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Berry effect in unmagnetized inhomogeneous cold plasmas

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The propagation of electromagnetic waves in an unmagnetized weakly inhomogeneous cold plasma is examined. We show that the inhomogeneity induces a gauge connection term in wave equation, which gives rise to Berry effects in the dynamics of polarized rays in the post geometric optics approximation. The polarization plane of a plane polarized ray rotates as a result of the geometric Berry phase, which is the Rytov rotation. Also, the Berry curvature causes the optical Hall effect, according to which, rays of left/right circular polarization deflect oppositely to produce a spin current directed across the direction of propagation.

1. Introduction. It has often been said that ninety nine percent of the matter in the Universe is in plasma state [1]. Because real plasmas have density inhomogeneity, the study of electromagnetic wave propagation in inhomogeneous plasmas has become important [2–4]. In this paper we examine the propagation of electromagnetic waves in an unmagnetized plasma via the post geometric optics approximation [5, 6]. The plasma is assumed to have weak stationary density inhomogeneity, and thermal motions are not considered. Different polarizations, the left and right circular polarizations, are degenerate in a homogeneous isotropic plasma medium [7]. In the presence of inhomogeneity, we show that this double degeneracy is lifted by a gauge connection term in the wave equation, which gives rise to Berry effects in the dynamics of polarized rays in the post geometric optics approximation. The polarization plane of a plane polarized wave rotates as a result of the geometric Berry phase, which is the Rytov–Vladimirskii rotation [8, 9]. This is, of course, in contrast to the well known Faraday rotation, which is a dynamical effect due to the interaction of light and the magnetic field in a medium. Berry phase is a non-integrable phase factor arising from the adiabatic transport of a system around a closed path in its parameter space [10]. Geometrically, it originates from parallel transport in the presence of a gauge connection in the parameter space [11]. When an electromagnetic wave travels in a weakly inhomogeneous medium, the direction of the wave vector varies slowly so that the parameter space in this case corresponds to the momentum space. The Berry curvature associated with the gauge connection in momentum space causes the optical Hall (or Magnus) effect [12, 13], according to which waves of left/right circular polarization deflect oppositely to produce a spin (polarization) current directed across the direction of propagation. Such Berry effects are very typical of spin transport and have been derived repeatedly for various particles [14–16], in particular, for photons in different inhomogeneous media [5, 6, 17–22].

The paper is organized as follows. In Section 2, we write the plasma wave equation in an operator form by introducing a Hamiltonian operator and show that, in the process of its diagonalization, a gauge connection emerges in the momentum space because of the inhomogeneity. In Section 3, we present the post-geometric approximation which is suitable for weak inhomogeneity, and focus on the circularly polarized states by projecting the Hamiltonian on the polarization subspace. Finally, in Section 4, we derive the Berry effects in the dynamics of polarized rays by considering the post-geometric optics Hamiltonian, and establish the Rytov and the optical Hall effects in the plasma medium.

2. Plasma wave equation and the Gauge connection. Consider an inhomogeneous plasma which can be generated by gravity or a position dependent electric field. Generally, both of them can exist and the balance equation of the forces in steady state is

$$\rho_{\alpha 0}(\mathbf{x}) q_\alpha \mathbf{E}_0(\mathbf{x}) + \rho_{\alpha 0}(\mathbf{x}) \mathbf{F}_{g\alpha} - m_\alpha \nabla p_{\alpha 0} = 0,$$

where $\rho_{\alpha 0}$ is unperturbed mass density, $\mathbf{F}_g$ is the gravitational force and $\alpha = e, i$ represent electrons and ions. Now, we want to derive the equation which determines the wave properties of an inhomogeneous unmagnetized plasma. The procedure is similar to the derivation of the wave equation in homogeneous plasma [7] but we should notice that the density $\rho_{\alpha 0}(\mathbf{x})$ is now a function of $\mathbf{x}$. Considering small harmonic perturbation about steady state

$$\rho_\alpha(\mathbf{x}) = \rho_{\alpha 0}(\mathbf{x}) + \rho_{\alpha 1}(\mathbf{x}) e^{-i\omega t},$$

