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Limiting energy density and a regular accelerating Universe in Riemann–Cartan spacetime

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Isotropic cosmology built in the Riemann–Cartan spacetime by using sufficiently general expression of gravitational Lagrangian is investigated. It is shown that cosmological equations obtained by certain restrictions on indefinite parameters of gravitational Lagrangian lead to limiting energy density at the beginning of cosmological expansion and all cosmological models filled with usual gravitating matter satisfying standard energy conditions are regular with respect to energy density, spacetime metrics with its time derivative and torsion functions. At asymptotics cosmological solutions of spatially flat models coincide with that of standard ΛCDM-model for accelerating Universe.

1. Introduction. The problem of the beginning of the Universe in time in the past – the problem of cosmological singularity (PCS) – remains as one of the most principal problems of relativistic cosmology and general relativity theory (GR). In accordance with Penrose–Hawking theorems about gravitational singularities the most part of cosmological solutions of GR are singular, if gravitating matter satisfies standard energy conditions. The behaviour of cosmological solutions near cosmological singularity was investigated in works by Belinsky V.A., Lifshits E.M., and Khalatnikov I.M. (see [1, 2] and Refs. herein). At the same time many attempts were undertaken with the purpose to solve the PCS in the frame of GR and existent candidates to quantum gravitation theory as well as of different generalizations of Einstein's gravitation theory, some particular regular cosmological solutions were obtained (see, for example [3, 4], review [5] and [6]). From physical point of view the appearance of gravitational singularities in gravitating systems with positive values of energy density and pressure is connected with the fact that the gravitational interaction in such systems in the frame of GR always has the character of attraction, which increases with the growth of energy density. Although the gravitational interaction in the case of gravitating systems with negative pressure in the frame of GR can be repulsive, the PCS can not be solved by considering corresponding models: the most part of cosmological solutions including inflationary solutions are singular.

As it was shown in a number of papers (see [7–13] and Refs. herein) the gravitation theory in 4-dimensional Riemann–Cartan spacetime U_4 – the Poncaré gauge theory of gravity (PGTG) – offers opportunities to solve the PCS and also to explain the acceleration of cosmological expansion at present epoch without introducing the notion of dark energy (DE). First of all it should be noted that in the framework of gauge approach to gravitation the PGTG is a necessary generalization of metric theory of gravity if the Lorentz group is included to gauge group, which corresponds to gravitational interaction[2)]. Let us to remind the most important physical results obtained in the frame of isotropic cosmology built in the frame of PGTG based on the gravitational Lagrangian $\mathcal{L}_g$ of general type including both a scalar curvature and different invariants quadratic in gravitational gauge field strengths – the curvature ($F_{\alpha\beta\mu\nu}$) and torsion ($S_{\alpha\mu\nu}$) tensors. Any homogeneous isotropic model (HIM) in the frame of PGTG is described by means of three functions of time – the scale factor of Robertson–Walker metrics R and two torsion functions S_1 and S_2 determining non-vanishing components of torsion tensor (unlike S_1 the torsion function S_2 is pseudoscalar with respect to spatial inversions). Two types of HIM were built and investigated: HIM with the only torsion function S_1 and HIM with two torsion functions. Isotropic cosmology based on HIM of the first type offers opportunities to solve the PCS [7–9]: all cosmological models filled with usual matter satisfying standard energy conditions (including inflationary models) are regular with respect to energy density, scale factor R with its time derivatives. However, the situation with DE in the case of these HIM becomes the same as in GR. Isotropic cosmology based on HIM with two torsion functions allows to build regular inflationary HIM and makes possible to

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[2)] From this point of view namely the PGTG but not metric theory of gravity corresponds to supergravity theory.

explain accelerating cosmological expansion at present epoch without introducing DE [10–13]. It is because the physical spacetime in the vacuum has the structure of de Sitter spacetime with non-vanishing torsion [13]. However, cosmological equations used in [10–12] do not exclude singular cosmological solutions, and the behaviour of cosmological solutions for flat HIM at asymptotics can differ from that of standard cosmological ΛCDM-model in dependence on initial conditions [14]. As it is shown in this Letter, by certain restrictions on indefinite parameters of gravitational Lagrangian PGTG allows to build totally regular isotropic cosmology for accelerating Universe, which quantitatively is in agreement at asymptotics with theory of standard cosmological ΛCDM-model.

