-range interaction assuming that the particles interact when located both on the same site. We discuss several regimes, among them one interesting weak Fock space disorder regime. In this regime we obtain a weak logarithmic scaling law. Numerical benchmark data support the absence of any strong enhancement of the two particle localization length.

1. Introduction. Quantum single particle dynamics in one-dimensional disordered lattices with uncorrelated random onsite energies exhibits Anderson localization [1]. The asymptotic spatial decay of an eigenvector is exponential and given by $A_l^{(\nu)} \sim e^{-l/\xi_1^\nu}$, where ξ_1^ν is the localization length of an eigenmode ν with the eigenvalue λ_ν, and the integer l counts the lattice site (see also e.g. [2]). The localization length is bounded from above.

In [3] Dorokhov studied one-dimensional continuous model with harmonic attraction between two particles placed in weak random potential. It was shown that defects induce transitions between the internal-quantization states leading to the possible increase of the two-particle localization length which is a measure for coherent propagation of two interacting particles. The interplay of disorder and interaction of two interacting particles (TIP), interacting in a random one-dimensional chain was later considered by Shepelyansky (Sh94) [4]. The conclusion was that two particles might propagate coherently over distances ξ_2 much larger than the single particle localization length ξ_1, if both particles are launched within a distance of ξ_1 from each other. Sh94 used an analogy between the two-particle eigenvalue problem and that of banded random matrices, and made an assumption about the scaling properties of overlap integrals which connect different noninteracting Fock eigenstates in the presence of interaction. He finally concluded that in the weak disorder limit $\xi_1 \to \infty$ the two-particle localization length ξ_2 will scale with ξ_1 as $\xi_2 \propto \xi_1^2 U^2$, where U is the interaction strength [4]. This result was further supported by Imry (Im95) in [5], where a Thouless-type scaling argument was replacing the banded random matrix analogy. Therefore, two interacting particles were predicted to explore a much larger space than noninteracting particles. Numerical calculations by Frahm et al (FR) [6] concluded that the scaling is probably weaker, namely $\xi_2 \propto \xi_1^{1.65}$, and raised doubts about the previously assumed scaling properties of overlap integrals. Using a Green function method adapted to the problem [7], a new scaling relation at the center of the band, $\xi_2 = \xi_1/2 + 0.074\xi_1^2|U|/(1+|U|)$, was obtained numerically in [8]. In particular, this implies that the enhancement effect will set in for weaker interactions than previously predicted. Later on, it was argued that the enhancement effect is probably due to finite-size effects and it should completely vanish for an infinite system [9]. Simulating the time dependent Schrödinger equation for two interacting particles [10], it was argued that the dynamics is characterized by two time scales, t_1 and t_2, set by, respectively, two localization lenghts, ξ_1 and ξ_2. Recently, two of us studied statistical properties of the overlap integrals perturbatively and numerically for weak disorder [11]. These results contradict previous assumptions of Sh94 and Im95 [4, 5], and if used within the previously applied theoretical schemes, predict a much weaker interaction induced increase of the localization length than previously discussed. Despite a number of studies, the problem of two interacting particles in a random potential remains therefore a completely open problem. At the same time this seemingly academic case can be both addressed by current techniques with ultracold interacting atoms [12], and is of fundamental importance for tackling the much more complicated case of many interacting particles in random potentials.

In the present work we first show that a nonperturbative strong localization length enhancement can be expected only in a regime of very weak disorder, with

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upper bounds on the disorder strength. This regime was not fully accessed in previous numerical scaling studies. We obtain upper bounds on the strength of the expected enhancement effect using correct scaling properties of overlap integrals. We then perform direct numerical measurements solving the corresponding eigenvalue problem and calculating the largest average localization length ξ_2 of exponentially decaying two-particle probability density function averaged over many disorder realizations. Finally we formulate a set of open issues which have to be addressed in the future.

2. Model. We consider the Bose–Hubbard hamiltonian with disorder

$$\hat{\mathcal{H}} \equiv \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\rm int},\ \hat{\mathcal{H}}_{\rm int} = \sum_l \left[\frac{U}{2}\hat{a}_l^+\hat{a}_l^+\hat{a}_l\hat{a}_l\right], \qquad (1)$$

$$\hat{\mathcal{H}}_0 = \sum_l \left[\epsilon_l \hat{a}_l^+\hat{a}_l + V(\hat{a}_{l+1}^+\hat{a}_l + \hat{a}_l^+\hat{a}_{l+1})\right],$$

and use the fixed boundary conditions. The hamiltonian (1) consists of non-interacting and interacting parts, $\hat{\mathcal{H}}_0$ and $\hat{\mathcal{H}}_{\rm int}$, where $\hat{a}_l^+$ and $\hat{a}_l$ are standard boson creation and annihilation operators on a lattice site l and U measures the interaction strength. The random on-site energies ϵ_l are chosen uniformly from the interval $[-W/2, W/2]$, with W and V denoting the disorder and hopping strengths, respectively.