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$$\mathbf{V}_\alpha = \mathbf{V}_{\alpha 1}(\mathbf{x})e^{-i\omega t},$$

$$\mathbf{E} = \mathbf{E}_0(\mathbf{x}) + \mathbf{E}_1(\mathbf{x})e^{-i\omega t},$$

$$\mathbf{B} = \mathbf{B}_1(\mathbf{x})e^{-i\omega t},$$

we linearize the two fluid and Maxwell's equations. The linearized momentum equation is

$$-i\omega\rho_{\alpha 0}(\mathbf{x})\mathbf{V}_{\alpha 1}(\mathbf{x}) =$$

$$= \frac{q_\alpha \rho_{\alpha 0}(\mathbf{x})}{m_\alpha}\mathbf{E}_1(\mathbf{x}) + \rho_{\alpha 1}(\mathbf{x})\frac{\nabla p_{\alpha 0}}{n_{\alpha 0}(\mathbf{x})} - \gamma\nabla\left(\frac{\rho_{\alpha 1}(\mathbf{x})p_{\alpha 0}}{\rho_{\alpha 0}(\mathbf{x})}\right).$$

In deriving the above, the balance equation and $p_{\alpha 1} = \gamma p_{\alpha 0}\rho_{\alpha 1}(\mathbf{x})e^{-i\omega t}/\rho_{\alpha 0}(\mathbf{x})$ have been used. After combining it with linearized Maxwell's equations, the wave equation will be

$$\nabla \times \nabla \times \mathbf{E_1} - \mathbf{k_0^2}\left(\mathbf{1} - \frac{\omega^2_{\mathbf{pe}}(\mathbf{x})}{\omega^2} - \frac{\omega^2_{\mathbf{pi}}(\mathbf{x})}{\omega^2}\right)\mathbf{E_1} =$$

$$= \frac{4\pi}{c^2}\sum_\alpha \frac{q_\alpha}{m_\alpha}\left\{\gamma\nabla\left[\frac{\rho_{\alpha 1}(\mathbf{x})p_{\alpha 0}}{\rho_{\alpha 0}(\mathbf{x})}\right] - \frac{\rho_{\alpha 1}(\mathbf{x})}{\rho_{\alpha 0}(\mathbf{x})}\nabla p_{\alpha 0}\right\},$$

where $k_0 = \omega/c$ and $\omega^2_{p\alpha}(\mathbf{x}) = 4\pi\rho_{\alpha 0}(\mathbf{x})q^2_\alpha$. The cold plasma approximation and assuming stationary and inhomogeneous density is usually used to describe wave propagation in inhomogeneous media [2, 23]. Under the cold plasma approximation, we can drop the right hand side of the equation and it takes the form

$$\nabla \times \nabla \times \mathbf{E}_1 = k_0^2\left[1 - \frac{\omega^2_{pe}(\mathbf{x})}{\omega^2} - \frac{\omega^2_{pi}(\mathbf{x})}{\omega^2}\right]\mathbf{E}_1. \quad (1)$$

Introducing

$$\epsilon(\mathbf{x},\omega) = \left[1 - \frac{\omega^2_{pe}(\mathbf{x})}{\omega^2} - \frac{\omega^2_{pi}(\mathbf{x})}{\omega^2}\right] \equiv n^2(\mathbf{x},\omega),$$

and the dimensionless "momentum" operator [6, 17], $\mathbf{p} = -ik_0^{-1}\nabla$, (1) reads

$$H\mathbf{E}_1 = 0, \quad (2)$$

where the Hamiltonian H is a matrix-valued differential operator with elements

$$H_{ij}(\mathbf{x},\mathbf{p},\omega) = [\mathbf{p}^2 - n^2(\mathbf{x},\omega)]\delta_{ij} - p_i p_j.$$

In inhomogeneous plasma media the refractive index is also a function of frequency as well as coordinate. The refractive index is real for frequencies approximately above the plasma frequency and is imaginary for frequencies below it which causes the first to remain and the later to be absorbed in the media.

The momentum operator obeys the standard commutation relations

$$[x_i, p_j] = ik_0^{-1}\delta_{ij},$$

which has an important consequence with regard to diagonalization of H. To this end, we build a unitary matrix $R(\mathbf{p})$ from the eigenvectors of the non-diagonal part $p_i p_j$, according to

$$R(\mathbf{p}) = \begin{pmatrix} \frac{p_y}{\sqrt{p_x^2+p_y^2}} & -\frac{p_x p_z}{p\sqrt{p_x^2+p_y^2}} & \frac{p_x}{p} \\ -\frac{p_x}{\sqrt{p_x^2+p_y^2}} & -\frac{p_y p_z}{p\sqrt{p_x^2+p_y^2}} & \frac{p_y}{p} \\ 0 & \frac{\sqrt{p_x^2+p_y^2}}{p} & \frac{p_z}{p} \end{pmatrix} \quad (3)$$

