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CPT, Lorentz invariance, mass differences, and charge non-conservation

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A non-local field theory which breaks discrete symmetries, including C, P, CP, and CPT, but preserves Lorentz symmetry, is presented. We demonstrate that at one-loop level the masses for particle and antiparticle remain equal due to Lorentz symmetry only. An inequality of masses implies breaking of the Lorentz invariance and non-conservation of the usually conserved charges.

1. Introduction. The interplay of Lorentz symmetry and CPT symmetry was considered in the literature for decades. The issue attracted an additional interest recently due to a CPT-violating scenario in neutrino physics with different mass spectrum of neutrinos and antineutrinos [1]. Theoretical frameworks of CPT breaking in quantum field theories, in fact in string theories, and detailed phenomenology of oscillating neutrinos with different masses of ν and $\bar{\nu}$ was further studied in papers [2].

On the other hand, it was argued in ref. [3] that violation of CPT automatically leads to violation of the Lorentz symmetry [3]. This might allow for some more freedom in phenomenology of neutrino oscillations.

Very recently this conclusion was revisited in our paper [4]. We demonstrated that field theories with different masses for particle and antiparticle are extremely pathological ones and can't be treated as healthy quantum field theories. Instead we constructed a class of slightly non-local Lorentz invariant field theories with the explicit breakdown of CPT symmetry and with the same masses for particle and antiparticle.

An example of such theory is a non-local QED with the Lagrangian $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{n.l}$, where $\mathcal{L}_0$ is the usual QED Lagrangian:

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}^2(x) + \bar{\psi}(x)[i\hat{\partial} - e\hat{A}(x) - m]\psi(x), \tag{1}$$

and $\mathcal{L}_{n.l}$ is a small non-local addition:

$$\mathcal{L}_{n.l}(x) = g\int dy\bar{\psi}(x)\gamma_\mu\psi(x)A_\mu(y)K(x-y). \tag{2}$$

Here $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$ is the electromagnetic field strength tensor, $A_\mu(x)$ is the four-potential, and $\psi(x)$ is the Dirac field for electrons.

Non-local form-factor $K(x-y)$ is chosen in such a way that it explicitly breaks T-invariance, e.g.

$$K(x-y) = \theta(x_0 - y_0)\theta[(x-y)^2]e^{-(x-y)^2/l^2}, \tag{3}$$

where l is a scale of the non-locality and the Heaviside functions $\theta(x_0 - y_0)\theta[(x-y)^2]$ are equal to the unity for the future light-cone and are identically zero for the past light-cone.

Non-local interaction, eq. (2), breaks T-invariance, preserves C- and P-invariance and, as a result, breaks CPT-invariance. This construction demonstrates that CPT-symmetry can be broken in Lorentz-invariant non-local field theory! The masses of an electron, m, and of a positron, $\tilde{m}$, remain identical to each other in this theory despite breaking of CPT-symmetry. The evident reason is that the interaction $\mathcal{L}_{n.l}(x)$ is C-invariant and its exact C-symmetry preserves the identity of masses and anti-masses.

In this note we would like to study further the relation between mass difference for a particle and an antiparticle and CPT-symmetry. We start from the standard local free field theory of electrons with the usual dispersion relation between energy and momentum:

$$p_\mu^2 = p_0^2 - \mathbf{p}^2 = m^2 = \tilde{m}^2 \tag{4}$$

and introduce a non-local interaction that breaks the whole set of discrete symmetries, i.e. C, P, CP, T, and CPT. So there is no discrete symmetry which preserves equality of m to $\tilde{m}$ in this case. Hence in principle the interaction can shift m from $\tilde{m}$. But an explicit one-loop calculation demonstrates that this is not true. So we conclude that it is Lorentz-symmetry that keeps the identity

$$m = \tilde{m}. \qquad (5)$$

This conclusion invalidates the experimental evidence for CPT-symmetry based on the equality of masses of particles and antiparticles. CPT may be strongly broken in a Lorentz invariant way and in such a case the masses must be equal. Another way around, if we assume that the masses are different, then Lorenz invariance must be broken. Lorentz and CPT violating theories would lead not only to mass difference of particles and antiparticles but to much more striking phenomena such as violation of gauge invariance, current non-conservation, and even to a breaking of the usual equilibrium statistics (for the latter see ref. [5]).