2.1. *One particle.* In this case the interaction term does not contribute. We use the basis $|l\rangle \equiv a_l^+|0\rangle$ with $l = 1, \ldots, N$ (N is the number of lattice sites). The eigenstates (also called single particle normal modes (NM)) $|\nu\rangle = \sum_l^N A_l^{(\nu)}|l\rangle$ are defined through the eigenvectors $A_l^{(\nu)} \sim e^{-|l|/\xi_1^\nu}$ with the eigenvalue problem

$$\lambda_\nu A_l^{(\nu)} = \epsilon_l A_l^{(\nu)} + V(A_{l+1}^{(\nu)} + A_{l-1}^{(\nu)}). \qquad (2)$$

The eigenvalues $-2V - W/2 \le \lambda \le 2V + W/2$ fill a band with a width $\Delta_1 = 4V + W$. The most extended NMs correspond to the band center $\lambda = 0$ with localization length

$$\xi_1(\lambda = 0, W) \approx 100(V^2/W^2), \qquad (3)$$

in the limit of weak disorder $W/V \le 4$ [2]. The average volume L which an eigenstate occupies has been estimated to be about $L \approx 3\xi_1$ for weak disorder [11].

2.2. *Two particles.* For $U = 0$ we construct orthonormalized two particle eigenstates as product states of single particle eigenstates in a corresponding Fock space

$$|\mu, \nu \ge \mu\rangle = \frac{|\mu\rangle|\nu\rangle}{\sqrt{1+\delta_{\mu,\nu}}},\ \hat{\mathcal{H}}_0|\mu,\nu\rangle = (\lambda_\mu + \lambda_\nu)|\mu,\nu\rangle. \qquad (4)$$

Then, we expand the eigenstates $|q\rangle$ of the interacting particle problem, $\hat{\mathcal{H}}|q\rangle = \lambda_q|q\rangle$, in systems of eigenstates for the noninteracting problem, $|q\rangle = \sum_{\nu,\mu\le\nu}^N \phi_{\mu\nu}^{(q)}|\mu,\nu\rangle$, where the coefficients $\phi_{\mu\nu}^{(q)}$ satisfy the eigenvalue problem

$$\lambda_q \phi_{\mu\nu}^{(q)} = \lambda_{\mu\nu}\phi_{\mu\nu}^{(q)} + 2U\sum_{\mu',\nu'} \bar{I}_{\mu\nu}^{\mu'\nu'}\phi_{\mu'\nu'}^{(q)}. \qquad (5)$$

Here $\lambda_{\mu\nu} \equiv \lambda_\mu + \lambda_\nu$ and therefore the noninteracting case $U = 0$ yields a band with width $\Delta_2 = 2\Delta_1$. The coefficients $\bar{I}_{\mu\nu}^{\mu'\nu'}$ are connected with the overlap integrals

$$I_{\mu\nu}^{\mu'\nu'} = \sum_l A_l^\mu A_l^\nu A_l^{\mu'} A_l^{\nu'} \qquad (6)$$

as follows: $\bar{I}_{\mu\nu}^{\mu'\nu'} = I_{\mu\nu}^{\mu'\nu'}/(\sqrt{1+\delta_{\mu\nu}}\sqrt{1+\delta_{\mu'\nu'}})$. The interacting case yields a single band for $U < \Delta_2$, but two bands separated by a gap for $U > \Delta_2$. Indeed, in the latter case two-particle bound states are renormalized out of the main band, and are mainly consisting of two particles occupying the same site [13]. Therefore, remaining band is due to states where the two particles can be anywhere but not on the same site. This is simply the limit of two noninteracting spinless fermions. The localization length of these two noninteracting fermions is of the same order as the single particle localization length. The localization length in the bound state band is even smaller, since the effective disorder strength in this band becomes $2W$, but the effective hopping is strongly suppressed.