where $p = |\mathbf{p}|$, of course. The unitary transformation $H \to R^\dagger H R$, $\mathbf{E}_1 \to R^\dagger \mathbf{E}_1$ applied to equation (2), then yields

$$H(\mathbf{x},\mathbf{p},\omega) = \mathbf{p}^2 1 - \Lambda - R^\dagger n^2 R, \quad (4)$$

where $\Lambda = \mathrm{diag}(0,0,\mathbf{p}^2)$ and 1 stands for the unit matrix. Of course, if $\mathbf{x}$ and $\mathbf{p}$ were classical (commuting) variables, the "potential" part of the Hamiltonian (5) would simply reduce to $n^2 1$, and H would be completely diagonalized. However, in view of their anticommutation, we have for the potential term in the momentum representation,

$$R^{-1}(\mathbf{p})n^2(ik_0^{-1}\nabla_\mathbf{p})R(\mathbf{p}) =$$

$$= n^2(1ik_0^{-1}\nabla_\mathbf{p} - k_0^{-1}\mathbf{A}) = n^2(\mathbf{x}1 - k_0^{-1}\mathbf{A}),$$

where

$$i\mathbf{A}(\mathbf{p}) = R^{-1}\nabla_\mathbf{p} R.$$

In deriving the above, we have made use of the identity [17, 18]

$$G^{-1}(x)f(\partial_x)G(x) = f(\partial_x + G^{-1}\partial_x G),$$

since $R^{-1}R = 1$, the vector matrix $\mathbf{A}$ is Hermitain. Thus

$$H(\mathbf{x},\mathbf{p},\omega) = \mathbf{p}^2 1 - \Lambda - n^2(\mathbf{x}1 - k_0^{-1}\mathbf{A}).$$

The canonical (generalized) coordinate conjugate to $\mathbf{p}$ is $\mathbf{x}$ which corresponds to the usual derivative $ik_0^{-1}\nabla_\mathbf{p}$.

In the absence of inhomogeneity, this canonical coordinate is the physical (observable) coordinate. With inhomogeneity present, however, the new coordinate, $\mathbf{x}1 - k_0^{-1}\mathbf{A}$, corresponds to $ik_0^{-1}\mathcal{D}_\mathbf{p}$, where $\mathcal{D}_\mathbf{p}$ is the covariant derivative defined by

$$\mathcal{D}_\mathbf{p} = 1\nabla_\mathbf{p} + i\mathbf{A}(\mathbf{p}).$$

The inhomogeneity can be, thus, viewed as inducing the non-Abelian gauge connection (potential) $\mathbf{A}(\mathbf{p})$ in the momentum space. Such a gauge potential is a well known feature of spin transport [6, 17, 18, 24, 25]. $\mathbf{A}$ is a *pure* gauge potential, i.e., the corresponding field strength (curvature), $\nabla_\mathbf{p} \times \mathbf{A} + i\mathbf{A} \times \mathbf{A}$, is identically zero. The covariant derivatives, thus, commute so that the new coordinates also commute. Furthermore, the latter satisfy the same commutation relations with $\mathbf{p}$ as the canonical coordinates $\mathbf{x}$.

We consider that the inhomogeneity is weak, thus the direction of the wave propagation varies slowly (infinitely slowly for adiabatic density variations). Let us take this direction to be along the z-axis so that E_{1z} is small (negligible in the adiabatic limit) compared to the other two components. Since we are considering polarization transport, we introduce the unitary matrix

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 \\ i & -i & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$$

and write the wave equation in the helicity basis via the unitary transformation

$$\begin{aligned} H \to V^\dagger H V &= \mathbf{p}^2 1 - \Lambda - n^2(\mathbf{x}1 - k_0^{-1}\tilde{\mathbf{A}}), \\ \mathbf{E_1} \to V^\dagger \mathbf{E_1} &= (E_{1+}, E_{1-}, E_{1z}), \end{aligned} \tag{5}$$

where $\tilde{\mathbf{A}} = V^\dagger \mathbf{A} V$, and $E_{1\sigma} = (E_{1x} - i\sigma E_{1y})/\sqrt{2}$ represents the two circularly polarized states with helicity $\sigma = \pm 1$. The elements $i, j = 1, 2$ of H, thus, correspond to the polarization subspace, on which we shall focus.