2. C, CP, and CPT violating QFT. To formulate a model we start with the standard QED Lagrangian:

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}(x)F_{\mu\nu}(x) + \bar{\psi}(x)[i\hat{\partial} - e\hat{A}(x) - m]\psi(x), \qquad (6)$$

and add the interaction of a photon, A_μ, with an axial current

$$\mathcal{L}_1 = g_1\bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)A_\mu(x) \qquad (7)$$

and with the electric dipole moment of an electron

$$\mathcal{L}_2 = g_2\bar{\psi}(x)\sigma_{\mu\nu}\gamma_5\psi(x)F_{\mu\nu}(x). \qquad (8)$$

The first interaction, $\mathcal{L}_1$, breaks C- and P-symmetry and conserves CP-symmetry. The second interaction breaks P- and CP-symmetry. Still the sum of Lagrangians

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 \qquad (9)$$

preserves CPT-symmetry. To break the CPT we modify the interaction $\mathcal{L}_1$ to a non-local one $\tilde{\mathcal{L}}_1$:

$$\mathcal{L}_1 \to \tilde{\mathcal{L}}_1(x) = \int dy g_1\bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)K(x-y)A_\mu(y). \qquad (10)$$

With this modification the model

$$\mathcal{L} = \mathcal{L}_0 + \bar{\mathcal{L}}_1 + \mathcal{L}_2 \qquad (11)$$

breaks all discrete symmetries.

3. One-loop calculation. In general to calculate high order perturbative contributions of a non-local interaction into S-matrix one has to modify the Dyson formulae for S-matrix with T-ordered exponential

$$S = T\left[\exp\left(i\int d^4x\mathcal{L}_{\text{int}}\right)\right] \qquad (12)$$

and the whole Feynman diagram techniques.

But in the first order in the non-local interaction one can work with the usual Feynman rules in the coordinate space. The only difference is that one of the vertices becomes non-local.

4. Mass and wave function renormalization for particle and antiparticle. We start with the standard free field theory for an electron, i.e.

$$\mathcal{L} = \bar{\psi}[i\hat{\partial} - m]\psi \qquad (13)$$

that fixes the usual dispersion law

$$p^2 = p_0^2 - \mathbf{p}^2 = m^2. \qquad (14)$$

The self-energy operator, $\Sigma(p)$, contributes both to the mass renormalization and to the wave function renormalization. In general one-loop effective Lagrangin can be written in the form:

$$\mathcal{L}_{\text{eff}}^{(1)} = \bar{\psi}[i(A\gamma_\mu + B\gamma_\mu\gamma_5)\partial_\mu - (m_1 + im_2\gamma_5)]\psi. \qquad (15)$$

It is useful to rewrite the same one-loop effective Lagrangian in terms of the field for antiparticle ψ_c:

$$\psi_c = (-i)[\bar{\psi}\gamma^0\gamma^2]^{\text{T}}, \qquad (16)$$

$$\mathcal{L}_{\text{eff}}^{(1)} = \bar{\psi}_c[i(A\gamma_5 - B\gamma_\mu\gamma_5)\partial_\mu - (m_1 + im_2\gamma_5)]\psi_c. \qquad (17)$$