For numerical purposes we expand the two particle eigenstates $|q\rangle$ in the local basis $|q\rangle = \sum_{m,l\le m}^N \mathcal{L}_{l,m}^{(q)}|l,m\rangle$, $|l,m\rangle \equiv a_l^+ a_m^+|0\rangle/(\sqrt{1+\delta_{lm}})$, where $\mathcal{L}_{l,m}^{(q)} = \langle l,m|q\rangle$ are the normalized eigenvectors. They satisfy

$$\phi_{\mu\nu} = \sum_{m,l\le m}^N \frac{A_m^{(\mu)}A_l^{(\nu)} + A_l^{(\mu)}A_m^{(\nu)}}{\sqrt{1+\delta_{lm}}\sqrt{1+\delta_{\mu\nu}}}\mathcal{L}_{l,m}^{(q)}. \qquad (7)$$

We will numerically compute the probability density function (PDF) of the number of particles in direct space $p_l = \langle q|\hat{a}_l^+\hat{a}_l|q\rangle/2$, which is given by

$$p_l^{(q)} = \frac{1}{2}\left(\sum_{k,l\le k}^N \mathcal{L}_{l,k}^{(q)2} + \sum_{m,l\ge m}^N \mathcal{L}_{m,l}^{(q)2}\right). \qquad (8)$$

3. Different scales for the two-particle problem. Since a single particle eigenstate occupies a volume L, there are of the order of L^2 two particle eigenstates which are residing in the same volume for $U = 0$. The overlap integrals built among these L^2 Fock states are nonzero (more precisely not exponentially weak) and

define the connectivity in the Fock space for nonzero U. The average eigenenergy spacing d of these connected Fock states is $d = \Delta_2/L^2$. It therefore defines an effective energy mismatch, i.e. an effective disorder strength $\bar{W} \equiv d$, in the Fock space. The effective hopping strength follows from (5) and is given by $\bar{V} = 2U\langle I\rangle$. Here $\langle I\rangle$ is an average overlap integral among all connected Fock states [11].

In analogy with Eq. (3) we can therefore obtain a localization length in Fock space for weak Fock space disorder $\bar{W} \lesssim 4\bar{V}$, which in real space is a measure in units of the single particle localization length:

$$\xi_2/\xi_1 \approx 100\bar{V}^2/\bar{W}^2 = 400\, U^2\langle I\rangle^2 L^4/\Delta_2^2\,. \qquad (9)$$

For strong Fock space disorder $\bar{W} \gg \bar{V}$ the volume $L \approx 1$, and two interacting particles are localized in the same way, therefore $\xi_2 \approx \xi_1$ in this case.

3.1 *Bounds on the weak Fock space disorder regime.* Let us now address the question whether we can enter the weak Fock space disorder regime for strong single particle disorder $W \gg V$. This seems possible at a first glance since we can increase the value of $\bar{V}$ by increasing U. However, in this limit $\langle I\rangle \sim V^2/W^2$. Therefore the needed interaction strength is $U \sim W^3/V^2$, since $\Delta_2 \sim W$. But an increase of the interaction strength beyond the band width Δ_2 leads to the separation of the energy spectrum into two bands - a bound state band with strongly localized particle pairs [13], and a noninteracting spinless fermion band which has no localization length increase as compared to the single particle case. The two conditions $U \lesssim W$ and $U \gtrsim W^3/V^2$ imply that $W \lesssim V$ is needed, which means that the single particle case must be in the regime of weak localization. Therefore $U \lesssim V$ is an upper bound for entering the weak Fock space disorder regime.

Lowering U further we will however again leave this regime and enter the perturbative one, which is again characterized by strong disorder in Fock space. Indeed, the energy renormalization of a given Fock state follows from (5) and is given by $2UI_0$, where I_0 is an average overlap integral of a Fock state with itself. Due to orthonormality of the single particle eigenfunctions it follows $I_0 \approx 1/L$. The perturbative regime holds as long as $UI_0 \lesssim d$. Inside the perturbative regime a Fock state is still a good approximation to an exact eigenstate, and therefore the two particle localization length is of the order of the single particle one. Therefore, the nonperturbative weak Fock space disorder regime is accessed for $\Delta_2/L \lesssim U \lesssim V$.

For any practical purposes we seek a strong enough interaction strength U, and this requires $U \approx V$ and $W < V$. In order to obtain any relevant scaling results upon variation of W one needs therefore to lower W significantly further such that $W \ll V$.