3. The post-geometric approximation. We now proceed to study the dynamics imposed by the Hamiltonian (5) based on the Taylor expansion of $n^2(\mathbf{x}1 - k_0^{-1}\tilde{\mathbf{A}})$. k_0^{-1} serves as a small parameter for the expansion, provided it is smaller than the length scale, L, of the density variations in the plasma. The zeroth order approximation (the geometric approximation), $k_0^{-1} \ll L$, thus holds appropriate for large L, i.e., for a practically homogeneous plasma. In the zeroth approximation the Hamiltonian (5) becomes diagonal,

$$H^{(0)}(\mathbf{x}, \mathbf{p}, \omega) = \mathrm{diag}(\mathbf{p}^2 - n^2, \mathbf{p}^2 - n^2, -n^2),$$

which implies a double degeneracy in the polarization subspace: electromagnetic waves with left/right circular polarizations have the same dispersion in a homogeneous isotropic plasma [7]. Furthermore, $E_{1z} = 0$, and the wave travels unrefracted along the z-direction, as expected.

In inhomogeneous plasmas, the refractive index gradient or equivalently the plasma frequency gradient removes the polarization degeneracy. In the first approximation (the post-geometric approximation), which is suitable for weak inhomogeneity, the Hamiltonian (5) takes the form

$$H^{(1)}(\mathbf{x}, \mathbf{p}, \omega) = H^{(0)}(\mathbf{x}, \mathbf{p}) + k_0^{-1}\nabla n^2 \cdot \tilde{\mathbf{A}}.$$

Calculating $\tilde{\mathbf{A}}$ via Appendix, the non-diagonal correction term is seen to couple E_σ to E_{1z}, too. Since E_{1z} is small (negligible in the adiabatic limit) this coupling can be ignored. Consequently, we can project $H^{(1)}$ on the polarization subspace with the result

$$H^{(1)}(\mathbf{x}, \mathbf{p}, \omega) = (\mathbf{p}^2 - n^2)1 + k_0^{-1}\nabla n^2 \cdot \mathbf{A}_\perp \sigma_3,$$

where $\mathbf{A}_\perp(\mathbf{p}) = \dfrac{p_3}{p(p_1^2 + p_2^2)}(-p_2, p_1, 0)$ and σ_3 is the Pauli matrix. The wave equation, thus, breaks down into two independent equations for the left/right circularly polarized waves according to $\mathcal{H}_\sigma E_\sigma = 0$, where

$$\mathcal{H}_\sigma = \frac{1}{2}[\mathbf{p}^2 - n^2(\mathbf{r}, \omega)] = \frac{1}{2}(\mathbf{p}^2 - n^2 + \sigma k_0^{-1}\nabla n^2 \cdot \mathbf{A}_\perp). \tag{6}$$

(The factor 1/2 has been introduced for later convenience.) As remarked, the refractive-index gradient is responsible for the removal of polarization degeneracy. $\mathbf{A}_\perp \sigma_3$, or equivalently its eigenvalue $\mathbf{A}_\perp \sigma$, is the (Abelian) gauge potential that emerges in the momentum space in the adiabatic approximation. The corresponding field strength (Berry curvature) and the physical coordinates are, thus, given by $\nabla_\mathbf{p} \times \mathbf{A}_\perp \sigma = -p^{-3}\mathbf{p}\sigma$ and $\mathbf{r} = \mathbf{x} - k_0^{-1}\mathbf{A}_\perp \sigma$ respectively. Note that because of the non-vanishing gauge field strength, which is the field of a magnetic monopole of charge $-\sigma$ situated at the origin of momentum space, the physical coordinates, now, do not commute:

$$[r_i, r_j] = i\sigma k_0^{-2}\varepsilon_{ijk}\frac{p_k}{p^3}.$$

4. Berry effects in the dynamics of polarized rays. Having constructed the Hamiltonian, the equations of motion of an electromagnetic ray can be obtained via the Hamilton's equations [26]. In the post geometric approximation, the semi-classical equations of motion of a circularly polarized ray are, therefore,

$$\dot{\mathbf{p}} = -\nabla_\mathbf{x}\mathcal{H}_\sigma, \quad \dot{\mathbf{x}} = \nabla_\mathbf{p}\mathcal{H}_\sigma,$$

where dot denotes derivative with respect to the ray parameter s, defined in terms of the ray length, l, by $dl = nds$. The physical coordinate $\mathbf{r} = \mathbf{x} - k_0^{-1}\mathbf{A}_\perp\sigma$ and the momentum $\mathbf{p} = k_0^{-1}\mathbf{k}$ are now considered classical, of course. Along the trajectory, they represent the ray's position and (dimensionless) wave vector, respectively. Thus, using (6),

$$\dot{\mathbf{p}} = \frac{1}{2}\nabla_\mathbf{r} n^2, \quad \dot{\mathbf{r}} = \mathbf{p} + \sigma k_0^{-1}\frac{\mathbf{p}\times\dot{\mathbf{p}}}{p^3}. \tag{7}$$