We see that the mass term is the same for ψ and for ψ_c, but the wave function renormalization is different: the coefficient in front of the pseudovector changes its sign. This change is unobservable since one can remove $B\gamma_\mu\gamma_5$ and $im_2\gamma_5$ terms by redefining of variables. Indeed

$$\bar{\psi}(A + B\gamma_5)\gamma_\mu\psi \equiv \bar{\psi}'\sqrt{A^2 + B^2}\gamma_\mu\psi', \qquad (18)$$

where

$$\psi = (\cosh\alpha + i\gamma_5\,\sinh\alpha)\psi', \qquad (19)$$

$$\tanh 2\alpha = B/A, \qquad (20)$$

and

$$\bar{\psi}(m_1 + i\gamma_5 m_2)\psi \equiv \sqrt{m_1^2 + m_2^2}\bar{\psi}'\psi', \qquad (21)$$

where

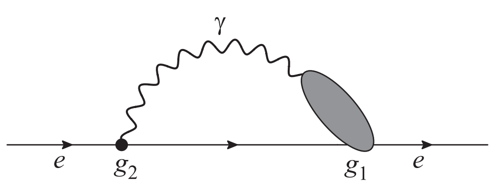
$$\psi = \exp(i\gamma_5\beta)\psi', \qquad (22)$$

$$\tan 2\beta = m_2/m_1. \qquad (23)$$

This simple observation is sufficient to conclude that technically there is no possibility to write one-loop corrections that produce different contributions for particle and antiparticle. Still it is instructive to check directly that the difference is zero.

5. Explicit one-loop calculation. We are looking for a one-loop contribution into self-energy operator $\Sigma(p)$ that breaks C-, CP-, and CPT-symmetries and that changes the chirality of the fermion line. It is clear that this contribution potentially can be different (opposite in sign) for particle ψ and antiparticle ψ_c.

To construct such contribution we need both anomalous interactions $\tilde{\mathcal{L}}_1$ and $\mathcal{L}_2$. Indeed interaction $\mathcal{L}_2$ changes chirality and breaks CP symmetry, while non-local interaction $\tilde{\mathcal{L}}_1$ breaks C and CPT and leaves the chirality unchanged. In combination they break all discrete symmetries and change chirality. There are two diagrams that are proportional to $g_1 g_2$ (see Figure).



The diagram contributing to the mass difference of electron and positron. The blob represents a non-local form-factor

We will calculate these diagrams in two steps. The first step is a pure algebraic one. Self-energy $\Sigma(p)$ is 4×4 matrix that was constructed from a product of three other 4×4 matrices, i.e. two vertices and one fermion propagator. Notice that any 4×4 matrix can be decomposed as a sum over complete set of 16 Dirac matrices. In this decomposition of $\Sigma(p)$ we need terms that are odd in C and changes chirality. Fortunately there is only one Dirac matrix with these properties. That is $\sigma_{\mu\nu}$. So

$$\Sigma(p) = \sigma_{\mu\nu} I_{\mu\nu}(p), \tag{24}$$

where $I_{\mu\nu}$ represents Feynman (divergent) integral. We could obtain eq. (24) after some long explicit algebraic transformation, but the net result is determined by the symmetry only.

The second step is the calculation of Feynman integrals. Again fortunately we do not need actual calculations. Indeed due to the Lorentz symmetry of the theory this $I_{\mu\nu}$ should be a tensor that depends only on the momentum of fermion line p. The general form for $I_{\mu\nu}$ is

$$I_{\mu\nu} = A g_{\mu\nu} + B p_\mu p_\nu. \tag{25}$$

As a result we get

$$\Sigma(p) = \sigma_{\mu\nu} I_{\mu\nu} \equiv 0 \tag{26}$$

and we conclude that the one-loop contribution into possible mass difference is identically zero[1)].

6. CPT and charge non-conservation. There is widely spread habit to parametrize CPT violation by attributing different masses to particle and antiparticle. This tradition is traced to an old time of the first observation of $K - \bar{K}$-mesons oscillation.

For K-mesons with a given momenta $\mathbf{q}$ the theory of oscillation is equivalent to a non-hermitian Quantum Mechanics (QM) with two degrees of freedom. Diagonal elements of 2×2 Hamiltonian matrix represent masses for particle and antiparticle. Their unequality breaks CPT-symmetry. Experimental bounds on mass difference are considered as bounds on the CPT-symmetry violation parameters. Such strategy has no explicit loop-holes and is still used for parametrization of CPT-symmetry violation in D and B meson oscillations.