3.2 *Overlap integrals revisited.* Sh94 and Im95 estimated the average overlap integral $\langle I\rangle_{\rm SI} \sim L^{-3/2}$ [4, 5] inside the weak Fock space disorder regime. This result is obtained in the following way. A single particle eigenstate occupies a volume $L \gg 1$. Due to normalization it follows $|A_l^{(\nu)}| \sim L^{-1/2}$. The crucial point was to assume that all terms inside one localization volume in the sum (6) have uncorrelated signs. This leads to the above estimate. However, in the limit of weak disorder and large localization length, the single particle eigenvectors inside a localization volume will appear similar to plane waves, with appreciable phase correlations between different sites, and also between different eigenstates. Some numerical studies by Römer et al. (R99) [14] even concluded that $\langle I\rangle_{\rm R} \sim L^{-2}$. This result essentially corresponds to the assumption that the eigenvectors are exact plane wave states inside a localization volume. It is this small difference in the exponent which separates a possible existing strong enhancement of the localization length from no effect at all.

In a recent work two of us performed a perturbation approach at the weak disorder limit and obtained that strong phase correlations will certainly modify the prediction of Sh94, Im95. At the same time corrections to the result of R99 are significant. As a final result we obtain $\langle I\rangle_{\rm SI} \sim -\ln(L)L^{-2}$ [11] – logarithmic corrections to the prediction of Römer et al. It is well-known that logarithmic corrections are rather resistent to numerical verifications, if no special trick or technique is used. Therefore, our numerical tests in a limited interval of W lead only to the clear result that the prediction of Sh94, Im95 is incorrect, and if $\langle I\rangle \sim 1/L^z$ is assumed, then $z \approx 1.7$. They were not sensitive to distinguish between this power law and a possible asymptotic $\langle I\rangle \sim -\ln(L)L^{-2}$ logarithmic law.

3.3. *Scaling of the localization length.* Combining the above predictions on the overlap integral scaling and the localization length scaling (9) we arrive at the following results in the weak Fock space disorder regime. Here we set $\Delta_2 = 8V$, take $W < 4V$ such that (3) holds. Then Sh94 and Im95 predict $\xi_2/\xi_1 \sim (U/V)^2\xi_1$ as derived using different methods in the original papers [4, 5]. According to R99 the whole effect is simply $\xi_2/\xi_1 \sim (U/V)^2$, i.e. no enhancement at all. Finally, our analytical estimate for the overlap integrals yields

$$\xi_2/\xi_1 \sim (\ln\xi_1)^2\,(U/V)^2\,. \qquad (10)$$

Note that the numerically estimated overlap integral dependence on L results in $\xi_2/\xi_1 \sim (U/V)^2\xi_1^{0.6}$.

4. Numerical technique. We estimate the largest average localization length ξ_2 of the probability density function $p_l \sim e^{-2l/\xi_2}$ [see Eq. (8)] using the following procedure (the prefactor 2 in the exponent takes care of the fact that densitites instead of wave functions are fitted). For a given realization we solve the eigenvalue problem and choose only those modes $\mathcal{L}^{(q)}_{l,m}$ which satisfy to the following selection rules:

- the center of masses

$$\bar{l}_q = \sum_{m,l\le m}^{N} l\mathcal{L}^{(q)2}_{l,m}, \quad \bar{m}_q = \sum_{m,l\le m}^{N} m\mathcal{L}^{(q)2}_{l,m} \qquad (11)$$

  satisfy to the inequalities $|\bar{l}_q - N/2| \le \xi_1$, $|\bar{m}_q - N/2| \le \xi_1$ (ζ is of the order of the corresponding average localization length for a single particle problem). Thus, we take into account only those modes for which the two particles reside in the same localization volume;

- the eigenvalues are near the bandwidth center. We assume that similar to the case of a single particle problem the most extended modes are with $\lambda_q \approx 0$;

- we project $\mathcal{L}^{(q)}_{l,m}$ onto the modes of the one-particle problem, calculate the amplitudes $\phi_{\mu\nu}$ in accordance with Eq. (7) and find the mode $|\mu_0, \nu_0\rangle$ with the largest amplitude, $\max_{\mu_0,\nu_0} \phi^2_{\mu\nu}$. Such a method allows us to identify the Fock state $|\mu_0, \nu_0\rangle$ which dominates all others. We then request that the eigenvalues λ_{μ_0} and λ_{ν_0} are close to the bandwidth center. Thus, we exclude possible cases when λ_q is close to the band center, but λ_{μ_0} and λ_{ν_0} are located at the two opposite band edges.