2. Homogeneous isotropic models in PGTG. We will consider the PGTG based on the following expression of gravitational Lagrangian corresponding to spacial parity conservation (definitions and notations of [10] are used below):

$$\mathcal{L}_g = [f_0 F + F^{\alpha\beta\mu\nu}(f_1 F_{\alpha\beta\mu\nu} + f_2 F_{\alpha\mu\beta\nu} + f_3 F_{\mu\nu\alpha\beta}) + + F^{\mu\nu}(f_4 F_{\mu\nu} + f_5 F_{\nu\mu}) + f_6 F^2 + + S^{\alpha\mu\nu}\left(a_1 S_{\alpha\mu\nu} + a_2 S_{\nu\mu\alpha}\right) + a_3 S^{\alpha}_{\mu\alpha} S^{\mu\beta}_{\beta}\Big]. \quad (1)$$

The Lagrangian (1) includes the parameter $f_0 = (16\pi G)^{-1}$ (G is Newton's gravitational constant, the light velocity $c = 1$) and a number of indefinite parameters: f_i $(i = 1, 2, ..., 6)$ and a_k $(k = 1, 2, 3)$. Gravitational equations for HIM with two torsion functions corresponding to gravitational Lagrangian (1) allow to obtain cosmological equations generalizing Friedmann cosmological equations of GR and equations for torsion functions given in general form in [13]. These equations contain five indefinite parameters:

$$a = 2a_1 + a_2 + 3a_3, \quad b = a_2 - a_1,$$
$$f = f_1 + f_2/2 + f_3 + f_4 + f_5 + 3f_6,$$
$$q_1 = f_2 - 2f_3 + f_4 + f_5 + 6f_6, \quad q_2 = 2f_1 - f_2,$$

and their mathematical structure and physical consequences depend essentially on restrictions on these parameters. Unlike metric gravitation theory, quadratic in the curvature terms of $\mathcal{L}_g$ do not lead to higher derivatives of R in cosmological equations; higher derivatives can appear because of terms of $\mathcal{L}_g$ quadratic in the torsion tensor; in order to exclude higher derivatives of R from cosmological equations we have to put the restriction $a = 0$ [13, 15]. It should be noted that isotropic cosmology with $a \neq 0$ possesses some principal problems: in particular, cosmological equations at physically available initial conditions lead in this case to not physical solutions [16] and do not exclude singular solutions; moreover, the presence of the seconde derivative of the Hubble parameter in cosmological equations leads to its oscillating behaviour at asymptotics [17]. The second restriction concerns the parameter q_2: if $q_2 \neq 0$, the equation for the torsion function S_2 is differential equation of the second order that leads to oscillating behaviour of the Hubble parameter [14]; by putting $q_2 = 0$ we will obtain physically necessary consequences. Below we will analyze the main relations of isotropic cosmology given in [13] in general case without using any restrictions on indefinite parameters by putting the following restrictions: $a = 0$ and $q_2 = 0$.