These, of course, reduce to the standard ray equations of geometric optics in the zeroth ("classical") approximation, $k_0^{-1} \to 0$, where the left/right circularly polarized rays follow the same trajectory. However, in the post-geometric (semi-classical) approximation, as seen from (7), the rays split due to the effect of Berry curvature of the momentum space (the magnetic monopole-like gauge field strength). The deflections from their classical (geometric optic) trajectories are given by

$$\delta\mathbf{r} = \sigma k_0^{-1}\int_C \frac{\mathbf{p}\times d\mathbf{p}}{p^3},$$

where C is the ray trajectory in momentum space. The resulting displacements are, therefore, opposite and locally orthogonal to the direction of propagation. This, which is a general feature of spin transport, constitutes the optical Hall effect in inhomogeneous plasmas.

The phase change suffered by the ray in the course of its propagation is given by

$$\phi = \omega t - \int \mathbf{k}\cdot d\mathbf{x} = \omega t - k_0\int \mathbf{p}\cdot d\mathbf{r} + \sigma\int_C \mathbf{A}_\perp\cdot d\mathbf{p},$$

where in the last integral, we have used the fact that $\mathbf{A}_\perp \cdot \mathbf{p} = 0$. This integral represents the geometric Berry phase, which is of opposite signs for the two polarizations. Therefore, the polarization plane of a plane polarized ray rotates through the angle

$$\int_C \mathbf{A}_\perp\cdot d\mathbf{p} = \int_C \cos\theta\, d\varphi, \tag{8}$$

where $\theta(\varphi)$ is the zenith (azimuth) angle in the spherical polar coordinates of the momentum space. This is the Rytov–Vladimirskii rotation.

5. Summary and conclusion. It was shown that if we consider the post geometric approximation in unmagnetized plasma media with stationary slowly varying density, an Abelian gauge field (Berry connection) will result in the ray Hamiltonian. Appearance of this gauge field leads to an additional displacement of the photon of distinct helicity in opposite directions normal to the ray. Also, for a linear polarization, the rotation of the polarization plane occurs in inhomogeneous unmagnetized plasmas. This rotation differs from the faraday rotation which is due to the presence of magnetic field in plasma media. In fact, it is shown theoretically that the rotation of polarization plane occurs even in unmagnetized plasma media as a result of Berry topological phase.

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Appendix. Components of gauge connection. Direct calculation via (3) yields the following expression for gauge connection components:

$$iA_1 = \begin{pmatrix} 0 & -\frac{p_y p_z}{p(p_x^2+p_y^2)} & \frac{p_y}{p\sqrt{p_x^2+p_y^2}} \\ \frac{p_y p_z}{p(p_x^2+p_y^2)} & 0 & -\frac{p_x p_z}{p^2\sqrt{p_x^2+p_y^2}} \\ -\frac{p_y}{p\sqrt{p_x^2+p_y^2}} & \frac{p_x p_z}{p^2\sqrt{p_x^2+p_y^2}} & 0 \end{pmatrix},$$

$$iA_2 = \begin{pmatrix} 0 & \frac{p_x p_z}{p(p_x^2+p_y^2)} & -\frac{p_x}{p\sqrt{p_x^2+p_y^2}} \\ -\frac{p_x p_z}{p(p_x^2+p_y^2)} & 0 & -\frac{p_y p_z}{p^2\sqrt{p_x^2+p_y^2}} \\ \frac{p_x}{p\sqrt{p_x^2+p_y^2}} & \frac{p_y p_z}{p^2\sqrt{p_x^2+p_y^2}} & 0 \end{pmatrix},$$

$$iA_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{p_x^2+p_y^2}}{p^2} \\ 0 & -\frac{\sqrt{p_x^2+p_y^2}}{p^2} & 0 \end{pmatrix}.$$

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