Having selected the modes $\mathcal{L}^{(q)}_{l,m}$, we compute their probability density functions p_l according to Eq. (8) and shift them such that their new center of mass are located at the center of a chain, $N/2$. Then, we compute logarithms of the PDFs, $\ln(p_l)$ and perform a statistical average of the PDFs over many disorder realizations as $\langle p_l\rangle = \exp[\langle \ln(p_l)\rangle]$. Finally, using a local regression smoothing technique, we obtain smooth functional dependencies of $\langle p_l\rangle$ and calculate the quantity $\alpha = 2|d(\ln\langle p_l\rangle)/dl|^{-1}$. In the limit of large l, $\alpha(l)$ should saturate at the average two particle localization length ξ_2.

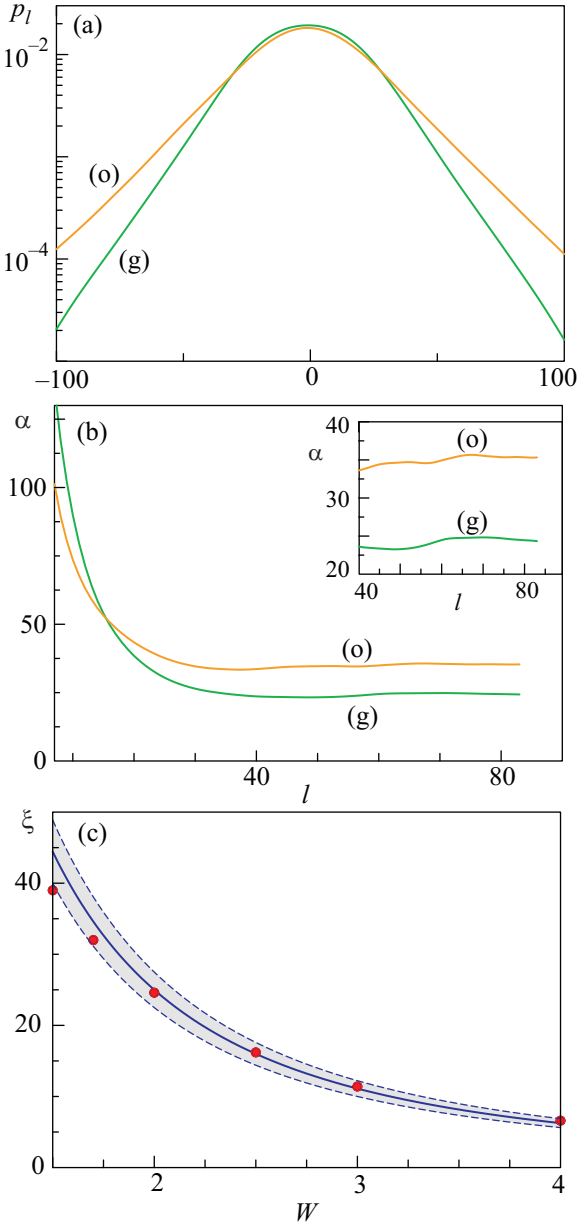

Fig. 1. (a) – Smoothed average probability distribution function $\langle p_l\rangle$ versus lattice site l in lin-log scale for $W = 2$, $U = 0$ ((g), green curve) and $W = 2$, $U = 0.2$ [(o), orange curve]. (b) – The corresponding quantity α (see text) versus l, with a zoom of the interval with saturated values of α (inset). (c) – The two-particle localization length ξ_2 versus W for the noninteracting case, $U = 0$ (red circles). Blue solid line: $\xi_1 = 100/W^2$. Dashed lines: maximal admissible error of 10% from the analytical formula. Gray area corresponds to the admissible values

5. Numerical results. The dimension of the Hilbert space p grows rapidly ($\sim N^2$) with the size of a chain, so that the maximal reachable size used in numerical computations, $N_{\max} = 234$. Thus, we inevitably face finite size effects for weak disorder. We start with the noninteracting case $U = 0$ for which ξ_2 must be exactly equal to ξ_1. We estimate the minimal value for the strength of disorder, respectively, maximal localization length, ξ_2, at which an error (caused by finite-size ef-

fects) is less than 10% (which is the maximal error we admit). We assume that this error depends only on the magnitude of ξ_2 but not on the interaction strength U. Thus, the largest tolerable values for ξ_2 found for the noninteracting case are also assumed to be the limiting values for the interacting case. For $U = 0$ the lower curve in Fig. 1a presents a smooth dependence of $\langle p_l \rangle$ on l. The corresponding quantity α (lower curve in Fig. 1b) saturates at large distances. The obtained localization length ξ_2 is finally shown in Fig. 1c and agrees well with the theoretical prediction, however systematic deviations accumulate for weak disorder. A recalculation of the same quantities for $U = 0.2$ in Fig. 1a, b shows that the method appears to be applicable to the interacting case as well. Finite size effects blurr our results substantially if $\xi_2 > 40$.