Cosmological equations generalizing Friedmann cosmological equations of GR take the following form:

$$\frac{k}{R^2} + (H - 2S_1)^2 - S_2^2 = = \frac{1}{6f_0 Z}\left[\rho - 6bS_2^2 + \frac{\alpha}{4}\left(\rho - 3p - 12bS_2^2\right)^2\right], \quad (2)$$

$$\dot{H} + H^2 - 2HS_1 - 2\dot{S}_1 = = -\frac{1}{12f_0 Z}\left[\rho + 3p - \frac{\alpha}{2}\left(\rho - 3p - 12bS_2^2\right)^2\right], \quad (3)$$

where ρ is the energy density, p is the pressure, $H = \dot{R}/R$ is the Hubble parameter (a dot denotes the differentiation with respect to time), the parameter $\alpha = f/3f_0^2$ $(f > 0)$ has inverse dimension of energy density, and $Z = 1 + \alpha\left(\rho - 3p - 12bS_2^2\right)$. The torsion function S_1 is determined by the following way:

$$S_1 = -\frac{\alpha}{4Z}[\dot{\rho} - 3\dot{p} + 12f_0\omega H S_2^2 - 12(2b - \omega f_0)S_2\dot{S}_2], \quad (4)$$

where dimensionless parameter $\omega = (2f - q_1)/f \neq 0$ is introduced. The torsion function S_2^2 satisfies algebraic quadratic equation, which gives the following root

$$S_2^2 = \frac{\rho - 3p}{12b} + \frac{1 - (b/2f_0)(1 + \sqrt{X})}{12b\alpha(1 - \omega/4)}, \quad (5)$$

where $X = 1 + \omega(f_0^2/b^2)[1 - b/f_0 - 2(1 - \omega/4)\alpha(\rho + 3p)]$ [3]. In order to reduce cosmological equations (2), (3) to closed form we have to specify the content of HIM and its equation of state. In connection with this it should be noted that the matter content and its equation of state change during cosmological evolution and the form of equation of state depends on coupling of matter with gravitational field. In the case of usual gravitating matter with energy density $\rho_m > 0$ and pressure $p_m \geq 0$

[3] It seems that the second root for S_2^2 with opposite sign before $\sqrt{X}$ in (5) does not lead to physically satisfactory theory.

coupled minimally with gravitation the equation of state can be written in usual form: $p_m = p_m(\rho_m)$ and the law of energy conservation takes the form as in GR:

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0. \quad (6)$$

We introduce at early stage of cosmological expansion the scalar field ϕ with potential $V = V(\phi)$ as component of gravitating matter with the purpose to investigate inflationary HIM. By minimal coupling with gravitation the equation for scalar field takes the usual form as in GR:

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi}. \quad (7)$$

Then the total energy density ρ and pressure p are the following:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V + \rho_m \quad (\rho > 0), \quad p = \frac{1}{2}\dot{\phi}^2 - V + p_m. \quad (8)$$

Now by using the formula (5) for torsion function S_2^2 and Eqs. (6)–(8) we transform the torsion function S_1 defined by (4) to the following form:

$$S_1 = -\frac{3f_0\omega\alpha}{4bZ}(HD + E), \quad (9)$$

where

$$D = \frac{1}{2}\left(3\frac{dp_m}{d\rho_m} - 1\right)(\rho_m + p_m) + \\ + \frac{1}{3}(\rho_m - 3p_m) + \frac{2}{3}\dot{\phi}^2 + \frac{4}{3}V - \frac{b}{6f_0\alpha(1-\omega/4)}\sqrt{X} + \\ + \frac{1-\omega(b/2f_0)}{2\sqrt{X}}\left[\left(3\frac{dp_m}{d\rho_m} + 1\right)(\rho_m + p_m) + 4\dot{\phi}^2\right] - \\ - \frac{\omega f_0(1 - b/f_0)}{b\alpha(1-\omega/4)},$$

$$E = \left(1 + \frac{1 - \omega(f_0/2b)}{\sqrt{X}}\right)\frac{\partial V}{\partial \phi}\dot{\phi},$$

$$Z = \frac{-\omega/4 + (b/2f_0)(1 + \sqrt{X})}{1 - \omega/4}. \quad (10)$$

By using the formulas for torsion functions we write the cosmological equations (2)–(3) in the following closed form:

$$\frac{k}{R^2} + \left[H\left(1 + \frac{3f_0\omega\alpha}{2bZ}D\right) + \frac{3f_0\omega\alpha}{2bZ}E\right]^2 = \\ = \frac{1}{6f_0 Z}\left[\rho + 6(f_0 Z - b)S_2^2 + \frac{[1-(b/2f_0)(1+\sqrt{X})]^2}{4\alpha(1-\omega/4)^2}\right], \quad (11)$$

$$(\dot{H} + H^2)\left(1 + \frac{3f_0\omega\alpha}{2bZ}D\right) + \\ + \frac{3f_0\omega\alpha}{2bZ}\left[H(\dot{D} - \frac{\dot{Z}}{Z}D + E) + \dot{E} - \frac{\dot{Z}}{Z}E\right] = \\ = -\frac{1}{12f_0 Z}\left[\rho + 3p - \frac{[1-(b/2f_0)(1+\sqrt{X})]^2}{2\alpha(1-\omega/4)^2}\right], \quad (12)$$

where the quantities S_2^2, D, E, Z have to be replaced according to (5) and (10).

3. Accelerating Universe with limiting energy density. By using obtained cosmological equations we will analyze properties of cosmological solutions at different stages of cosmological evolution. Simultaneously we will find by what restrictions on indefinite parameters α, b and ω physical consequences are the most satisfactory and correspond to observational cosmological data. At first we will consider the evolution of cosmological models at asymptotics, where energy densities are small. If the value of dimensionless parameter ω is sufficiently small $|\omega| \ll 1$, the following estimations are valid: $X \to 1$, $Z \to b/f_0$, $S_1 \to 0$ and the torsion function S_2^2 approximately is equal to:

$$S_2^2 = \frac{\rho - 3p}{12b} + \frac{1 - b/f_0}{12\alpha b}. \quad (13)$$

As a result cosmological equations (2)–(3) take the form of Friedmann cosmological equations with effective cosmological constant induced by pseudoscalar torsion function S_2:

$$\frac{k}{R^2} + H^2 = \frac{1}{6f_0}\left[\rho(f_0/b) + \frac{1}{4}\alpha^{-1}(1 - b/f_0)^2(f_0/b)\right], \quad (14)$$

$$\dot{H} + H^2 = \\ = -\frac{1}{12f_0}\left[(\rho + 3p)(f_0/b) - \frac{1}{2}\alpha^{-1}(1 - b/f_0)^2(f_0/b)\right]. \quad (15)$$

Effective cosmological constant in eqs. (14)–(15) by certain relation for indefinite parameters α and b coincides with cosmological constant of ΛCDM-model that allows to explain accelerating cosmological expansion at present epoch [10, 11]. The value of parameter b determines the contribution of dark matter to energy density in the Universe [11]. If dark matter exists, the value of parameter b is close to f_0 being less than f_0 and the value of α^{-1} corresponds to the scale of high energy densities. By taking into account the role of dark matter in galaxies and their accumulations in the frame of GR, we have to conclude that the investigation of dark matter problem in the frame of PGTG assumes the study of inhomogeneous gravitating systems at astrophysical scale.

Now we will analyze the behaviour of cosmological solutions in the beginning of cosmological expansion. First of all important physical consequences follow from formula (5) for S_2^2-function. If the parameter ω is positive ($0 < \omega \ll 1$), because the value of X can not be negative, we obtain principal constraint for admissible energy densities and pressures:

$X = 1 + \omega(f_0^2/b^2)[1-(b/f_0) - 2(1-\omega/4)\alpha(\rho+3p)] \geq 0$ or by taking into account smallness of ω the following relation:

$$X = 1 - 2(f_0^2/b^2)\omega\alpha(\rho+3p) \geq 0. \qquad (16)$$

In the case of systems filled with usual matter with energy density ρ_m $(p_m = p_m(\rho_m))$ without scalar fields the equality defined by (16) determines a limiting (maximum) energy density $\rho_{\max}$ [4]. When energy density ρ_m is comparable with $\rho_{\max}$, the gravitational interaction has the character of repulsion ensuring the regularity of such systems. The order of $\rho_{\max}$ is determined by the value of $(\omega\alpha)^{-1}$. In the frame of classical theory the value of $\rho_{\max}$ has to be less than the Planckian energy density. In the case of systems including also scalar fields, for which energy density and pressure are defined by (8), the relation $X \geq 0$ determines in space of matter parameters $(\rho_m, \phi, \dot\phi)$ domain of their admissible values. This domain is limited by surface L defined by $X = 1 + \omega(f_0^2/b^2)[1 - b/f_0 - 2(1-\omega/4)\alpha(\rho_m + 3p_m + 2\dot\phi^2 - 2V)] = 0$. Moreover, it is necessary to take into account that additional restriction on admissible values of matter parameters $(\rho_m, \phi, \dot\phi)$ follows from positivity of expression (5) for S_2^2. Now we will analyze the behaviour of cosmological solutions near the limiting energy density or limiting surface L, where $X \ll 1$. With this purpose we consider the expression of the Hubble parameter H following from cosmological equation (11):

$$H_{\pm} = \left[-\frac{3f_0\omega\alpha}{2bZ}E \pm \left(\frac{1}{6f_0Z}\left\{ \rho + 6(f_0Z - b)S_2^2 + \frac{[1-(b/2f_0)(1+\sqrt{X})]^2}{4\alpha(1-\omega/4)^2} \right\} - \frac{k}{R^2} \right)^{1/2} \right] \times \left(1 + \frac{3f_0\omega\alpha}{2bZ}D\right)^{-1}. \qquad (17)$$

Similarly to isotropic cosmology based on HIM with the only torsion function [7–9], at asymptotics, where energy densities are sufficiently small, H_--solutions correspond to cosmological compression and H_+-solutions – to cosmological expansion, and the transition from H_-- to H_+-solution takes place by reaching the limiting energy density or limiting surface L (see below)[5]. In order to obtain some physical characteristics of such transitions the formula (17) for $H_{\pm}$ can be simplified by taking into account the smallness of parameter ω $(\omega \ll 1)$ and also that at considering extreme conditions: $\alpha^{-1} \ll \rho$, $X \ll 1$, $\rho \sim (\omega\alpha)^{-1}$. In the case of models filled with usual matter without scalar fields in linear approximation with respect to $\sqrt{X}$, where the quantity X is defined by (16), it is easy to obtain from (17) the Hubble parameter and its time derivative in the following form:

$$H_{\pm} = \pm\frac{2b^2}{3f_0^2\omega\alpha}\frac{\sqrt{X}[(1/4b)(\rho_m+p_m) - (k/R^2)]^{1/2}}{(3dp_m/d\rho_m+1)(\rho_m+p_m)},$$
$$\dot H = \frac{4b^2}{3f_0^2\omega\alpha}\frac{(1/4b)(\rho_m+p_m)-(k/R^2)}{(3dp_m/d\rho_m+1)(\rho_m+p_m)}. \qquad (18)$$

By reaching a limiting energy density $(X=0)$ the Hubble parameter vanishes and the value of its time derivative is the same for H_-- and H_+-solution and it is positive that corresponds to a bounce. By using obtained expression for the Hubble parameter it is easy to show that by given equation of state $p_m = p_m(\rho_m)$ the evolution of scale factor $R(t)$ near a bounce takes the following form: $R(t) = R_{\min} + r_1t^2 + ...$, where $t=0$ corresponds to a bounce, $R_{\min}$ is minimum value of R depending on limiting energy density and given equation of state, the value of $r_1 > 0$ is expressed by $\dot H$ at a bounce. It should be noted that in considered case the condition $S_2^2 > 0$ is valid, if equation of state of gravitating matter satisfies the following condition $p_m \leq \rho_m/3$, and it does not lead to additional restrictions on indefinite parameters.