Quantum Field Theory (QFT) deals not with one mode for a given momenta but rather with an infinite sum over all momenta. The set of plane waves with all possible momenta for particle and antiparticle is a complete set of orthogonal modes and an arbitrary field operator can be decomposed over this set.

Naive generalization of CPT-conserving QFT to CPT-violating QFT was to attribute different masses for particle and antiparticle [1, 2]). Say for a complex scalar field they use the infinite sum [1, 2]

$$\phi(x) = \sum_{\mathbf{q}} \left[a(\mathbf{q}) \frac{1}{\sqrt{2E}} e^{-i(Et-\mathbf{qx})} + b^{+}(\mathbf{q}) \frac{1}{\sqrt{2\tilde{E}}} e^{i(\tilde{E}t-\mathbf{qx})} \right], \tag{27}$$

where $(a(\mathbf{q}), a^{+}(\mathbf{q}))$, $(b(\mathbf{q}), b^{+}(\mathbf{q}))$ are annihilation and creation operators, and (m, E) and $(\tilde{m}, \tilde{E})$ are masses and energies of particle and antiparticle respectively.

Greenberg [3] found that this construction runs into trouble. The dynamic of fields determined according to eq. (27) cannot be a Lorentz-invariant one.

We'd like to notice that for charged particles (say for electrons and positrons) similar generalization of the field theory breaks not only the Lorentz symmetry but the electric charge conservation as well. The reason is

1) Recently our former collaborators published a paper where they demonstrated that for a particle with a non-standard dispersion law the quantity which they define as mass can be different for particle and antiparticle [6].

very simple. For the standard QED the operator of electric charge $\hat{Q}(t)$ can be written in the form

$$\hat{Q}(t) = \sum_{\mathbf{q}} \left[a^{+}(\mathbf{q})a(\mathbf{q}) - b^{+}(\mathbf{q})b(\mathbf{q})\right]. \qquad (28)$$

Operator $\hat{Q}(t)$ is a diagonal one, i.e. there are no mixed terms with different momenta. The modes with different momenta are orthogonal to each other and disappear after integration over space. This is a technical explanation why one can construct a time-independent operator.

If one shifts the mass of electron from the mass of positron the situation drastically changes. For electron the modes with different momenta are still orthogonal to each other. The same is true for the modes of positron, they are also orthogonal among themselves. But there is no reason for wave function of electron with mass m be orthogonal to wave functions for positron with mass $\tilde{m}$. As a result one obtaines

$$Q(t) = \sum_{\mathbf{q}} \left[a^{+}(\mathbf{q})a(\mathbf{q}) - b^{+}(\mathbf{q})b(\mathbf{q})\right] + \\ + C\sum_{\mathbf{q}} \frac{(E-\tilde{E})}{\sqrt{4E\tilde{E}}}\left[b(\mathbf{q})a(-\mathbf{q})e^{-i(E+\tilde{E})t} + \text{h.c.}\right], \qquad (29)$$

where constant C depends on the sorts of particles and on the definition of the charge.

We can conclude from this equation that non-conservation of charge exhibits itself only in annihilation processes but not in the scattering processes. So there is no immediate problem with the Coulomb law. Nevertheless non-conservation of this type is also absolutely excluded by the experiment. In a case of charge-nonconservation annihilation of particle and antiparticle with a creation of the infinite number of soft massless photons creates a terrible infrared problem. Infrared catastrophe can not be avoided by usual summation over infrared photons. On the other hand, as is argued in ref. [7], the electron decay might be exponentially suppressed due to vanishing of the corresponding form-factor created by virtual longitudinal photons.

Similar arguments lead to the conclusion that conservation of energy cannot survive as well in a theory with different masses of particles and antiparticles.

7. Conclusion. We have shown that in the framework of a Lorentz invariant field theory it is impossible to have different masses of particles and antiparticles, even if CPT (together with C and P) invariance is broken. On the other hand, unequal masses of particles and antiparticles imply breaking of the Lorentz invariance. Moreover, in such theories charge and energy conservation seem to be broken as well.

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