Let us discuss our results for nonzero interaction. The ratio ξ_2/ξ_1 grows with increasing interaction constant U, as shown for different values of W in Fig. 2. This growth is stronger, the weaker the disorder strength

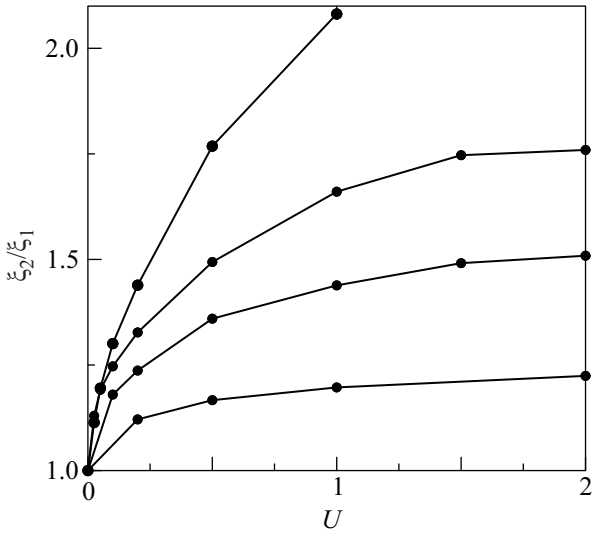

Fig. 2. The ratio ξ_2/ξ_1 versus interaction constant U for different values of disorder $W = 2, 2.5, 3, 4$ (from top to bottom)

is. For our data, the ratio did not substantially exceed the value 2. However, it seems plausible that for $W < 2$ (which is not treatable with our current technique), stronger enhancement effects could be observed.

The central result is plotted in Fig. 3. Here we plot ξ_2 versus ξ_1 on log-log scales. We try to fit data for a fixed value of U and different values of W using power law estimates. Both ξ_1 and ξ_2 vary less than an order of magnitude, while a safe power law fit needs at least two orders of magnitude variations on each variable. Nevertheless we bound the obtained variations with two lines

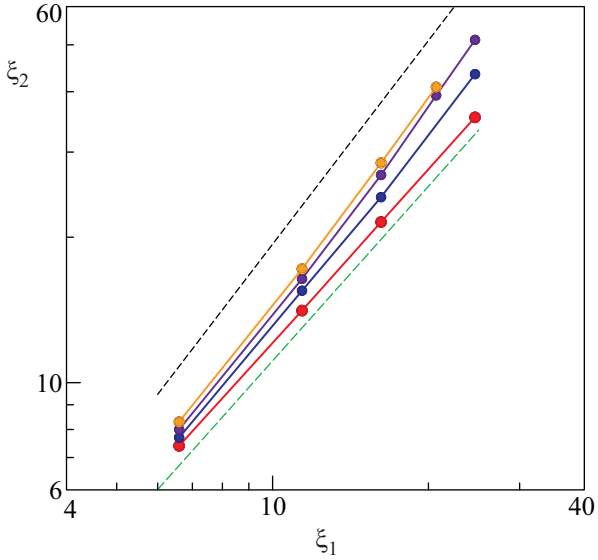

Fig. 3. The two-particle localization length ξ_2 versus one particle localization length ξ_1 for $U = 1.5, 1.0, 0.5, 0.2$ (from top to bottom) in log-log scale. Dashed straight lines are power laws ξ_1^α with the exponents $\alpha = 1.4$ (upper line) and $\alpha = 1.3$ (lower line). The size of a chain is $N = 234$

$\xi_2 \sim \xi_1^{1.3}$ and $\xi_2 \sim \xi_1^{1.4}$. Such a scaling is much weaker than the any of the above predicted power laws. It is possible that we observe the onset of the logarithmic scaling obtained from perturbation theory (10).