In the case of models including also scalar fields the Hubble parameter does not vanish by reaching a limiting surface L and according to (17) its value on surface L is:

$$H_L = \frac{-2(\partial V/\partial\phi)\dot\phi}{(3dp_m/d\rho_m+1)\,(\rho_m+p_m)+4\dot\phi^2}. \qquad (19)$$

The bounce in this case takes place in points of extremum surface in space of matter parameters $(\rho_m, \phi, \dot\phi)$, equation of which we obtain by setting $H=0$ in cosmological equation (11). It is necessary to note that the condition $S_2^2 > 0$ (see (5), (8)) in considered case leads to additional restrictions on indefinite parameters, in particular the value of α^{-1} corresponds to the scale of high energy densities. By given potential $V(\phi)$ the numerical analysis allows to obtain more detailed restrictions on indefinite parameters. Cosmological solutions can be found by numerical integration of Eqs. (12), (6) and (7) by choosing initial conditions on extremum surface. Similarly to inflationary cosmological models with the only torsion function (see [7, 8]), if initial value of scalar field is sufficiently large regular cosmological

[4] In the frame of PGTG with gravitational Lagrangian (1) the conclusion about existence of limiting mass density was obtained in the case of HIM with the only torsion function S_1 in [15]. Later the hypothesis about existence in the nature of limiting mass density equal to the Planckian one was discussed in [18].

[5] In the case of HIM of closed type $(k=+1)$ there are also solutions without reaching the limiting surface L (see [7]).

solution contains transition stage from compression to expansion, inflationary stage with slow-rolling behaviour of scalar field and post-inflationary stage with oscillating scalar field. Similarly to our works [7, 8] regular Big Bang scenario can be built on the base of such inflationary cosmological models. All cosmological solutions are regular with respect to energy density, the scale factor R and the Hubble parameter H by virtue of existence of limiting energy density. Unlike HIM with the only torsion function S_1, in the case of considered HIM with two torsion functions the torsion does not diverge by reaching limiting surface L (or limiting energy density): the torsion function S_2 is continuous and the torsion function S_1 undergoes a finite jump by reaching the surface L (or limiting energy density).

4. Conclusion. We see that the PGTG allows to build totally regular isotropic cosmology for accelerating Universe. It is achieved by virtue of the change of gravitational interaction in comparison with GR and Newton's theory of gravity at cosmological scale. These changes are provoked by more complicated structure of physical spacetime, namely by spacetime torsion. In connection with this the following question appears: what situation takes place in the case of gravitating systems at other spatial scales (galaxies, stars, solar system), what possible role the torsion plays in these systems? First of all, the torsion should be important in gravitating systems at astrophysical scale (galaxies and their accumulations), by investigation of which the notion of dark matter was introduced more than 70 years ago. The conclusion obtained in this paper about possible existence of limiting energy density can be of principal meaning for theory of massive superdense stars, where gravitational repulsion effect at extreme conditions has to prevent a collapse. Concerning the solar system, usual gravitational effects including relativistic corrections can be obtained in this case in the frame of PGTG, because any vacuum solution of GR (in particular, the vacuum Schwarzschild solution) together with vanishing torsion is exact solution of PGTG independently on values of indefinite parameters of gravitational Lagrangian (1) [19]. However, from physical point of view such solutions have certain limits of their applicability, because the physical spacetime in the vacuum in accelerating Universe in the frame of PGTG is de Sitter spacetime with non-vanishing torsion (but not Minkowski spacetime) [13].

Physical phenomena discussed above in the frame of PGTG have essentially non-linear origin. In particular, this concerns properties of accelerating Universe at asymptotics, where principal role plays the structure of physical spacetime in the vacuum having also nonlinear origin. This leads to necessity to re-examine results of some investigations fulfilled earlier in the frame of PGTG by applying usual approximative method, in particular the particle content of PGTG (see [13]).

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