Let us compare our benchmark results (obtained from exact diagonalization, without any finite size fitting, and with a maximum error of 10%) with the results using a Green function (GF) approach [7, 8]. This method is not exact. First, it measures the decay of the two particle wavefunction along the diagonal (cf. Fig. 4), instead of the PDF used here. As a consequence, at $U = 0$ GF-measures a length $L_2 = \xi_1/2$. Indeed, at $U = 0$ the wave function is more elongated along the main axes and compressed along the diagonal (cf. Fig. 4). However, at nonzero U and sufficiently weak disorder, the wavefunction elongates along the diagonal (cf. Fig. 4). Therefore, in this limit one expects that $L_2 \to \xi_2$. This nontrivial crossover feature adds to and blurres any straightforward fitting procedure. Second, the GF-suffers from finite size effects, and the actual data for L_2 are obtained from a finite size fit [8]. We tested the quality of such fits in our calculations, and dropped this method since it is way too incorrect in order to extract scaling features. To give an example, we take $U = 1$ and find in our benchmark stady $\xi_2(W = 3) = 16.4 \pm 2$, $\xi_2(W = 2.5) = 27 \pm 2$, and $\xi_2(W = 2) = 51 \pm 4$. The corresponding numbers from [8] area $L_2(W = 3) = 12.5$, $L_2(W = 2.5) = 20$,

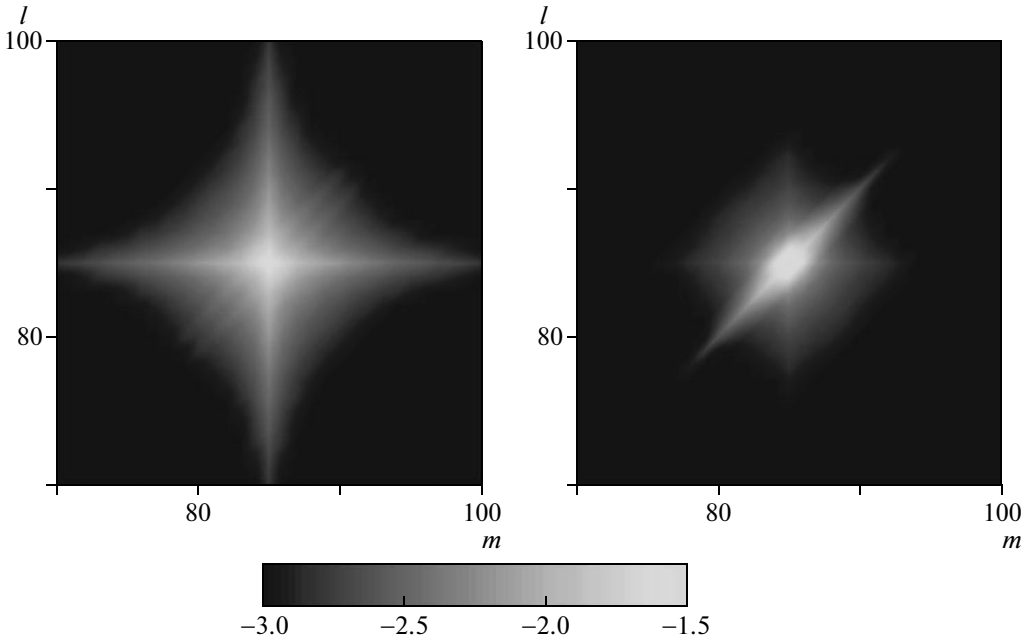

Fig. 4. $\log(|c_{l,m}|^2)$ versus l and m averaged over time and 5000 disorder realizations. For better visualization results are unfolded from the irreducible triangle shaped state space onto a square with $c_{l,m} = c_{m,l}$ for $m \leq l$. The strength of disorder $W = 2.5$ and the interaction constants $U = 0$ (a) and $U = 2$ (b). Particles are initially located on the same site at the center of a chain with $N = 170$ sites

$L_2(W = 2) = 41$. Therefore even at disorder $W = 2$ and $U = 1$ the GF-numbers are suffering from the abovementioned crossover. What is left in the GF-analysis [8] is a little window $1 < W < 1.75$. Such a small window is not enough in order to extract meaningful scaling data.

5.1. *Averaged evolution of two particles.* In order to visualize the effect of interaction on the localization of two particles, we solve the time dependent Schrödinger equation $i\partial_t|\Psi(t)\rangle = \hat{\mathcal{H}}|\Psi(t)\rangle$. We expand $|\Psi(t)\rangle$ in terms of the orthonormal states $|l,m\rangle$ $(l \leq m)$ as $|\Psi(t)\rangle = \sum_{m,l\leq m}^{N} c_{l,m}(t)|l,m\rangle$, where the coefficients $\mathcal{L}_{l,m}^{(q)}$ are $c_{l,m}(t) = \sum_{q=1}^{p} \varphi_q \mathcal{L}_{l,m}^{(q)} e^{-i\lambda_q t}$. Here φ_q are the amplitudes of NMs related with the initial amplitudes $c_{l,m}(0) = \langle l,m|\Psi(0)\rangle$ of the two-particle states as $\varphi_q = \sum_{m,l\leq m}^{N} c_{l,m}(0)\mathcal{L}_{l,m}^{(q)}$. We launch two particles on the same site, $l_0 = m_0$, or adjacent sites, $l_0 = m_0 - 1$, such that the initial amplitude $c_{l,m}(0) = \delta_{l,l_0}\delta_{m,m_0}$. We calculate then the averaged in time square amplitude $\langle|c_{l,m}|^2\rangle_t$, which is given by

$$\langle|c_{l,m}|^2\rangle_t \equiv \lim_{T\to\infty} \frac{\int_0^T |c_{l,m}|^2}{T} dt = \sum_{q=1}^{p} |\varphi_q|^2 \mathcal{L}_{l,m}^{(q)2}. \quad (12)$$

We further average $\langle|c_{l,m}|^2\rangle_t$ over 5000 disorder realizations. In addition we perform an averaging with respect to initial conditions, by keeping the same disorder potential, and taking different neighboring sites as an initial location of the particles. Finally, we compute the average probability density function $\langle p_l\rangle$ using $p_l = \frac{1}{2}\left(\sum_{k,l\leq k}^{N} |c_{lk}|^2 + \sum_{m,l\geq m}^{N} |c_{ml}|^2\right)$. Note that the averaged in time two-particle wavefunction $|c_{lm}|^2$ for a single disorder realization has many spots at different locations due to resonances. This feature is smeared out, once the averaging with respect to disorder realizations is performed as is seen in Fig. 4a, b. For the noninteracting case the obtained distribution is elongated along the main axes. This happens because the two particles are not correlated, and it is much more probable for them to occupy different space regions. However for $U = 2$ the distribution is elongated along the diagonal. This implies that the two particles are exploring more states when being close to each other.

6. Summary. In summary, we discussed the possible regimes of two interacting particles in a random potential. The most interesting case of a weak Fock space disorder regime was analyzed, and scaling laws were discussed. These results, as well as the numerical data presented as well, show that the localization length enhancement effect is much weaker than previously assumed. Further numerical studies are needed in order to substantiate these results. However the current techniques are not of use for weaker disorder strength. Therefore new computational approaches are needed in

order to reach disorder values as low as $W = 0.1$, which may be enough to test the predicted weak logarithmic scaling.

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1. P. W. Anderson, Phys. Rev. **109**, 1492 (1958).
2. B. Kramer and A. MacKinnon, Rep. Prog. Phys. **56**, 1469 (1993).
3. O. N. Dorokhov, Zh. Eksp. Teor. Fiz. **98**, 646 (1990).
4. D. L. Shepelyansky, Phys. Rev. Lett. **73**, 2607 (1994).
5. Y. Imry, Europhys. Lett. **30**, 405 (1995).
6. K. Frahm, A. Müller-Groeling, J.-L. Pichard, and D. Weinmann, Europhys. Lett. **31**, 169 (1995).
7. F. von Oppen, T. Wettig, and J. Müller, Phys. Rev. Lett. **76**, 491 (1996).
8. K. M. Frahm, Eur. Phys. J. B **10**, 371 (1999).
9. R. A. Römer and M. Schreiber, Phys. Rev. Lett. **78**, 515 (1997).
10. S. De Toro Arias, X. Waintal, and J.-L. Pichard, Eur. Phys. J. B **10**, 149 (1999).
11. D. O. Krimer, S. Flach, Phys. Rev. E **82**, 046221 (2010).
12. K. Winkler et al., Nature **441**, 853 (2006).
13. A. C. Scott, J. C. Eilbeck, and H. Gilhoj, Physica D **78**, 194 (1994).
14. R. A. Römer, M. Schreiber, and T. Vojta, Phys. Stat. Sol. **211**, 681 (1999).